International Journal of Mathematical Archive-2(11), 2011, Page: 2163-2165
Available online through www.ijma.info ISSN 2229 – 5046

Gπ- Closed Sets in Bi ˇCech Closure Spaces

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(Received on: 12-10-11; Accepted on: 30-10-11)

ABSTRACT

In this paper, we introduce the concepts of gπ- closed and gπ- open sets in BiCˇech closure space and study some of their properties.

Key words: *BiCˇech closure operator, BiCˇech closure spaces, BiCˇech gπ closed sets.*

2000 Mathematics subject classification: *54A05, 54E55.*

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1. INTRODUCTION:

Cech closure spaces were introduced by E. ˇCech [1]. In ˇ Cech's approach the operator satisfies idempotent condition among kuratowski axioms. This condition need not hold for every set A of X. When this condition is also true, the operator becomes topological closure operator. Thus the concept of closure space is the generalization of a topological space. In this paper, we introduce the concept of (k_1, k_2)- gπ closed sets, $_{g\pi}C_0$ bi- ˇ Cech space and study their basic properties.

2. PRELIMINARIES:

Definition: 2.1 [2] Two functions k_1 and k_2 from power set X to itself are called bi- ˇ Cech closure operators (simply biclosure operator) for X if they satisfy the follo wing properties.

(i) $k_1(\varphi) = \varphi$ and $k_2(\varphi) = \varphi$
(ii) $A \subset k_1(A)$ and $A \subset k_2(A)$ for any set $A \subset X$
(iii) $k_1(A \cup B) = k_1(A) \cup k_1(B)$ and $k_2(A \cup B) = k_2(A) \cup k_2(B)$ for any $A, B \subset X$ (X, k_1, k_2) is called bi- ˇ Cech closure space.

Example: 2.2 Let X = {a, b, c} and let k_1 and k_2 be defined as $k_1(\{a\}) = \{a\}$,
$k_1(\{b\}) = k_1(\{c\}) = k_1(\{b, c\}) = \{b, c\}$, $k_1(\{a, b\}) = k_1(\{a, c\}) = k_1(\{X\}) = X$, and $k_1(\varphi) = \varphi$.
$k_2(\{a\}) = \{a\}$, $k_2(\{b\}) = \{b, c\}$, $k_2(\{c\}) = k_2(\{a, c\}) = \{a, c\}$, $k_2(\{a, b\}) = k_2(\{b, c\}) = k_2\{X\}) = X$, $k_2(\varphi) = \varphi$.

Now, (X, k_1, k_2) is a bi- ˇ Cech closure space.

Definition: 2.3 [3] A subset A in a bi- ˇ Cech closure space (X, k_1, k_2) is said to be

1. k_i-regular open if $A = \mathrm{int}_{ki}(k_i(A))$, i = 1, 2
2. k_i -regular closed if $A = k_i(\mathrm{int}_{ki}(A))$, i = 1, 2
3. k_i-semi open if $A \subseteq k_i(\mathrm{int}_{ki}(A))$, i = 1, 2
4. k_i-semi closed if $\mathrm{int}_{ki}(k_i(A)) \subseteq A$, i = 1, 2

The finite union of k_{i-} regular open sets is said to be k_i π-open.

The complement of a k_i π -open set is said to be k_i π -closed. The smallest k_i π closed set containing A is called $k_i - \pi$ closure of A and it is denoted by $k_{\pi\, cli}(A)$.

3. (k_1, k_2) - gπ closed sets:

Definition: 3.1 A subset A is a bi- ˇ Cech closure space (X, k_1, k_2) is said to be (k_1, k_2)- gπ closed if $k_{\pi\, cl2}(A) \subseteq U$ whenever $A \subseteq U$ and U is k_1- open set in X.

Example: 3.2 In example 2.2, the (k_1, k_2) - gπ closed sets are X, φ, {a},{c},{a, b},{b, c},{a, c}.

Theorem: 3.3 If A and B are (k_1, k_2) - gπ closed sets and so is $A \cup B$.

Proof: Let A and B be two (k_1, k_2)- gπ closed sets. Let U be k_1-open set in X. Let $(A \cup B) \subseteq U$, we have $A \subseteq U$ and $B \subseteq U$. Then $k_{\pi\, cl2}(A) \subseteq U$ and $k_{\pi\, cl2}(B) \subseteq U$ implies $(k_{\pi\, cl2}(A) \cup k_{\pi\, cl2}(B)) \subseteq U$. Hence $k_{\pi\, cl2}(A \cup B) \subseteq U$.

Thus
$A \cup B$ is (k_1, k_2)- gπ closed set.

Theorem: 3.4 If A is (k_1, k_2)- gπ closed set, then $k_{\pi\, cl2}(A)$-A contains no nonempty k_1 - closed sets.

Proof: Let A be (k_1, k_2)- gπ closed. Let U be k_1 - closed contained in $k_{\pi\, cl2}(A)$-A.

Now, $U \subseteq k_{\pi\, cl2}(A)$ and $U \subseteq A^c$. Now, $U \subseteq A^c$ then $A \subseteq U^c$. Since U is k_1-closed. $k_{\pi\, cl2}(A) \subseteq U^c$. Consequently $U \subseteq [k_{\pi\, cl2}(A)]^c$. As $U \subseteq k_{\pi\, cl2}(A) \cap [k_{\pi\, cl2}(A)]^c = \varphi$, U = φ. Hence $k_{\pi\, cl2}(A)$-A contains no non-empty k_1-closed sets.

Theorem: 3.5 If A is (k_1, k_2)- gπ closed set, then $k_{\pi\, cl1}(x) \cap A \neq \varphi$ holds for each $x \in k_{\pi\, cl2}(A)$.

Proof: Let A be (k_1, k_2)- gπ closed set. Suppose $k_{\pi\, cl1}(x) \cap A = \varphi$, for some $x \in k_{\pi\, cl2}(A)$, we have $A \subseteq [k_{\pi\, cl1}(x)]^c$.

Now, $k_{\pi\, cl1}(x)$ is k_1- π-closed. Therefore $[k_{\pi\, cl1}(x)]^c$ is k_1- π open. Thus $[k_{\pi\, cl1}(x)]^c$ is k_1- open. Since A is (k_1, k_2)- gπ closed set, we have $k_{\pi\, cl2}(A) \subseteq [k_{\pi\, cl1}(x)]^c$ implies $k_{\pi\, cl2}(A) \cap k_{\pi\, cl1}(x) = \varphi$. Then $x \notin k_{\pi\, cl2}(A)$ is a contradiction.

Hence $k_{\pi\, cl1}(x) \cap A \neq \varphi$ holds for each $x \in k_{\pi\, cl2}(A)$.

Theorem: 3.6 Let (X, k_1, k_2) be bi- ˇ Cech closure space. For each x in X, {x} is k_1 - closed or $\{x\}^c$ is (k_1, k_2)- gπ closed set.

Proof: Let (X, k_1, k_2) be bi- ˇ Cech closure space. Suppose that {x} is not k_1- closed, $\{x\}^c$ is not k_1- open. Therefore, the only k_1- open set containing $\{x\}^c$ is X. Thus, $\{x\}^c \subseteq X$. Now, $k_{\pi\, cl2}[\{x\}^c] \subseteq k_{\pi\, cl2}(X) = X$.

Hence $\{x\}^c$ is (k_1, k_2) - gπ closed set.

Theorem: 3.7 Let A be (k_1, k_2) - gπ closed set and if A is k_1 - open then $A = k_{\pi\, cl2}(A)$.

Proof: Let A be (k_1, k_2) - gπ closed subset of a bi ˇ C ech closure spaces (X, k_1, k_2) and let A be k_1 - open set. Then $k_{\pi\, cl2}(A) \subseteq U$ whenever $A \subseteq U$ and U is k_1 - open set in X. Since A is k_1 - open and $A \subseteq A$, we have $k_{\pi\, cl2}(A) \subseteq A$. But always, $A \subseteq k_{\pi\, cl2}(A)$. Thus, $A = k_{\pi\, cl2}(A)$.

Theorem: 3.8 Let $A \subseteq Y \subseteq X$ and suppose that A is (k_1, k_2)- gπ closed in (X, k_1, k_2). Then A is (k_1, k_2) - gπ closed relative to Y.

Proof: Let S be any k_1 - open set in Y such that $A \subseteq S$. Then $S = U \cap Y$ for some U is k_1 - open in X.. Therefore $A \subseteq U \cap Y$ implies $A \subseteq U$. Since A is (k_1, k_2) - gπ closed set in X, we have $k_{\pi cl2}(A) \subseteq U$. Hence $Y \cap k_{\pi cl2}(A) \subseteq Y \cap U = S$. Thus A is ($k_1$, k_2) - gπ closed set relative to Y.

4. $_{g\pi}C_0$ bi-ˇC ech spaces:

Definition: 4.1 A bi- ˇ Cech closure space (X, k_1, k_2) is said to be a $_{g\pi}C_0$ bi-ˇ Cech space if for every gπ -open subset U of (X, k_1), $x \in U$ implies $k_2(\{x\}) \subseteq U$.

Theorem: 4.2 A bi- ˇ Cech closure space (X, k_1, k_2) is a $_{g\pi}$**C_0** bi-ˇC ech space if and only if for every gπ -closed subset F of (X, k_1) such that $x \notin F$, $k_2(\{x\}) \cap F = \varphi$.

Proof: Let F be a $g\pi$ -closed subset of (X, k_1) and let $x \notin F$. Since $x \in X-F$ and $X - F$ is a $g\pi$ -open subset of (X, k_1), $k_2(\{x\}) \subseteq X - F$. Consequently $k_2(\{x\}) \cap F = \varphi$.

Conversely, let U be a $g\pi$ -open subset of (X, k_1) and let $x \in U$. Since $X - U$ is a $g\pi$ -closed subset of (X, k_1) and $x \notin X - U$, $k_2(\{x\}) \cap (X - U) = \varphi$. Consequently $k_2(\{x\}) \subseteq U$.

Hence, (X, k_1, k_2) is a ${}_{g\pi}C_0$ bi- ˇ Cech space.

Theorem: 4.3 Let $\{(X_i, k_i^1, k_i^2): i \in I\}$ be a family of bi-ˇC ech closure spaces. If $\prod_{i \in I} (X_i, k_i^1, k_i^2)$ is an ${}_{g\pi}C_0$ bi- ˇ Cech space , then (X_i, k_i^1, k_i^2) is an ${}_{g\pi}C_0$ bi- ˇ Cech space for each $i \in I$.

Proof: Suppose that $\prod_{i \in I} (X_i, k_i^1, k_i^2)$ is an ${}_{g\pi}C_0$ bi- ˇ Cech space . Let $j \in I$ and let G be an $g\pi$ -open subset of (X_j, k_j^1) such that $x_j \in G$. Then $G \times \prod_{i \neq j, i \in I} X_i$ is an $g\pi$ -open subset of $\prod_{i \in I} (X, k_i^1)$ such that $(x_i)_{i \subset I} \in G \times \prod_{i \neq j, i \in I} X_i$

Since $\prod_{i \subset I} (X_i, k_i^1, k_i^2)$ is an ${}_{g\pi}C_0$ bi- ˇ Cech space, $\prod_{i \subset I} k_i^2 \pi_i(\{(x_i)_{i \subset I}\}) \subseteq G \times \prod_{i \neq j, i \in I} X_i$.

Consequently, $k_j^2\{x_j\} \subseteq G$.

Hence (X_i, k_i^1, k_i^2) is an ${}_{g\pi}C_0$ bi- ˇ Cech space.

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