G\pi - Closed Sets in Bi ÑČech Closure Spaces

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ABSTRACT

In this paper, we introduce the concepts of g\pi - closed and g\pi - open sets in BiČech closure space and study some of their properties.

Key words: ÑČech closure operator, ÑČech closure spaces, ÑČech g\pi closed sets.

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1. INTRODUCTION:

Čech closure spaces were introduced by E. ÑČech [1]. In ÑČech’s approach the operator satisfies idempotent condition among kuratowski axioms. This condition need not hold for every set A of X. When this condition is also true, the operator becomes topological closure operator. Thus the concept of closure space is the generalization of a topological space. In this paper, we introduce the concept of (k_1, k_2)- g\pi closed sets, g\pi C_0 bi- ÑČech space and study their basic properties.

2. PRELIMINARIES:

Definition: 2.1 [2] Two functions k_1 and k_2 from power set X to itself are called bi- ÑČech closure operators (simply biclosure operator) for X if they satisfy the following properties.

(i) k_1 (\emptyset) = \emptyset and k_2 (\emptyset) = \emptyset
(ii) A \subseteq k_1 (A) and A \subseteq k_2 (A) for any set A \subseteq X
(iii) k_1 (A \cup B) = k_1 (A) \cup k_1 (B) and k_2 (A \cup B) = k_2 (A) \cup k_2 (B) for any A, B \subseteq X (X, k_1, k_2) is called bi- ÑČech closure space.

Example: 2.2 Let X = \{a, b, c\} and let k_1 and k_2 be defined as k_1 (\{a\}) = \{a\}, k_1 (\{b\}) = k_1 (\{c\}) = k_1 (\{b, c\}) = \{b, c\}, k_1 (\{a, b\}) = k_1 (\{a, c\}) = k_1 (\{X\}) = X, and k_2 (\emptyset) = \emptyset.

k_2 (\{a\}) = \{a\}, k_2 (\{b\}) = \{b, c\}, k_2 (\{c\}) = k_2 (\{a, c\}) = k_2 (\{a\}, k_2 (\{a, b\}) = k_2 (\{b, c\}) = k_2 (\{X\}) = X, k_2 (\emptyset) = \emptyset.

Now, (X, k_1, k_2) is a bi- ÑČech closure space.

Definition: 2.3 [3] A subset A in a bi- ÑČech closure space (X, k_1, k_2) is said to be

1. k_i-regular open if A = int_ki(k_i(A)), i = 1, 2
2. k_i-regular closed if A = ki(int_ki(A)), i = 1, 2
3. k_i-semi open if A \subseteq ki(int_ki(A)), i = 1, 2
4. k_i-semi closed if int_ki(k_i(A)) \subseteq A, i = 1, 2

The finite union of k_i regular open sets is said to be k_i \pi-open.

The complement of a k_i \pi-open set is said to be k_i \pi-closed. The smallest k_i \pi closed set containing A is called k_i- \pi closure of A and it is denoted by k_\pi cl (A).

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3. \((k_1, k_2)\) - \(\pi\) closed sets:

**Definition:** 3.1 A subset \(A\) is a bi-\(^{\pi}\) Čech closure space \((X, k_1, k_2)\) is said to be \((k_1, k_2)\)-\(\pi\) closed if \(k_{\pi, cl2}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is \(k_1\)-open set in \(X\).

**Example:** 3.2 In example 2.2, the \((k_1, k_2)\) - \(\pi\) closed sets are \(X, \varnothing, \{a\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}\).

**Theorem:** 3.3 If \(A\) and \(B\) are \((k_1, k_2)\) - \(\pi\) closed sets and so is \(A \cup B\).

**Proof:** Let \(A\) and \(B\) be two \((k_1, k_2)\) - \(\pi\) closed sets. Let \(U\) be \(k_1\)-open set in \(X\). Let \((A \cup B) \subseteq U\), we have \(A \subseteq U\) and \(B \subseteq U\). Then \(k_{\pi, cl2}(A) \subseteq U\) and \(k_{\pi, cl2}(B) \subseteq U\) implies \((k_{\pi, cl2}(A) \cup k_{\pi, cl2}(B)) \subseteq U\). Hence \(k_{\pi, cl2}(A \cup B) \subseteq U\).

Thus \(A \cup B\) is \((k_1, k_2)\) - \(\pi\) closed set.

**Theorem:** 3.4 If \(A\) is \((k_1, k_2)\) - \(\pi\) closed set, then \(k_{\pi, cl2}(A)\)-\(A\) contains no nonempty \(k_1\) - closed sets.

**Proof:** Let \(A\) be \((k_1, k_2)\) - \(\pi\) closed set. Let \(U\) be \(k_1\) - closed contained in \(k_{\pi, cl2}(A)\)-\(A\).

Now, \(U \subseteq k_{\pi, cl2}(A)\) and \(U \subseteq A\). Now, \(U \subseteq A\) then \(A \subseteq U\). Since \(U\) is \(k_1\)-closed. \(k_{\pi, cl2}(A) \subseteq U\). Consequently \(U \subseteq [k_{\pi, cl2}(A)]^c\). As \(U \subseteq k_{\pi, cl2}(A) \cap [k_{\pi, cl2}(A)]^c\). \(U \neq \varnothing\). Hence \(k_{\pi, cl2}(A)\)-\(A\) contains no non-empty \(k_1\)-closed sets.

**Theorem:** 3.5 If \(A\) is \((k_1, k_2)\) - \(\pi\) closed set, then \(k_{\pi, cl}(x) \cap A \neq \varnothing\) holds for each \(x \in k_{\pi, cl2}(A)\).

**Proof:** Let \(A\) be \((k_1, k_2)\) - \(\pi\) closed set. Suppose \(k_{\pi, cl}(x) \cap A \neq \varnothing\) for some \(x \in k_{\pi, cl2}(A)\), we have \(A \subseteq [k_{\pi, cl}(x)]^c\).

Now, \(k_{\pi, cl}(x)\) is \(k_1\) - \(\pi\) closed. Therefore \([k_{\pi, cl}(x)]^c\) is \(k_1\) - \(\pi\) open. Thus \([k_{\pi, cl}(x)]^c\) is \(k_1\) - \(\pi\) open. Since \(A\) is \((k_1, k_2)\) - \(\pi\) closed set, we have \(k_{\pi, cl2}(A) \subseteq [k_{\pi, cl}(x)]^c\). \(k_{\pi, cl2}(A) \cap k_{\pi, cl}(x) = \varnothing\). Hence \(k_{\pi, cl2}(A)\) is a contradiction.

**Theorem:** 3.6 Let \((X, k_1, k_2)\) be bi-\(^{\pi}\) Čech closure space. For each \(x\) in \(X\), \(\{x\}\) is \(k_1\) - closed or \(\{x\}\) is \((k_1, k_2)\) - \(\pi\) closed set.

**Proof:** Let \((X, k_1, k_2)\) be bi-\(^{\pi}\) Čech closure space. Suppose that \(\{x\}\) is not \(k_1\) - closed. \(\{x\}\) is not \(k_1\) - open. Therefore, the only \(k_1\) - open set containing \(\{x\}\) is \(X\). Thus, \(\{x\}\) is \(\pi\) closed. \(k_{\pi, cl2}([\{x\}]) \subseteq k_{\pi, cl2}(X) = X\).

**Theorem:** 3.7 Let \(A\) be \((k_1, k_2)\) - \(\pi\) closed set and if \(A\) is \(k_1\) - open then \(A = k_{\pi, cl2}(A)\).

**Proof:** Let \(A\) be \((k_1, k_2)\) - \(\pi\) closed subset of a bi-\(^{\pi}\) Čech closure spaces \((X, k_1, k_2)\) and let \(A\) be \(k_1\) - open set. Then \(k_{\pi, cl2}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is \(k_1\) - open set in \(X\). Since \(A\) is \(k_1\) - open and \(A \subseteq A\), we have \(k_{\pi, cl2}(A) \subseteq A\). But always, \(A \subseteq k_{\pi, cl2}(A)\). Thus, \(A = k_{\pi, cl2}(A)\).

**Theorem:** 3.8 Let \(A \subseteq Y \subseteq X\) and suppose that \(A\) is \((k_1, k_2)\) - \(\pi\) closed in \((X, k_1, k_2)\). Then \(A\) is \((k_1, k_2)\) - \(\pi\) closed relative to \(Y\).

**Proof:** Let \(S\) be any \(k_1\) - open set in \(Y\) such that \(A \subseteq S\). Then \(S = U \cap Y\) for some \(U\) in \(k_1\) - open in \(X\). Therefore \(A \subseteq U \cap Y\) implies \(A \subseteq U\). Since \(A\) is \((k_1, k_2)\) - \(\pi\) closed set in \(X\), we have \(k_{\pi, cl2}(A) \subseteq U\). Hence \(Y \cap k_{\pi, cl2}(A) \subseteq Y \cap U\). Thus \(A\) is \((k_1, k_2)\) - \(\pi\) closed set relative to \(Y\).

4. \(\pi\) \(C_0\) bi-\(^{\pi}\) Čech spaces:

**Definition:** 4.1 A bi-\(^{\pi}\) Čech closure space \((X, k_1, k_2)\) is said to be a \(\pi\) \(C_0\) bi-\(^{\pi}\) Čech space if for every \(\pi\) -open subset \(U\) of \((X, k_1)\), \(x \in U\) implies \(k_1(\{x\}) \subseteq U\).

**Theorem:** 4.2 A bi-\(^{\pi}\) Čech closure space \((X, k_1, k_2)\) is a \(\pi\) \(C_0\) bi-\(^{\pi}\) Čech space if and only if for every \(\pi\) -closed subset \(F\) of \((X, k_1)\) such that \(x \in F\), \(k_1(\{x\}) \subseteq F\).
Proof: Let $F$ be a $g\pi$-closed subset of $(X, k_1)$ and let $x \not\in F$. Since $x \in X\setminus F$ and $X\setminus F$ is a $g\pi$-open subset of $(X, k_1)$, $k_2 (\{x\}) \subseteq X\setminus F$. Consequently $k_2 (\{x\}) \cap F = \emptyset$.

Conversely, let $U$ be a $g\pi$-open subset of $(X, k_1)$ and let $x \in U$. Since $X\setminus U$ is a $g\pi$-closed subset of $(X, k_1)$ and $x \not\in X\setminus U$, $k_2 (\{x\}) \cap (X\setminus U) = \emptyset$. Consequently $k_2 (\{x\}) \subseteq U$.

Hence, $(X, k_1, k_2)$ is a $g\pi C_0$ bi-ˇCech space.

**Theorem 4.3** Let $\{(X_i, k_i^1, k_i^2) : i \in I\}$ be a family of bi-ˇCech closure spaces. If $\prod_{i=1}^\infty (X_i, k_i^1, k_i^2)$ is an $g\pi C_0$ bi-ˇCech space, then $(X_i, k_i^1, k_i^2)$ is an $g\pi C_0$ bi-ˇCech space for each $i \in I$.

Proof: Suppose that $\prod_{i=1}^\infty (X_i, k_i^1, k_i^2)$ is an $g\pi C_0$ bi-ˇCech space. Let $j \in I$ and let $G$ be an $g\pi$-open subset of $(X_j, k_j^1)$. Since $\prod_{i \in I, i \neq j} X_i$ is an $g\pi$-open subset of $\prod_{i=1}^\infty (X_i, k_i^1)$ such that $\prod_{i \in I, i \neq j} X_i \subseteq G \times \prod_{i \in I, i \neq j} X_i$.

Since $\prod_{i=1}^\infty (X_i, k_i^1, k_i^2)$ is an $g\pi C_0$ bi-ˇCech space, $\prod_{i=1}^\infty k_i^2 \prod_{i \in I, i \neq j} \{((x_i)_{i=1})\} \subseteq G \times \prod_{i \in I, i \neq j} X_i$.

Consequently, $k_j^2 \{x_j\} \subseteq G$.

Hence $(X_i, k_i^1, k_i^2)$ is an $g\pi C_0$ bi-ˇCech space.

**REFERENCES:**


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