\( \tilde{g}_\alpha \)-WEAKLY GENERALIZED CONTINUOUS FUNCTIONS

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ABSTRACT

In this paper we introduce and study of \( \tilde{g}_\alpha \)- weakly generalized continuous functions and \( \tilde{g}_\alpha \)- weakly generalized irresolute functions also obtain some properties of such functions.

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1. INTRODUCTION:

S. Jafari, M. llelis Thivagar and N. Rebecca Paul [19] introduced and studied \( g_\alpha \)-closed sets. M. Maria Singam, G. Anitha [13] introduced the class \( g_\alpha \)-Weakly generalized closed sets. By using such sets we introduce new forms of functions called \( \tilde{g}_\alpha \)-Weakly generalized continuous functions and \( \tilde{g}_\alpha \)-Weakly generalized irresolute functions. We obtain properties of such functions.

2. PRELIMINARIES:

Throughout this paper (\( X, \tau \)) , (\( Y, \sigma \)) and (\( Z, \eta \)) represent non empty topological space on which no separation axiom is defined unless otherwise mentioned. For a subset \( A \) of a space \( \text{Cl}(A) \) and \( \text{Int}(A) \) denote the closure and interior of \( A \) respectively.

**Definition 1.1:** A subset \( A \) of a space \( X \) is called

1. a semi-open set [10] if \( A \subseteq \text{cl}(\text{int}(A)) \)
2. a pre-open set [15] if \( A \subseteq \text{int}(\text{cl}(A)) \)
3. an \( \alpha \)-open set [17] if \( A \subseteq \text{int}(\text{cl}(\text{int}(A))) \)
4. a regular open[20] if \( A = \text{int}(\text{cl}(A)) \)
5. a semi-preopen set [1] if \( A \subseteq \text{cl}(\text{int}(\text{cl}(A))) \)

The complement of a semi-open (pre open, \( \alpha \)-open, regular open, semi-preopen) set is called a semi-closed (resp. pre-closed, \( \alpha \)-closed, regular closed, semi-preclosed) set.

**Definition 1.2:** A subset \( A \) of a space \( X \) is called

1. a generalized closed set(g-closed)[9] if \( \text{cl}(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is open in \( (X, \tau) \).
2. a weakly generalized closed set(wg-closed)[16] if \( \text{Cl}(\text{Int}(A)) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is open in \( (X, \tau) \).
3. a semi generalized closed set(sg-closed)[4] if \( \text{scl}(A) \subseteq U \), whenever \( A \subseteq U \) and \( U \) is semi open in \( (X, \tau) \).
4. a semi-pre generalized closed set(spcl)([4] if \( \text{spcl}(A) \subseteq U \), whenever \( A \subseteq U \) and \( U \) is w-open in \( (X, \tau) \).
5. a generalized \( \alpha \)-closed set (g\( \alpha \)-closed) if \( \alpha \text{cl}(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is \( \alpha \)-open in \( (X, \tau) \).
6. a generalized \( \alpha \)-closed set (g\( \alpha \)-closed) if \( \alpha \text{cl}(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is \( \alpha \)-open in \( (X, \tau) \).
7. an \( \alpha \)-generalized closed set (\( \alpha \)-g-closed) [12] if \( \alpha \text{cl}(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is open in \( (X, \tau) \).
8. a \( \# \) g-closed set[22] if \( \text{cl}(A) \subseteq U \), whenever \( A \subseteq U \) and \( U \) is w-open in \( (X, \tau) \).
9. a \( \# \) g-semi closed set[23] if \( \text{scl}(A) \subseteq U \), whenever \( A \subseteq U \) and \( U \) is \( \# \)-g-open in \( (X, \tau) \).

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10. a \( \tilde{a}_g \) -closed\[19\] if \( \alpha \text{cl}(A) \subseteq U \), whenever \( A \subseteq U \) and \( U \) is \# gs-open in \((X, \tau)\).

11. a \( \tilde{a}_g \) -Weakly generalized closed set(\( \tilde{a}_g \) wg-closed) \[13\] if \( \text{Cl}(\text{Int}(A)) \subseteq U \), whenever \( A \subseteq U \), \( U \) is \( \tilde{a}_g \) -open in \((X, \tau)\).

The complements of the above sets are called their respective open sets.

**Definition 1.3:** A function \( f : (X, \tau) \rightarrow (Y, \sigma) \) is called

1. \( \alpha \) -continuous \[14\] if \( f^{-1}(V) \) is \( \alpha \) -closed in \((X, \tau)\) for every closed set \( V \) in \((Y, \sigma)\).
2. semi continuous \[10\] if \( f^{-1}(V) \) is semi closed in \((X, \tau)\) for every closed set \( V \) in \((Y, \sigma)\).
3. \( g \) \( \alpha \) -continuous \[3\] if \( f^{-1}(V) \) is \( g \) \( \alpha \) -closed in \((X, \tau)\) for every closed set \( V \) in \((Y, \sigma)\).
4. sg-continuous \[21\] if \( f^{-1}(V) \) is sg-closed in \((X, \tau)\) for every closed set \( V \) in \((Y, \sigma)\).
5. \( \alpha \) g-continuous \[5\] if \( f^{-1}(V) \) is \( \alpha \) g-closed in \((X, \tau)\) for every closed set \( V \) in \((Y, \sigma)\).
6. \( g \alpha \) -continuous \[5\] if \( f^{-1}(V) \) is \( g \alpha \) -closed in \((X, \tau)\) for every closed set \( V \) in \((Y, \sigma)\).
7. gs-continuous \[6\] if \( f^{-1}(V) \) is gs-closed in \((X, \tau)\) for every closed set \( V \) in \((Y, \sigma)\).
8. gsp-continuous \[7\] if \( f^{-1}(V) \) is gsp-closed in \((X, \tau)\) for every closed set \( V \) in \((Y, \sigma)\).
9. completely-continuous \[2\] if \( f^{-1}(V) \) is regular closed in \((X, \tau)\) for every closed set \( V \) in \((Y, \sigma)\).
10. \( \alpha \) g-continuous \[8\] if \( f^{-1}(V) \) is \( \alpha \) g-closed in \((X, \tau)\) for every closed set \( V \) in \((Y, \sigma)\).
11. \( g \alpha \) -irresolute \[8\] if \( f^{-1}(V) \) is \( g \alpha \) -closed in \((X, \tau)\) for every \( g \alpha \) -closed set \( V \) in \((Y, \sigma)\).

**Proposition 1.4:** If a subset \( A \) of a topological space \((X, \tau)\) is a regular closed, then it is \( \tilde{a}_g \) wg-closed but not conversely.

**Proof:** Suppose a subset \( A \) of a topological space \( X \) is regular closed. Let \( G \) be a \( \tilde{a}_g \) -open set containing \( A \). Then \( G \supseteq \text{cl}(A) = \text{cl}(\text{int}(A)) \) since \( A \) is regular closed. Hence \( A \) is \( \tilde{a}_g \) wg-closed in \((X, \tau)\).

Converse of the above theorem need not be true as seen in the following example.

**Example 1.5:** Let \( X = \{a, b, c\} \) and \( \tau = \{\phi, \{a\}, \{b, c\}, X\} \). In this topological space the subset \( \{b\} \) is \( \tilde{a}_g \) wg-closed but it is not regular closed.

**Proposition 1.6:** If a subset \( A \) of a topological space \((X, \tau)\) is a \( g \alpha \) - closed, then it is \( \tilde{a}_g \) wg-closed but not conversely.

**Proof:** Suppose \( A \) is \( g \alpha \) - closed subset \( X \) and let \( G \) be a \( \alpha \) -open set containing \( A \). Since every \( \alpha \) -open set is \( g \alpha \) - open. Hence \( G \) is \( \tilde{a}_g \) -open set containing \( A \).

\[ G \supseteq \text{cl}(\text{Int}(\text{cl}(A))) \supseteq \text{cl}(\text{int}(A)) \text{Thus } A \text{ is } \tilde{a}_g \text{-wg-closed in } (X, \tau). \]

Converse of the above theorem need not be true as seen in the following example.

**Example 1.7:** Let \( X = \{a, b, c\} \) and \( \tau = \{\phi, \{a, c\}, X\} \). In this topological space the subset \( \{a\} \) is \( \tilde{a}_g \) wg-closed but it is not \( g \alpha \) - closed.

**Proposition 1.8:** If a subset \( A \) of a topological space \((X, \tau)\) is a \( \alpha \) g-closed, then it is gsp-closed but not conversely.

**Proof:** Let \( A \) be a \( \tilde{a}_g \) wg-closed subset \( X \) and \( G \) be an open set containing \( A \) in \((X, \tau)\). Then \( G \supseteq \text{cl}(A) \supseteq \text{cl}(\text{int}(A)) \). Since every open set is \( \tilde{a}_g \) -open. Hence \( G \) is \( \tilde{a}_g \) -open set containing \( A \). \( G \supseteq (\text{int}(\text{cl}(A))) \) which implies \( A \cup G \supseteq A \cup \text{int}(\text{cl}(A)) \). That is \( G \supseteq \text{spcl}(A) \). Thus \( A \) is gsp-closed in \((X, \tau)\).

Converse of the above theorem need not be true as seen in the following example.
Example 1.9: Let \( X = \{a, b, c\} \) and \( \tau = \{\emptyset, \{a\}, \{c\}, \{a, c\}, X\} \). In this topological space the subset \( \{a\} \) is gsp closed but not \( \tilde{g}_\alpha \)-wg-closed.

2. \( \tilde{g}_\alpha \)-CONTINUOUS FUNCTIONS:

We have introduced the following definition

**Definition 2.1:** A function \( f : (X, \tau) \rightarrow (Y, \sigma) \) is said to be \( \tilde{g}_\alpha \)-continuous if \( f^{-1}(V) \) is \( \tilde{g}_\alpha \)-closed in \( (X, \tau) \) for every closed set \( V \) of \( (Y, \sigma) \).

Example 2.2: Let \( X = \{a, b, c\} = Y, \tau = \{\emptyset, \{a\}, \{b, c\}, X\} \) and \( \sigma = \{\emptyset, \{a\}, Y\} \). Define a function \( f : (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = b, f(b) = c, f(c) = a \). Then \( f \) is \( \tilde{g}_\alpha \)-continuous since the inverse image of the closed set \( \{b, c\} \) in \( (Y, \sigma) \) is \( \{a, b\} \) which is in \( \tilde{g}_\alpha \)-wg-closed in \( (X, \tau) \).

**Theorem 2.3:** Every continuous map is \( \tilde{g}_\alpha \)-continuous but not conversely.

**Proof:** Let \( V \) be a closed set in \( (Y, \sigma) \). Since \( f \) is continuous, then \( f^{-1}(V) \) is closed in \( (X, \tau) \). By theorem 3.2 of [13], every closed set is \( \tilde{g}_\alpha \)-closed. Then \( f^{-1}(V) \) is \( \tilde{g}_\alpha \)-closed in \( (X, \tau) \). Hence \( f \) is \( \tilde{g}_\alpha \)-continuous.

Example 2.4: Let \( X = \{a, b, c\} = Y, \tau = \{\emptyset, \{a, b\}, X\} \) and \( \sigma = \{\emptyset, \{a\}, Y\} \). Define a function \( f : (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = b, f(b) = a, f(c) = c \). Then \( f \) is \( \tilde{g}_\alpha \)-continuous but not \( \alpha \)-continuous.

**Theorem 2.7:** Every \( \alpha \)-continuous function is \( \tilde{g}_\alpha \)-continuous but not conversely.

**Proof:** Let \( V \) be a closed set in \( (Y, \sigma) \). Since \( f \) is \( \alpha \)-continuous, then \( f^{-1}(V) \) is \( \alpha \)-closed in \( (X, \tau) \). By theorem 3.11 of [13], every \( \alpha \)-closed set is \( \tilde{g}_\alpha \)-closed. Then \( f^{-1}(V) \) is \( \tilde{g}_\alpha \)-closed in \( (X, \tau) \). Hence \( f \) is \( \tilde{g}_\alpha \)-continuous.
Theorem 2.11: Every completely continuous function is $\tilde{g}_a$ mg-continuous but not conversely.

**Proof:** Let $V$ be a closed set in $(Y, \sigma)$. Since $f$ is completely continuous function, then $f^{-1}(V)$ is regular closed in $(X, \tau)$. By Proposition 1.4, every regular closed set is $\tilde{g}_a$ mg-closed. Then $f^{-1}(V)$ is $\tilde{g}_a$ mg-closed in $(X, \tau)$. Hence $f$ is $\tilde{g}_a$ mg-continuous.

Example 2.12: Let $X = \{a, b, c\} = Y$, $\tau = \{\phi, \{a\}, \{b, c\}, X\}$ and $\sigma = \{\phi, \{a\}, Y\}$. Define a function $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = b$, $f(b) = a$, $f(c) = c$. Then $f$ is $\tilde{g}_a$ mg-continuous but not regular continuous function.

Theorem 2.13: Every $\tilde{g}_a$ mg-continuous is gsp-continuous but not conversely.

**Proof:** Let $V$ be a closed set in $(Y, \sigma)$. Since $f$ is $\tilde{g}_a$ mg-continuous function, then $f^{-1}(V)$ is $\tilde{g}_a$ mg-closed in $(X, \tau)$. By Proposition 1.8, every $\tilde{g}_a$ mg-closed set is gsp closed. Then $f^{-1}(V)$ is gsp closed in $(X, \tau)$. Hence $f$ is gsp continuous.

Example 2.14: Let $X = \{a, b, c\} = Y$, $\tau = \{\phi, \{a\}, \{b, c\}, X\}$ and $\sigma = \{\phi, \{b, c\}, Y\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be the identity function. Then $f$ is gsp continuous but not $\tilde{g}_a$ mg-continuous.

Theorem 2.15: Every $\tilde{g}_a$ mg-continuous is wg-continuous but not conversely.

**Proof:** Let $V$ be a closed set in $(Y, \sigma)$. Since $f$ is $\tilde{g}_a$ mg-continuous function, then $f^{-1}(V)$ is $\tilde{g}_a$ mg-closed in $(X, \tau)$. By Theorem 3.9 of [13], every $\tilde{g}_a$ mg-closed set is wg closed. Then $f^{-1}(V)$ is wg closed in $(X, \tau)$. Hence $f$ is wg continuous.

Example 2.16: Let $X = \{a, b, c\} = Y$, $\tau = \{\phi, \{a\}, \{b, c\}, X\}$ and $\sigma = \{\phi, \{b, c\}, Y\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be the identity function. Then $f$ is wg continuous but not $\tilde{g}_a$ mg-continuous.

Remark 2.17: The following examples show that semi continuous and $\tilde{g}_a$ mg-continuous functions are independent.

Example 2.18: Let $X = \{a, b, c\} = Y$, $\tau = \{\phi, \{a\}, \{b, c\}, X\}$ and $\sigma = \{\phi, \{a\}, Y\}$ defined $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = c$, $f(b) = a$, $f(c) = b$. Then $f$ is $\tilde{g}_a$ mg-continuous but not semi continuous.

Example 2.19: Let $X = \{a, b, c\} = Y$, $\tau = \{\phi, \{a\}, \{b, c\}, X\}$ and $\sigma = \{\phi, \{b, c\}, Y\}$ defined $f : (X, \tau) \rightarrow (Y, \sigma)$ be the identity function. Then $f$ is semi continuous but not $\tilde{g}_a$ mg-continuous.

Remark 2.20: The following examples show that g-continuous and $\tilde{g}_a$ mg-continuous functions are independent.

Example 2.21: Let $X = \{a, b, c\} = Y$, $\tau = \{\phi, \{a\}, X\}$ and $\sigma = \{\phi, \{a, b\}, Y\}$ defined $f : (X, \tau) \rightarrow (Y, \sigma)$ be the identity function. Then $f$ is $\tilde{g}_a$ mg-continuous but not g-continuous.

Example 2.22: Let $X = \{a, b, c\} = Y$, $\tau = \{\phi, \{b, c\}, \{c\}, X\}$ and $\sigma = \{\phi, \{b\}, Y\}$ defined $f : (X, \tau) \rightarrow (Y, \sigma)$ be the identity function. Then $f$ is g-continuous but not $\tilde{g}_a$ mg-continuous.

Remark 2.23: The following examples show that sg-continuous and $\tilde{g}_a$ mg-continuous functions are independent.

Example 2.24: Let $X = \{a, b, c\} = Y$, $\tau = \{\phi, \{a, c\}, X\}$ and $\sigma = \{\phi, \{a\}, \{b, c\}, Y\}$ defined $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = c$, $f(b) = b$, $f(c) = a$. Then $f$ is $\tilde{g}_a$ mg-continuous but not sg-continuous.
Example 2.25: Let \( X = \{a, b, c\} = Y, \quad \tau = \{\phi, [a], [a, c]\}, X \) and \( \sigma = \{\phi, [a, b]\}, Y \) defined \( f: (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = b, f(b) = a, f(c) = c \). Then \( f \) is \( \sigma \)-continuous but not \( \bar{g}_a \)-\( \sigma \)-continuous functions are independent.

Remark 2.26: The following examples show that \( \sigma \)-continuous and \( \bar{g}_a \)-\( \sigma \)-continuous functions are independent.

Example 2.27: Let \( X = \{a, b, c\} = Y, \quad \tau = \{\phi, [a], [a, b]\}, X \) and \( \sigma = \{\phi, [a, b]\}, Y \) define \( f: (X, \tau) \rightarrow (Y, \sigma) \) be the identity function. Then \( f \) is \( \sigma \)-continuous function but not \( \bar{g}_a \)-\( \sigma \)-continuous function;

Example 2.28: Let \( X = \{a, b, c, d\} = Y, \quad \tau = \{\phi, [b, c], [a, b, c]\}, X \) and \( \sigma = \{\phi, [a, c, d]\}, Y \) define \( f: (X, \tau) \rightarrow (Y, \sigma) \) be the identity function. Then \( f \) is \( \bar{g}_a \)-\( \sigma \)-continuous but not \( \sigma \)-continuous.

Remark 2.29: The following examples show that \( \sigma \)-continuous and \( \bar{g}_a \)-\( \sigma \)-continuous functions are independent.

Example 2.30: Let \( X = \{a, b, c\} = Y, \quad \tau = \{\phi, [a], [a, c]\}, X \) and \( \sigma = \{\phi, [a, b]\}, Y \) define \( f: (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = a, f(b) = c, f(c) = b \). Then \( f \) is \( \sigma \)-continuous but not \( \bar{g}_a \)-\( \sigma \)-continuous.

Example 2.31: Let \( X = \{a, b, c, d\} = Y, \quad \tau = \{\phi, [b, c], [b, c, d]\}, [a, b, c]\}, X \) and \( \sigma = \{\phi, [a, c, d]\}, Y \) define \( f: (X, \tau) \rightarrow (Y, \sigma) \) be the identity function. Then \( f \) is \( \bar{g}_a \)-\( \sigma \)-continuous but not \( \sigma \)-continuous.

Remark 2.32: The composition of two \( \bar{g}_a \)-\( \sigma \)-continuous map need not be \( \bar{g}_a \)-\( \sigma \)-continuous.

Example 2.33: Let \( X = Y = Z = \{a, b, c\}, \quad \tau = \{\phi, [a, b]\}, X \), \( \sigma = \{\phi, [a, b]\}, Z \) defined \( \phi: (X, \tau) \rightarrow (Y, \sigma) \) by \( \phi(a) = a, \phi(b) = b \) and Define \( \psi: (Y, \sigma) \rightarrow (Z, \eta) \) by \( \psi(a) = b, \psi(b) = a, \psi(c) = c \). Then \( \phi, \psi \) are \( \bar{g}_a \)-\( \sigma \)-continuous. But \( \phi \circ \psi: (X, \tau) \rightarrow (Z, \eta) \) is not \( \bar{g}_a \)-\( \sigma \)-continuous.

3. \( \bar{g}_a \)-\( \sigma \)-IRRESOLUTE FUNCTIONS

Definition 3.1: A function \( f: (X, \tau) \rightarrow (Y, \sigma) \) is said to be \( \bar{g}_a \)-irresolute if \( f^{-1}(V) \) is \( \bar{g}_a \)-\( \sigma \)-closed in \( (X, \tau) \) for every \( \bar{g}_a \)-\( \sigma \)-closed set \( V \) of \( (Y, \sigma) \).

Theorem 3.2: Every \( \bar{g}_a \)-\( \sigma \)-irresolute map is \( \bar{g}_a \)-\( \sigma \)-continuous.

Proof: Let \( f: (X, \tau) \rightarrow (Y, \sigma) \) be a \( \bar{g}_a \)-\( \sigma \)-irresolute map and \( V \) be a closed set of \( (Y, \sigma) \).

Since every closed set is \( \bar{g}_a \)-\( \sigma \)-closed set by theorem 3.2 of [13], \( V \) is \( \bar{g}_a \)-\( \sigma \)-closed. Since \( f \) is a \( \bar{g}_a \)-\( \sigma \)-irresolute, \( f^{-1}(V) \) is a \( \bar{g}_a \)-\( \sigma \)-closed set of \( (X, \tau) \). Hence \( f \) is \( \bar{g}_a \)-\( \sigma \)-continuous.

Remark 3.3: \( \bar{g}_a \)-\( \sigma \)-continuous map need not be \( \bar{g}_a \)-\( \sigma \)-irresolute map.

Example 3.4: Let \( X = \{a, b, c\} = Y, \quad \tau = \{\phi, [a, b], [a, c]\}, X \) and \( \sigma = \{\phi, [a, c]\}, Y \) define \( f: (X, \tau) \rightarrow (Y, \sigma) \) be the identity function. Then \( f \) is \( \bar{g}_a \)-\( \sigma \)-continuous but not \( \bar{g}_a \)-\( \sigma \)-irresolute map.

Theorem 3.5: Let \( f: (X, \tau) \rightarrow (Y, \sigma) \) be an \( \bar{g}_a \)-irresolute and closed map. Then \( f(A) \) is \( \bar{g}_a \)-\( \sigma \)-closed of \( (Y, \sigma) \) for every \( \bar{g}_a \)-\( \sigma \)-closed set \( A \) of \( (X, \tau) \).

Proof: Let \( A \) be a \( \bar{g}_a \)-\( \sigma \)-closed in \( (X, \tau) \).Let \( U \) be any \( \bar{g}_a \)-\( \sigma \)-open set of \( (Y, \sigma) \) such that \( f(A) \subseteq U \) then \( A \subseteq f^{-1}(U) \).Since \( f \) is \( \bar{g}_a \)-irresolute then \( f^{-1}(U) \) is \( \bar{g}_a \)-\( \sigma \)-open set of \( (X, \tau) \).
By hypothesis, A is $\tilde{g}_a$-wg-closed and $f^{-1}(U)$ is $\tilde{g}_a$-open set containing A.

then $\text{cl}(\text{int}(A)) \subseteq f^{-1}(U)$ which implies $f(\text{cl}(\text{int}(A))) \subseteq U$.

Now, $\text{cl}(\text{int}(f(A))) \subseteq \text{cl}(f(\text{cl}(\text{int}(A)))) \subseteq f(\text{cl}(\text{int}(A))) \subseteq U$.

Hence $\text{cl}(\text{int}(f(A))) \subseteq U$. Hence f(A) is $\tilde{g}_a$-wg-closed in $(Y, \sigma)$.

**Theorem 3.6:** If a function $f: (X, \tau) \rightarrow (Y, \sigma)$ is $\tilde{g}_a$-irresolute and $\tilde{g}_a$-wg-closed and A is a $\tilde{g}_a$-wg-closed set of $(X, \tau)$, then $A$ is $\tilde{g}_a$-wg-closed in $(Y, \sigma)$.

**Proof:** Let F be closed subset of A. Then F is $\tilde{g}_a$-wg-closed. By theorem 3.5 $f_A(F) = f(F)$ is $\tilde{g}_a$-wg-closed in $(Y, \sigma)$. Hence $f_A: A \rightarrow Y$ is $\tilde{g}_a$-wg-closed function.

**Theorem 3.7:** Let $f: (X, \tau) \rightarrow (Y, \sigma)$ and $g: (Y, \sigma) \rightarrow (z, \eta)$ be such that $g \circ f : (X, \tau) \rightarrow (z, \eta)$ is $\tilde{g}_a$-wg-closed function.

(i) If f is continuous and injective then g is $\tilde{g}_a$-wg-closed.

(ii) If g is $\tilde{g}_a$-wg-irresolute and injective then f is $\tilde{g}_a$-wg-closed.

**Proof:** Let F be closed set of $(Y, \sigma)$. Since f is continuous, $f^{-1}(F)$ is closed in X. $g \circ f(F)$ is $\tilde{g}_a$-wg-closed in $(z, \eta)$. Hence $g(F)$ is $\tilde{g}_a$-wg-closed in $(z, \eta)$. Thus g is $\tilde{g}_a$-wg-closed.

Proof of (ii) is similar to proof (i).

**Theorem 3.8:** Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a bijection function such that the image of every $\tilde{g}_a$-open in $(X, \tau)$ is $\tilde{g}_a$ open in $(Y, \sigma)$ and $\tilde{g}_a$-continuous then f is $\tilde{g}_a$-wg-irresolute.

**Proof:** Let F be a $\tilde{g}_a$-wg-closed set in $(Y, \sigma)$. Let $f^{-1}(F) \subseteq U$ where U is $\tilde{g}_a$-open set in $(X, \tau)$. Since f is $\tilde{g}_a$-continuous and $\text{cl}(\text{int}(F))$ is closed in $(Y, \sigma)$ then $f^{-1}(\text{cl}(\text{int}(F)))$ is $\tilde{g}_a$-wg closed in $(X, \tau)$. Since $f^{-1}(\text{cl}(\text{int}(F))) \subseteq U$ and $f^{-1}(\text{cl}(\text{int}(F)))$ is $\tilde{g}_a$-wg closed. We have $\text{cl}(\text{int}(f^{-1}(\text{cl}(\text{int}(F)))) \subseteq U$ and so $\text{cl}(\text{int}(f^{-1}(\text{cl}(\text{int}(F)))) \subseteq U$. Hence f is $\tilde{g}_a$-wg-irresolute.

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