International Journal of Mathematical Archive-2(11), 2011, Page: 2142-2148
IJMA Available online through www.ijma.info ISSN 2229 – 5046

RANDOM FIXED POINT THEOREMS OF RANDOM MULTIVALUED OPERATORS ON POLISH SPACE

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(Received on: 16-10-11; Accepted on: 01-11-11)

ABSTRACT

In this paper, We established Some Common Fixed Point Theorems for Random operator, in polish spaces, by using rational expressions.

Keywords: *Polish Space, Random Operator, Random Multivalued Operator, Random Fixed Point, Measurable mapping.*

AMS Subject Classification: *47H10, 54H25.*

1. INTRODUCTION:

Probabilistic functional analysis has emerged as one of the important mathematical disciplines in view of its role in analyzing Probabilistic models in the applied sciences. The study of fixed point of random operator forms a central topic in this area. Random fixed point theorem for contraction mappings in Polish spaces and random Fixed point theorems are of fundamental importance in probabilistic functional analysis. There study was initiated by the Prague school of Probabilistics, in 1950, with their work of Spacek [15] and Hans [5,6]. For example survey is refer to Bharucha-Reid [4]. Itoh [8] proved several random fixed point theorems and gave their applications to Random differential equations in Banach spaces. Random coincidence point theorems and random fixed point theorems are stochastic generalization of classical coincidence point theorems and classical fixed point theorems. Sehgal and Singh [14], Papageorgiou [12], Rhoades, Sessa, Khan [13] and Lin [11] have proved differential Stochastic version of well known Schauder's fixed point theorem. Recently, Beg and Shahzad [3] studied the structure of common fixed point and random coincidence Points of a pair of compatible random operators.

2. PRELIMINARIES:

Definition: 2.1 A metric space (X,d) is said to be a Polish Space, if it satisfying following conditions:

(1) X, is complete,
(2) X is separable,

Before we describe our next hierarchy of set of reals of ever increasing complexity,

we would like to consider a class of metric spaces under which we can unify 2^{ω}, ω^{ω}, $\mathcal{R}$ and there products. This will be helpful in formulating this hierarchy. ($as\ well\ as\ future\ ones$)

Recall that a metric space (X,d) is complete if whenever $(x_n\colon n\in\omega)$ is a sequence of member of X, such that for every $\epsilon>0$ there is an N, such that $m,n\geq N$ implies $d(x_n,x_m)<\epsilon$, there is a single $x\ in\ X$ such that $\lim_{n<\omega} x_n = x$. It is easy to see that 2^{ω}, ω^{ω} are polish space, So in fact is ω under the discreate topology, whose metric is given by letting $d(x,y)=1$ when $x\neq y$ and $d(x,y)=0$ when $x=y$.

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Let (X,d) be a Polish space that is a separable complete metric space and (Ω,q) be Measurable space. Let 2^x be a family of all subsets of X and $CB(X)$ denote the family of all nonempty bounded closed subsets of X. A mapping $T:\Omega \to 2^X$ is called measurable if for any open subset C of X, $T^{-1}(C) = \{\omega \in \Omega: f(\omega) \cap C \neq \phi \in q$. A mapping $\xi:\Omega \to X$ is said to be measurable selector of a measurable mapping $T:\Omega \to 2X$, if ξ is measurable and for any $\omega \in \Omega$, $\xi(\omega) \in T(\omega)$. A mapping $f:\Omega \times X \to X$ is called random operator, if for any $x \in X, f(\cdot,x)$ is measurable. A Mapping $T:\Omega \times X \to CB(X)$ is a random multivalued operator, if for every $x \in X, T(\cdot,x)$ is measurable . A measurable mapping $\xi:\Omega \to X$ is called random fixed point of a random multivalued operator $T:\Omega \times X \to CB(X)$ $(f:\Omega \times X \to X)$, if for every $\omega \in \Omega$, $\xi(\omega) \in T(\omega,\xi(\omega))$, $f(\omega),\xi(\omega) = \xi(\omega))$. Let $T:\Omega \times X \to CB(X)$ be a random operator And $\{\xi_n\}$ a sequence of measurable mappings , $\xi_n:\Omega \to X$. The sequence $\{\xi_n\}$ is said to be asymptotically T-regular if $d(\xi_n(\omega), T(\omega,\xi_n(w)) \to 0$.

3. MAIN RESULT:

Theorem 3.1: Let X be a Polish space. Let $T,S:\Omega \times X \to CB(X)$ be two continuous random multivalued operators, If there exists measurable mappings $a,b:\Omega \to (0,1)$, such that, it satisfy followings condition,

$$H(S(\omega,X),T(\omega,y)) \le a(\omega)\left\{d(y,T(\omega,y))d(x,S(\omega,x)).d(x,y)\right\}^{\frac{1}{3}} \tag{3.1.1}$$

For each $x,y \in X$ and $\omega \in \Omega$ *and* $a \in R^+$ *with* $a(\omega) < 1$. Then there exists a common random fixed point of S *and* T. (*Here* H *represent the Housdroff metric on* $CB(X)$ *induced by the metric* d)

Proof: Let $\xi_0: \Omega \to X$ be an arbitrary measurable mapping and choose a measurable mapping $\xi:\Omega \to X$ such that $\xi_1(\omega) \in S(\omega,\xi_0(\omega))$ for each $\omega \in \Omega$. then for each $\omega \in \Omega$,

$$H\left(S(\omega,\xi_0(\omega)),T(\omega,\xi_1(\omega))\right) \le a(\omega)\ \left\{d\left(\xi_1(\omega),T(\omega,\xi_1(\omega))\right).d\left(\xi_0(\omega),S(\omega,\xi_0(\omega))\right).d(\xi_0(\omega),\xi_1(\omega))\right\}^{\frac{1}{3}}$$

Further, there exists a measurable mapping $\xi_2:\Omega \to X$ such that for all $\omega \in \Omega$, $\xi_2(\omega) \in T(\omega,\xi_1(\omega))$ and,

$$d(\xi_1(\omega),\xi_2(\omega)) = H(S(\omega,\xi_0(\omega)),T(\omega,\xi_1(\omega)))$$

$$d(\xi_1(\omega),\xi_2(\omega)) \le a(\omega)\left\{d(\xi_1(\omega),\xi_2(\omega)).d(\xi_0(\omega),\xi_1(\omega)).d(\xi_0(\omega),\xi_1(\omega))\right\}^{\frac{1}{3}}$$

$$d(\xi_1(\omega),\xi_2(\omega)) \le (a(\omega))^{\frac{3}{2}}\ d(\xi_0(\omega),\xi_1(\omega))$$

Let $k = (a(\omega))^{\frac{3}{2}} < 1$ then,

$$d(\xi_1(\omega),\xi_2(\omega)) \le k.\ d(\xi_0(\omega),\xi_1(\omega))$$

By Beg and Shahzad [2, lemma2.3], we obtain a measurable mapping $\xi_3:\Omega \to X$, such that for all $\omega \in \Omega$, $\xi_3(\omega) \in S(\omega,\xi_2(\omega))$ and,

$$d(\xi_2(\omega),\xi_3(\omega)) = H(S(\omega,\xi_1(\omega)),T(\omega,\xi_2(\omega)))$$

$$d(\xi_2(\omega),\xi_3(\omega)) \le a(\omega)\left\{d(\xi_2(\omega),\xi_3(\omega)).d(\xi_1(\omega),\xi_2(\omega)).d(\xi_1(\omega),\xi_2(\omega))\right\}^{\frac{1}{3}}$$

$$d(\xi_2(\omega),\xi_3(\omega)) \le k\ d(\xi_1(\omega),\xi_2(\omega))$$

$$d(\xi_2(\omega),\xi_3(\omega)) \le k^2.\ d(\xi_0(\omega),\xi_1(\omega))$$

Similarly, proceeding in the same way: by induction, we produce a sequence of measurable mapping $\xi_n:\Omega \to X$, such that for $n > 0$ and any $\omega \in \Omega$, $\xi_{2n+1}(\omega) \in S(\omega,\xi_{2n}(\omega))$, $\xi_{2n+2}(\omega) \in T(\omega,\xi_{2n+1}(\omega))$ and hence,

$$d(\xi_n(\omega),\xi_{n+1}(\omega)) \le k^n.\ d(\xi_0(\omega),\xi_1(\omega))$$

Furthermore, for $m > n$,

$$d(\xi_n(\omega),\xi_m(\omega)) \le d(\xi_n(\omega),\xi_{n+1}(\omega)) + d(\xi_{n+1}(\omega),\xi_{n+2}(\omega)) + \dots\dots\dots + d(\xi_{m-1}(\omega),\xi_m(\omega))$$

$$d(\xi_n(\omega),\xi_m(\omega)) \le [k^n + k^{n+1} + \dots\dots + k^{m-1}]\, d(\xi_0(\omega),\xi_1(\omega))$$

$$d(\xi_n(\omega),\xi_m(\omega)) \le [1 + k + \dots\dots + k^{m-n-1}].\,k^n.\,d(\xi_0(\omega),\xi_1(\omega))$$

$$d(\xi_n(\omega),\xi_m(\omega)) \le \left[\frac{k^n}{1-k}\right].\,d(\xi_0(\omega),\xi_1(\omega))$$

$$d(\xi_n(\omega),\xi_m(\omega)) \to 0 \;\; as \;\; n,m \to \infty$$

It follows that, $\{\xi_n(\omega)\}$ is a Cauchy sequence and there exists a measurable mapping $\xi : \Omega \to X$ such that $\xi_n(\omega) \to \xi(\omega)$, for each $\omega \in \Omega$. It further implies that, $\xi_{2n+1}(\omega) \to \xi(\omega)$ and $\xi_{2n+2}(\omega) \to \xi(\omega)$. Thus we have for any $\omega \in \Omega$,

$$d\left(\xi(\omega), S(\omega,\xi(\omega))\right) \le d(\xi(\omega),\xi_{2n+2}(\omega)) + d\left(\xi_{2n+2}(\omega), S(\omega,\xi(\omega))\right)$$

$$d\left(\xi(\omega), S(\omega,\xi(\omega))\right) \le d(\xi(\omega),\xi_{2n+2}(\omega)) + H(S(\omega,\xi(\omega)), T(\omega,\xi_{2n+1}(\omega)))$$

By using (3.1.1) $and,\; as\;\; n \to \infty$, we have

$$d\left(\xi(\omega), S(\omega,\xi(\omega))\right) \le 0$$

Hence $\xi(\omega) \in S(\omega,\xi(\omega))$ for $\omega \in \Omega$. similarly, for any $\omega \in \Omega$,

$$d\left(\xi(\omega), T(\omega,\xi(\omega))\right) \le d(\xi(\omega),\xi_{2n+1}(\omega)) + d\left(\xi_{2n+1}(\omega), T(\omega,\xi(\omega))\right)$$

$$d\left(\xi(\omega), T(\omega,\xi(\omega))\right) \le d(\xi(\omega),\xi_{2n+1}(\omega)) + H(S(\omega,\xi_{2n}(\omega)), T(\omega,\xi(\omega)))$$

By using (3.1.1) $and,\; as\;\; n \to \infty$, we have

$$d\left(\xi(\omega), T(\omega,\xi(\omega))\right) \le 0$$

Hence $\xi(\omega) \in T(\omega,\xi(\omega))$ for $\omega \in \Omega$. similarly, for any $\omega \in \Omega$,

Theorem 3.2: Let X be a Polish space. Let $T,S : \Omega \times X \to CB(X)$ be two continuous random multivalued operators, If there exists measurable mappings $a,b : \Omega \to (0,1)$, such that, it satisfy followings condition,

$$H(S(\omega,X),T(\omega,y)) \le a(\omega)\ \max\{d(x,y),\ d(y,T(\omega,y)), d(x,S(\omega,x))\} \quad + b(\omega)\{d(x,T(\omega,y)) + d(y,S(\omega,x))\} \tag{3.1.2}$$

For each $x\,,y \in X$ and $\omega \in \Omega$ $and\;\; a,b \in R^+ \;with\;\; a(\omega) + b(\omega) < 1$. Then there exists a common random fixed point of $S\;\; and\;\; T$.$(Here\;\; H\; represent\;\; the\; Housdroff\;\; metric\; on\; CB(X)\;\; induced\; by\; the\; metric\;\; d\,)$

Proof: Let $\xi_0 :\ \Omega \to\ X$ be an arbitrary measurable mapping and choose a measurable mapping $\xi: \Omega \to X$ such that $\xi_1(\omega) \in S(\omega,\xi_0(\omega))$ for each $\omega \in \Omega$. then for each $\omega \in \Omega$,

$$H\left(S(\omega,\xi_0(\omega)), T(\omega,\xi_1(\omega))\right) \le a(\omega)\ \max\left\{\begin{array}{c} d(\xi_0(\omega),\xi_1(\omega)),\ d\left(\xi_1(\omega), T(\omega,\xi_1(\omega))\right), \\ d\left(\xi_0(\omega), S(\omega,\xi_0(\omega))\right) \end{array}\right\} + b(\omega)\left\{d\left(\xi_0(\omega), T(\omega,\xi_1(\omega))\right) + d\left(\xi_1(\omega), S(\omega,\xi_0(\omega))\right)\right\}$$

Further, there exists a measurable mapping $\xi_2 : \Omega \to X$ such that for all $\omega \in \Omega$, $\xi_2(\omega) \in T(\omega,\xi_1(\omega))$ and,

$$d(\xi_1(\omega),\xi_2(\omega)) = H(S(\omega,\xi_0(\omega)), T(\omega,\xi_1(\omega)))$$

$$d\big(\xi_1(\omega),\xi_2(\omega)\big) \le a(\omega)\ \max\begin{Bmatrix} d\big(\xi_0(\omega),\xi_1(\omega)\big), d\left(\xi_1(\omega),T\big(\omega,\xi_1(\omega)\big)\right), \\ d\left(\xi_0(\omega),S\big(\omega,\xi_0(\omega)\big)\right)\end{Bmatrix}$$

$$+\ b(\omega)\left\{d\left(\xi_0(\omega),T\big(\omega,\xi_1(\omega)\big)\right)+d\left(\xi_1(\omega),S\big(\omega,\xi_0(\omega)\big)\right)\right\}$$

$$d\big(\xi_1(\omega),\xi_2(\omega)\big) \le a(\omega)\ \max\begin{Bmatrix} d\big(\xi_0(\omega),\xi_1(\omega)\big), d\big(\xi_1(\omega),\xi_2(\omega)\big) \\ , d\big(\xi_0(\omega),\xi_1(\omega)\big)\end{Bmatrix}$$

$$+\ b(\omega)\big\{d\big(\xi_0(\omega),\xi_2(\omega)\big)+d\big(\xi_1(\omega),\xi_1(\omega)\big)\big\}$$

$$d\big(\xi_1(\omega),\xi_2(\omega)\big) \le\ [\,a(\omega)+b(\omega)]\,.\,d(\xi_0(\omega),\xi_1(\omega))$$

Let $k =\ [\,a(\omega)\ +\ b(\omega)]\ < 1$ then,

$$d\big(\xi_1(\omega),\xi_2(\omega)\big) \le k.\ d(\xi_0(\omega),\xi_1(\omega))$$

By Beg and Shahzad [lemma2.3], we obtain a measurable mapping $\xi_3\ :\Omega\to\ X$, such that for all $\omega\in\Omega$, $\xi_3(\omega)\in S\big(\omega,\xi_2(\omega)\big)$ and,

$$d\big(\xi_2(\omega),\xi_3(\omega)\big) = H\big(S(\omega,\xi_1(\omega)),T(\omega,\xi_2(\omega))\big)$$

$$d\big(\xi_1(\omega),\xi_2(\omega)\big) \le a(\omega)\ \max\begin{Bmatrix} d\big(\xi_1(\omega),\xi_2(\omega)\big), d\big(\xi_2(\omega),\xi_3(\omega)\big), \\ d\big(\xi_1(\omega),\xi_2(\omega)\big)\end{Bmatrix}$$
$$+\ b(\omega)\big\{d\big(\xi_1(\omega),\xi_3(\omega)\big)+d\big(\xi_2(\omega),\xi_2(\omega)\big)\big\}$$

$$d\big(\xi_2(\omega),\xi_3(\omega)\big) \le [\,a(\omega)\ \ +\ \ b(\omega)]\,d(\xi_1(\omega),\xi_2(\omega))$$

Let $k =\ [\,a(\omega)\ +\ b(\omega)]\ \ < 1$ then,

$$d\big(\xi_2(\omega),\xi_3(\omega)\big) \le\ k^2.\ d(\xi_0(\omega),\xi_1(\omega))$$

Similarly, proceeding in the same way: by induction, we produce a sequence of measurable mapping $\xi_n\ :\Omega\to\ X$, such that for $n>0$ and any $\omega\in\Omega$, $\xi_{2n+1}(\omega)\in S\big(\omega,\xi_{2n}(\omega)\big)$, $\xi_{2n+2}(\omega)\in T\big(\omega,\xi_{2n+1}(\omega)\big)$

And hence,

$$d\big(\xi_n(\omega),\xi_{n+1}(\omega)\big) \le k^n.\ d(\xi_0(\omega),\xi_1(\omega))$$

Furthermore, for $m>n$,

$$d\big(\xi_n(\omega),\xi_m(\omega)\big)\ \le d\big(\xi_n(\omega),\xi_{n+1}(\omega)\big)\ +\ d\big(\xi_{n+1}(\omega),\xi_{n+2}(\omega)\big)+ \dots\dots\dots..+\ d\big(\xi_{m-1}(\omega),\xi_m(\omega)\big)$$

$$d\big(\xi_n(\omega),\xi_m(\omega)\big)\ \le\ [k^n+k^{n+1}+\ \dots\dots+k^{m-1}]\ d\big(\xi_0(\omega),\xi_1(\omega)\big)$$

$$d\big(\xi_n(\omega),\xi_m(\omega)\big) \le [1+k+\ \dots\dots+k^{m-n-1}].\,k^n.\,d\big(\xi_0(\omega),\xi_1(\omega)\big)$$

$$d\big(\xi_n(\omega),\xi_m(\omega)\big) \le \left[\frac{k^n}{1-k}\right].\,d\big(\xi_0(\omega),\xi_1(\omega)\big)$$

$$d\big(\xi_n(\omega),\xi_m(\omega)\big) \to\ \ 0\ \ as\ \ n,m\to\infty$$

It follows that, $\{\xi_n(\omega)\}$ is a Cauchy sequence and there exists a measurable mapping $\xi:\Omega\to X$ such that $\xi_n(\omega)\to\xi(\omega)$, for each $\omega\in\Omega$. It further implies that, $\xi_{2n+1}(\omega)\to\xi(\omega)$ and $\xi_{2n+2}(\omega)\to\xi(\omega)$. Thus we have for any $\omega\ \in\Omega$,

$$d\left(\xi(\omega),S\big(\omega,\xi(\omega)\big)\right) \le d\big(\xi(\omega),\xi_{2n+2}(\omega)\big)\ +d\left(\xi_{2n+2}(\omega),S\big(\omega,\xi(\omega)\big)\right)$$

$$d\left(\xi(\omega), S\left(\omega, \xi(\omega)\right)\right) \leq d\left(\xi(\omega), \xi_{2n+2}(\omega)\right) + H\left(S(\omega, \xi(\omega)), T(\omega, \xi_{2n+1}(\omega))\right)$$

By using (3.1.2). and As $n \to \infty$, we have

$$d\left(\xi(\omega), S\left(\omega, \xi(\omega)\right)\right) \leq 0$$

Hence $\xi(\omega) \in S\left(\omega, \xi(\omega)\right)$ for $\omega \in \Omega$. similarly, for any $\omega \in \Omega$,

$$d\left(\xi(\omega), T\left(\omega, \xi(\omega)\right)\right) \leq d\left(\xi(\omega), \xi_{2n+1}(\omega)\right) + d\left(\xi_{2n+1}(\omega), T\left(\omega, \xi(\omega)\right)\right)$$

$$d\left(\xi(\omega), T\left(\omega, \xi(\omega)\right)\right) \leq d\left(\xi(\omega), \xi_{2n+1}(\omega)\right) + H\left(S(\omega, \xi_{2n}(\omega)), T(\omega, \xi(\omega))\right)$$

As $n \to \infty$, we have

$$d\left(\xi(\omega), T\left(\omega, \xi(\omega)\right)\right) \leq 0$$

Hence $\xi(\omega) \in T\left(\omega, \xi(\omega)\right)$ for $\omega \in \Omega$. similarly, for any $\omega \in \Omega$,

Theorem 3.3: Let X be a Polish space. Let $T, S : \Omega \times X \to CB(X)$ be two continuous random multivalued operators, If there exists measurable mappings $a, b, c : \Omega \to (0,1)$, such that, it satisfy followings condition,

$$H\left(S(\omega, X), T(\omega, y)\right) \leq a(\omega)\left[d\left(y, S(\omega, x)\right) + d\left(x, T(\omega, y)\right)\right] \\ + b(\omega)\left[d\left(y, T(\omega, y)\right) + d\left(x, S(\omega, x)\right)\right] + c(\omega) d(x, y) \tag{3.1.3}$$

For each $x, y \in X$ *and* $\in \Omega$ *and* $a, b, c \in R^{+}$ *with* $a(\omega) + b(\omega) + c(\omega) < 1$ *and* $0 < \frac{a(\omega) + b(\omega) + c(\omega)}{1 - a(\omega) - b(\omega)} < 1$. *Then there exists a common random fixed point of* S *and* T . (*Here* H *represent the Housdroff metric on* $CB(X)$ *induced by the metric* d)

Proof: Let $\xi_0 : \Omega \to X$ be an arbitrary measurable mapping and choose a measurable mapping $\xi : \Omega \to X$ such that $\xi_1(\omega) \in S\left(\omega, \xi_0(\omega)\right)$ for each $\omega \in \Omega$. then for each $\omega \in \Omega$,

$$H\left(S\left(\omega, \xi_0(\omega)\right), T\left(\omega, \xi_1(\omega)\right)\right) \leq a(\omega)\left\{d\left(\xi_1(\omega), S\left(\omega, \xi_0(\omega)\right)\right) + d\left(\xi_0(\omega), T\left(\omega, \xi_1(\omega)\right)\right)\right\} \\ + b(\omega) \left\{d\left(\xi_1(\omega), T\left(\omega, \xi_1(\omega)\right)\right) + d\left(\xi_0(\omega), S\left(\omega, \xi_0(\omega)\right)\right)\right\} \\ + c(\omega) d(\xi_0(\omega), \xi_1(\omega))$$

Further, there exists a measurable mapping $\xi_2 : \Omega \to X$ such that for all $\omega \in \Omega$, $\xi_2(\omega) \in T(\omega, \xi_1(\omega))$ and,

$$d\left(\xi_1(\omega), \xi_2(\omega)\right) = H\left(S(\omega, \xi_0(\omega)), T(\omega, \xi_1(\omega))\right)$$

$$d\left(\xi_1(\omega), \xi_2(\omega)\right) \leq a(\omega)\left\{d\left(\xi_1(\omega), S\left(\omega, \xi_0(\omega)\right)\right) + d\left(\xi_0(\omega), T\left(\omega, \xi_1(\omega)\right)\right)\right\} \\ + b(\omega) \left\{d\left(\xi_1(\omega), T\left(\omega, \xi_1(\omega)\right)\right) + d\left(\xi_0(\omega), S\left(\omega, \xi_0(\omega)\right)\right)\right\} \\ + c(\omega) d(\xi_0(\omega), \xi_1(\omega))$$

$$d\left(\xi_1(\omega), \xi_2(\omega)\right) \leq a(\omega)\left\{d\left(\xi_1(\omega), \xi_1(\omega)\right) + d\left(\xi_0(\omega), \xi_2(\omega)\right)\right\} \\ + b(\omega) \left\{d\left(\xi_1(\omega), \xi_2(\omega)\right) + d\left(\xi_0(\omega), \xi_1(\omega)\right)\right\} + c(\omega) d(\xi_0(\omega), \xi_1(\omega))$$

$$d\left(\xi_1(\omega), \xi_2(\omega)\right) \leq \left[\frac{a(\omega) + b(\omega) + c(\omega)}{1 - a(\omega) - b(\omega)}\right] . d(\xi_0(\omega), \xi_1(\omega))$$

Let $k = \left[\frac{a(\omega) + b(\omega) + c(\omega)}{1 - a(\omega) - b(\omega)}\right] < 1$ then,

$$d\left(\xi_1(\omega), \xi_2(\omega)\right) \leq k.\ d(\xi_0(\omega), \xi_1(\omega))$$

By Beg and Shahzad [2, lemma2.3], we obtain a measurable mapping $\xi_3 : \Omega \to X$, such that for all $\omega \in \Omega$, $\xi_3(\omega) \in S\left(\omega, \xi_2(\omega)\right)$ and,

$$d(\xi_2(\omega), \xi_3(\omega)) = H(S(\omega, \xi_1(\omega)), T(\omega, \xi_2(\omega)))$$

$$d(\xi_2(\omega), \xi_3(\omega)) \leq a(\omega)\{d(\xi_2(\omega), \xi_2(\omega)) + d(\xi_1(\omega), \xi_3(\omega))\} + b(\omega)\ \{d(\xi_2(\omega), \xi_3(\omega)) + d(\xi_1(\omega), \xi_2(\omega))\} + c(\omega)d(\xi_1(\omega), \xi_2(\omega))$$

$$d(\xi_2(\omega), \xi_3(\omega)) \leq \left[\frac{a(\omega) + b(\omega) + c(\omega)}{1 - a(\omega) - b(\omega)}\right] d(\xi_1(\omega), \xi_2(\omega))$$

Let $k = \left[\frac{a(\omega) + b(\omega) + c(\omega)}{1 - a(\omega) - b(\omega)}\right] < 1$ then,

$$d(\xi_2(\omega), \xi_3(\omega)) \leq k^2.\ d(\xi_0(\omega), \xi_1(\omega))$$

Similarly, proceeding in the same way: by induction, we produce a sequence of measurable mapping $\xi_n : \Omega \to X$, such that for $n > 0$ and any $\omega \in \Omega$, $\xi_{2n+1}(\omega) \in S(\omega, \xi_{2n}(\omega))$, $\xi_{2n+2}(\omega) \in T(\omega, \xi_{2n+1}(\omega))$

And hence,

$$d(\xi_n(\omega), \xi_{n+1}(\omega)) \leq k^n.\ d(\xi_0(\omega), \xi_1(\omega))$$

Furthermore, for $m > n$,

$$d(\xi_n(\omega), \xi_m(\omega)) \leq d(\xi_n(\omega), \xi_{n+1}(\omega)) + d(\xi_{n+1}(\omega), \xi_{n+2}(\omega)) + \ldots\ldots\ldots.. + d(\xi_{m-1}(\omega), \xi_m(\omega))$$

$$d(\xi_n(\omega), \xi_m(\omega)) \leq [k^n + k^{n+1} + \ldots\ldots + k^{m-1}]\, d(\xi_0(\omega), \xi_1(\omega))$$

$$d(\xi_n(\omega), \xi_m(\omega)) \leq [1 + k + \ldots\ldots + k^{m-n-1}].\, k^n.\, d(\xi_0(\omega), \xi_1(\omega))$$

$$d(\xi_n(\omega), \xi_m(\omega)) \leq \left[\frac{k^n}{1-k}\right].\, d(\xi_0(\omega), \xi_1(\omega))$$

$$d(\xi_n(\omega), \xi_m(\omega)) \to 0 \ as\ n, m \to \infty$$

It follows that, $\{\xi_n(\omega)\}$ is a Cauchy sequence and there exists a measurable mapping $\xi : \Omega \to X$ such that $\xi_n(\omega) \to \xi(\omega)$, for each $\omega \in \Omega$. It further implies that, $\xi_{2n+1}(\omega) \to \xi(\omega)$ and $\xi_{2n+2}(\omega) \to \xi(\omega)$. Thus we have for any $\omega \in \Omega$,

$$d\left(\xi(\omega), S(\omega, \xi(\omega))\right) \leq d(\xi(\omega), \xi_{2n+2}(\omega)) + d\left(\xi_{2n+2}(\omega), S(\omega, \xi(\omega))\right)$$

$$d\left(\xi(\omega), S(\omega, \xi(\omega))\right) \leq d(\xi(\omega), \xi_{2n+2}(\omega)) + H(S(\omega, \xi(\omega)), T(\omega, \xi_{2n+1}(\omega)))$$

By using (3.1.3). and As $n \to \infty$, we have

$$d\left(\xi(\omega), S(\omega, \xi(\omega))\right) \leq 0$$

Hence $\xi(\omega) \in S(\omega, \xi(\omega))$ for $\omega \in \Omega$. similarly, for any $\omega \in \Omega$,

$$d\left(\xi(\omega), T(\omega, \xi(\omega))\right) \leq d(\xi(\omega), \xi_{2n+1}(\omega)) + d\left(\xi_{2n+1}(\omega), T(\omega, \xi(\omega))\right)$$

$$d\left(\xi(\omega), T(\omega, \xi(\omega))\right) \leq d(\xi(\omega), \xi_{2n+1}(\omega)) + H(S(\omega, \xi_{2n}(\omega)), T(\omega, \xi(\omega)))$$

As $n \to \infty$, we have

$$d\left(\xi(\omega), T(\omega, \xi(\omega))\right) \leq 0$$

Hence $\xi(\omega) \in T(\omega, \xi(\omega))$ for $\omega \in \Omega$. similarly, for any $\omega \in \Omega$,

ACKNOWLEDGEMENT:

We Heartily gratefull to Dr. S.S, Rajput Head of Department, (Mathematics), Govt. P. G Collage, Gadarwara for the motivation and Guidance during the preparation of this article.

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