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BOUNDEDNESS AND ASYMPTOTIC BEHAVIOR OF SECOND ORDER INTEGRODIFFERENTIAL EQUATION WITH RETARDED ARGUMENT

D. B. DHAIGUDE[1] & J. V. PATIL[2]*

[1]Department of Mathematics, Dr.Babasaheb Ambedkar Marathwada,University, Aurangabad – 431 004, India

E-mail: dnyanraja@gmail.com

[2]Department of Mathematics,Vasantrao Naik college, Aurangabad,431 005, India

E-mail: jv.patil29@gmail.com

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ABSRACT

The integral inequality with retarded argument is obtained and the qualitative properties such as boundedness and asymptotic behavior of solutions of certain class of second order integrodifferential equation with retarded argument is studied as an application of this integral inequality. The results are illustrated by suitable examples.

KEYWORDS: *integral inequality, retarded argument, asymptotic behavior.*

SUBJECT CLASSIFICATION: *26D15*

INTRODUCTION:

The purpose of the paper is to study the boundedness and asymptotic behavior of solutions of second order integrodifferential equations with retarded argument of the form,

$$(r(t)x^{'})^{'} + a(t)x = f[t, x(\alpha(t)), \int_{t_0}^{t} k(t,s,x(\alpha(s)ds)] \tag{1.1}$$

by comparing with the solutions of the second order linear differential equation,

$$(r(t)x^{'})^{'} + a(t)x = 0 \tag{1.2}$$

In equation (1.1) and (1.2) we will assume that $f \in C(I \times R \times R, R)$ where $I = [t_0, T)$ and R denotes the set of real numbers, $k(t,s,x(\alpha)) \in C(I \times I \times R, R)$, $r(t) > 0$, $r(t)$ and $a(t)$ are continuous function defined on $I = [t_0, T)$. $I(t) \in (I, R_+)$ be non increasing with $\alpha(t) = t - l(t) \geq 0$ on I . $l^{'}(t) < 1$, $l(t_0) = 0$. $\alpha(t) \in C^{'}(I, I)$, $\alpha(t) \leq t$ Boundedness and asymptotic behavior of solution of equations (1.1) and (1.2) with $r(t) = 1$, $f(t, x(\alpha(t))), \int_{t_0}^{t} k(t,s,x(\alpha(s)))ds = b(t)x$ have been previously studied in [2] and [3]. In recent years there have been several extensions of the results given in [2] and [3] see for examples [1],[4],[5],[8],[9], and some of the references given in.Recenty boundedness of solutions of particular case of integrodifferential equation of (1.1) have been studied by Pachpatte in [6].In this paper we study boundedness and asymptotic behaviour of (1.1) by using retarded integral inequality. We organize the paper as follows. Section 2 is devoted for linear integral inequality with retarded argument.This inequality is successfully applied to study the qualitative behavior such as boundedness and asymptotic behavior of solution of integrodifferential equation with retarded argument (1.1) in section 3 .The last section consists of illustrative examples .

****Corresponding author: J. V. PATIL[2]*,*E-mail: jv.patil29@gmail.com***

2 NONLINEAR INTEGRAL INEQUALITY WITH RETARDED ARGUMENT:

The integral inequality for p = 1 is establish by Pachpatte in [7,pp-149] Now we prove the following integral inequality with retarded argument for p, $p \in (0,1)$.

Theorem: 2.1 *Let* $u(t)$, $a(t)$ *and* $b(t)$ *be nonnegative continuous functions defined on* I *, and* $\alpha(t) \in C^1(I,I)$ *be nondecreasing with* $\alpha(t) \le t$ *on* $I = [t_0, T)$ *and* K *be a real constant.* suppose that the inequality,

$$u(t) = K + \int_{\alpha(t_0)}^{\alpha(t)} a(s)[u(s) + \int_{\alpha(t_0)}^{s} b(\sigma)u^p d\sigma]ds, for\, t \in I \tag{2.1}$$

where $p \in (0,1)$ is a constant . Then

$$u(t) \le K[1 + \int_{\alpha(t_0)}^{\alpha(t)} a(s)(E_0(s))^{\frac{1}{1-p}} exp\int_{\alpha(t_0)}^{s} a(s)ds] \tag{2.2}$$

where,

$$E_0(t) = 1 + K^{p-1}(1-p)\int_{\alpha(t_0)}^{\alpha(t)} b(s)exp(-(1-p)\int_{\alpha(t_0)}^{\alpha(s)} a(\sigma)d\sigma)ds \tag{2.3}$$

proof: Let $K > 0$ let us assume that $z(t)$ is right side of (2.1), $u(t) \le z(t)$

$$z'(t) = a(\alpha(t))[u(\alpha(t)) + \int_{\alpha(t_0)}^{\alpha(t)} b(\sigma)u^p(\sigma)d\sigma]\alpha'(t) \tag{2.4}$$

$$z'(t) \le a(\alpha(t))[z(\alpha(t)) + \int_{\alpha(t_0)}^{\alpha(t)} b(\sigma)z^p(\sigma)d\sigma]\alpha'(t)$$

$$z'(t) \le a(\alpha(t))[z(t) + \int_{\alpha(t_0)}^{\alpha(t)} b(\sigma)z^p(\sigma)d\sigma]\alpha'(t) \tag{2.5}$$

Let

$$v(t) = z(t) + \int_{\alpha(t_0)}^{\alpha(t)} b(\sigma)z^p(\sigma)d\sigma, \tag{2.6}$$

such that $z(t) \le v(t), v(t_0) = z(t_0) = K$,

Differentiating (2.6) with respective t,we have,

$$v'(t) = z'(t) + b(\alpha(t))z^p(\alpha(t))\alpha'(t)$$

$$v'(t) \le a(\alpha(t)v(t)\alpha'(t) + b(\alpha(t))\alpha'(t)v^p(t)$$

solving above inequality by Bernoulli's equation , we have ,

$$v^{-p}v'(t) - a(\alpha(t))\alpha'(t)v^{1-p}(t) \le b(\alpha(t))\alpha'(t) \tag{2.7}$$

solving above inequality by Bernoulli's equation , we have , Put,

$$m(t) = (v(t))^{1-p} \tag{2.8}$$

$$m'(t) = (1-p)v^{-p}v'(t)$$

$$m'(t) = (1-p)\frac{v'(t)}{v^p}$$

And $$\frac{m'(t)}{(1-p)} - a(\alpha(t))\alpha'(t)m(t) \le b(\alpha(t))\alpha'(t)$$

$$m(t_0) = (v(t_0))^{1-p} = K^{1-p}$$

As

$$\frac{m'(t)}{(1-p)} - a(\alpha(t))\alpha'(t)m(t) \le b(\alpha(t))\alpha'(t)$$

$$m'(t) - (1-p)a(\alpha(t))\alpha'(t)m(t) \le (1-p)b(\alpha(t))\alpha'(t)$$

multiply both sides by

$$exp(-(1-p)\int_{t_0}^{t} a(\alpha(s))\alpha'(s)ds)$$

we have

$$m'(t)exp(-(1-p)\int_{t_0}^{t} a(\alpha(s))\alpha'(s)ds) - (1-p)a(\alpha(t))\alpha'(t)m(t)exp(-(1-p)\int_{t_0}^{t} a(\alpha(s))\alpha'(s)ds)$$
$$\le (1-p)b(\alpha(t))\alpha'(t)exp(-(1-p)\int_{t_0}^{t} a(\alpha(s))\alpha'(s)ds)$$

$$\frac{d}{dt}[m(t)exp(-(1-p)\int_{t_0}^{t} a(\alpha(s))\alpha'(s)ds)] \le (1-p)b(\alpha(t))\alpha'(t)exp(-(1-p)\int_{t_0}^{t} a(\alpha(s))\alpha'(s))ds$$

Setting t=s and integrating above inequality from t_0 to t we have,

$$m(t)exp(-(1-p)\int_{t_0}^{t} a(\alpha(s))\alpha'(s)ds) - m(t_0) \le \int_{t_0}^{t} (1-p)b(\alpha(s))\alpha'(s)exp(-(1-p)\int_{t_0}^{s} a(\alpha(s))\sigma'(\sigma)d\sigma)ds$$

$$m(t) \le \{m(t_0) + (1-p)\int_{t_0}^{t} b(\alpha(s))\alpha'(s)exp(-(1-p)\int_{t_0}^{s} a(\alpha(\sigma))\alpha'(\sigma)d\sigma)ds\} \times exp(-(1-p)\int_{t_0}^{s} a(\alpha(s))\alpha'(s))ds$$

$$m(t) \le \{K^{-p+1} + (1-p)\int_{t_0}^{t} b(\alpha(s))\alpha'(s)exp(-(1-p)\int_{t_0}^{s} a(\alpha(\sigma))\alpha'(\sigma)d\sigma\})ds\} \times exp(-(1-p)\int_{t_0}^{s} a(\alpha(s))\alpha'(s))ds$$

By taking change of variable on right hand side of the above inequality,

$$m(t) \le \frac{\{1 + (1-p)K^{p-1}\int_{\alpha(t_0)}^{\alpha(t)} b(s)exp(-(1-p)\int_{\alpha(t_0)}^{s} a(\sigma)d\sigma)ds\} \times exp((1-p)\int_{\alpha(t_0)}^{\alpha(t)} a(s)ds)}{K^{p-1}}$$

$$m(t) \le K^{1-p}E_0(t)exp((1-p)\int_{\alpha(t_0)}^{\alpha(t)} a(s)ds)$$

$$E_0(t) = 1 + K^{p-1}(1-p)\int_{\alpha(t_0)}^{\alpha(t)} b(s)exp(-(1-p)\int_{\alpha(t_0)}^{s} a(\sigma)d\sigma)ds$$
$$As, m(t) = (v(t))^{1-p}$$
$$(v(t))^{1-p} \le K^{1-p}(E_0(t))exp(1-p)\int_{\alpha(t_0)}^{\alpha(t)} a(s)ds$$
$$(v(t))^{1-p} \le K^{1-p}(E_0(t))exp(1-p)\int_{\alpha(t_0)}^{\alpha(t)} a(s)ds$$
$$(v(t)) \le K(E_0(t))^{\frac{1}{1-p}} exp\int_{\alpha(t_0)}^{\alpha(t)} a(s)ds$$

As

$$z'(t) \le a(\alpha(t))v(t)\alpha'(t)$$

Taking t = s and integrating from t_0 to t and change of variable , we have

$$z(t) - z(t_0) \le \int_{t_0}^{t} a(\alpha(s))v(s)\alpha^{'}(s)ds$$

$$z(t) \le z(t_0) + \int_{t_0}^{t} a(\alpha(s))v(s)\alpha^{'}(s)ds$$

$$z(t) \le K + \int_{\alpha(t_0)}^{\alpha(t)} a(s)K(E_0(t))^{\frac{1}{1-p}} exp(\int_{\alpha(t_0)}^{s} a(\sigma)d\sigma)ds$$

As $u(t) \le z(t)$, we get desired result.

If $K \ge 0$,we carry out above procedure with $K+\varepsilon$ instead of K , where $\varepsilon > 0$ is arbitrary small constant , and subsequently pass the limit as $\varepsilon \to 0$ to obtain desired result.

REMARK: Put $\alpha(t) = t, \alpha(t_0) = 0$, we get the non linear inequality due to Pachpatte, [see,Theorem 2.7.1 in[5]]

In this section we discuss the applications of integral inequality obtain in section 2. The boundedness and asymptotic behaviour of solution of second order integrodifferential equation with retarded argument is studied.

3 APPLICATIONS:

Theorem: 3.1 . Let the function k and f in (1.1) satisfy the conditions,

$$| k(t,s,x) | \le h(s) | x |^{p} \tag{3.1}$$

$$| f(t,x,z) | \le g(t)(| x | + | z |) \tag{3.2}$$

where g and h are real valued continuous functions defined on I

and $\int_{t_0}^{t} g(s)ds < \infty$, $\int_{t_0}^{t} h(s)ds < \infty$ and let

$$M^{'} = \max_{t\in I} \frac{1}{1-l^{'}(t)}$$

$MM^{'2} = N$, let $\alpha(t) = t - l(t)$, $\alpha(t) \le t$. Let $\overline{a}(t) = Ng(t+l(t))$, $\overline{b}(t) = h(t+l(t))$. If $x_1(t)$ and $x_2(t)$ are bounded solutions of equation (1.2) then the corresponding solution x(t) of equation (1.1) is bounded on I .

Proof : Let $x_1(t)$ and $x_2(t)$ be solution of (1.2) such that ,

$$r(t)[x_1(t)x_2^{'}(t) - x_1^{'}(t)x_2(t)] = 1 \tag{3.3}$$

Suppose that x(t) is any solution of (1.1) then on using variation of constants formula x(t) may be expressed as,

$$x(t) = c_1x_1(t) + c_2x_2(t) + \int_{t_0}^{t} [x_1(s)x_2(t) - x_1(t)x_2(s)] \times f(s, x(\alpha(s)), \int_{t_0}^{s} k(s,\sigma, x(\alpha(\sigma)d\sigma))ds) \tag{3.4}$$

where c_1 and c_2 are constant . From (3.1),(3.2) and (3.4) we have ,

$$| x(t) | \le C + \int_{t_0}^{t} Mg(s) | x(\alpha(s)) | ds + \int_{t_0}^{t} Mg(s) \int_{t_0}^{s} (h(\sigma) | x(\alpha(\sigma) |^{p} \ d\sigma))ds \tag{3.5}$$

where C and M are upper bounds for $| c_1x_1(t) + c_2x_2(t) |$ and $| x_1(s)x_2(t) - x_1(t)x_2(s) |$ respectively . Making change of variable on right hand side of (3.5) we have ,

$$|x(t)|\le C+\int_{\alpha(t_0)}^{\alpha(t)} MM'\, g(s+l(s))|x(s)|ds+\int_{\alpha(t_0)}^{\alpha(t)} MM'^2\, g(s+l(s))(\int_{\alpha(t_0)}^{s} h(\sigma+l(\sigma))|x(\sigma)|^p\, d\sigma)ds \quad (3.6)$$

$$|x(t)|\le C+\int_{\alpha(t_0)}^{\alpha(t)} Ng(s+l(s))[|x(s)|+\int_{\alpha(t_0)}^{s} h(\sigma+l(\sigma))|x(\sigma)|^p\, d\sigma]ds$$

$$|x(t)|\le C+\int_{\alpha(t_0)}^{\alpha(t)} \overline{a}(s)[|x(s)|+\int_{\alpha(t_0)}^{s} \overline{b}(\sigma)|x(\sigma)|^p\, d\sigma]ds \quad (3.7)$$

Applying Theorem (2.1) in section 2 to (3.7)

$$|x(t)|\le C+[1+\int_{\alpha(t_0)}^{\alpha(t)} \overline{a}(s)(E_0(t))^{\frac{1}{1-p}} exp(\int_{\alpha(t_0)}^{s} \overline{a}(s))]ds$$

where

$$E_0(t)=1+C^{p-1}(1-p)\int_{\alpha(t_0)}^{\alpha(t)} \overline{b}(s)exp(-(1-p)\times\int_{\alpha(t_0)}^{s} \overline{a}(\sigma)d\sigma)ds$$

for $o<p<1$, $t\in I$.

Theorem 3.2 : Let the function k and f in (1.1) satisfy the conditions,

$$|k(t,s,x)|\le h(s)|x|^p \quad (3.8)$$

$$|(t,x,z)|\le g(t)(|x|+exp(-\beta t+l(t))|z|) \quad (3.9)$$

where $\beta>0$, g and h are real valued continuous functions defined on $I=[t_o,T)$ and $\int_{t_0}^{t} h(s)<\infty$, $\int_{t_0}^{t} g(s)<\infty$

If $x_1(t)$ and $x_2(t)$ be solution of (1.2) such that $|x_i(t)|\le M_i exp(-\beta t)$ $i=1,2$ where $M_i>0$ for i = 1,2 are constants and $\alpha(t)=t-l(t), l(t)\le t on$ I , satisfy all conditions stated in Theorem 2 .Let $\overline{a}(t)=Ng(s+l(s))exp(-2\beta s-\beta l(s))$ and $\overline{b}(t)=h(\sigma+l(\sigma))exp(-\beta\sigma)$ then the corresponding solution x(t) of (1.1) approach zero as $t\to\infty$.

Proof : Let $x_1(t)$ and $x_2(t)$ be solution of (1.2) such that,

$$r(t)[x_1(t)x_2'(t)-x_1'(t)x_2(t)]=1$$

Suppose that x(t) is any solution of (1.1) then on using variation of constants formula x(t) may be expressed as,

$$x(t)=c_1x_1(t)+c_2x_2(t)+\int_{t_0}^{t}[x_1(s)x_2(t)-x_1(t)x_2(s)]\times f(s,x(\alpha(s)),\int_{t_0}^{s} k(s,\sigma,x(\alpha(\sigma)d\sigma))ds)$$

where c_1 and c_2 are constant .

$$|x(t)|\le Cexp(-\beta t)+exp(-\beta t)\int_{t_0}^{t} Mg(s)exp(-\beta s)|x(\alpha(s))|ds$$
$$+exp(-\beta t)\int_{t_0}^{t} Mg(s)exp(-2\beta s+l(s))\times\int_{t_0}^{s} h(\sigma)|x(\alpha(\sigma)|^p\, d\sigma)ds$$

$$|x(t)|\le Cexp(-\beta t)+exp(-\beta t)\int_{t_0}^{t} Mg(s)exp(-2\beta s+\beta l(s))exp(\beta s-\beta l(s))|x(\alpha(s))|ds$$

$$+exp(-\beta t)\int_{t_0}^{t} Mg(s)exp(-2\beta s+\beta l(s))\times\int_{t_0}^{s} h(\sigma)exp(-\beta\sigma+\beta l(\sigma))exp(\beta\sigma-\beta l(\sigma))|x(\alpha(\sigma)|^p)d\sigma ds$$

$$exp(\beta t)|x(t)|\le C+\int_{t_0}^{t} Mg(s)exp(-2\beta s+\beta l(s))exp(\beta s-\beta l(s))|x(\alpha(s))|ds$$
$$+\int_{t_0}^{t} Mg(s)exp(-2\beta s+\beta l(s))\times\int_{t_0}^{s} h(\sigma)exp(-\beta\sigma+\beta l(\sigma))exp(\beta\sigma-\beta l(\sigma))|x(\alpha(\sigma))|^p\, d\sigma)ds$$

Making change of variable in above inequality, we have,

$$exp(\beta t)\,|\,x(t)\,|\leq C+\int_{\alpha(t_0)}^{\alpha(t)} MM'\,g(s+l(s))exp(-2\beta(s+l(s))+\beta exp(\beta s)\,|\,x(s)\,|\,ds$$
$$+\int_{\alpha(t_0)}^{\alpha(t)} MM'\,g(s+l(s))exp(-2\beta(s+l(s)+\beta l(s))$$
$$\times\int_{\alpha(t_0)}^{s} h(\sigma+l(\sigma))exp(-\beta\sigma)exp(\beta\sigma)\,|\,x(\sigma\,|^p\ d\sigma)ds$$

$$exp(\beta t)\,|\,x(t)\,|\leq C+\int_{\alpha(t_0)}^{\alpha(t)} Ng(s+l(s))exp(-2\beta s-\beta l(s))exp(\beta s)\,|\,x(\alpha(s))\,|\,ds+$$
$$\int_{\alpha(t_0)}^{\alpha(t)} Ng(s+l(s))exp(-2\beta s-\beta l(s))(\int_{\alpha(t_0)}^{s} h(\sigma+l(\sigma))exp(-\beta\sigma)exp(\beta\sigma)\,|\,x(\sigma)\,|^p\,|\,d\sigma)ds$$

$$exp(\beta t)\,|\,x(t)\,|\leq C+\int_{\alpha(t_0)}^{\alpha(t)} Ng(s+l(s))exp(-2\beta s-\beta l(s))[exp(\beta s)\,|\,x(\phi(s))\,|\times\int_{\alpha(t_0)}^{s} h(\sigma+l(\sigma))$$
$$exp(-\beta\sigma)exp(\beta\sigma)\,|\,x(\sigma)\,|^p\,|\,d\sigma]ds \tag{3.10}$$

$$exp(\beta t)\,|\,x(t)\,|\leq C+\int_{\alpha(t_0)}^{\alpha(t)} \overline{a}(s)exp(\beta s)[\,|\,x(s)\,|+\int_{\alpha(t_0)}^{s} exp(\beta t)\,|\,x(t)\,|\,\overline{b}(\sigma)exp(\beta\sigma)\,|\,x(\sigma)\,|^p\ d\sigma]ds \tag{3.11}$$

Taking $u(t)=exp(\beta t)\,|\,x(t)\,|$, applying Theorem 1 to inequality (3.11) we have ,

$$(E_0(t))=1+C^{1-p}(1-p)\int_{\alpha(t_0)}^{\alpha(t)} \overline{b}(s)exp(-(1-p)\int_{\alpha(t_0)}^{s}\overline{a}(\sigma))d\sigma)ds$$

$$exp(\beta t)\,|\,x(t)\,|\leq C[1+\int_{\alpha(t_0)}^{\alpha(t)}(E_0(s))^{\frac{1}{1-p}}]exp(\int_{\alpha(t_0)}^{\alpha(t)}\overline{a}(s)ds \tag{3.12}$$

for $o<p<1$, $t\in I$.

4 EXAMPLES:

In this section we discuss some simple examples to illustrate our results .

A :- consider the second order integrodifferential equation with retarded argument,

$$(1-t^2)x''-2tx'+2x=t^2x(t-\frac{3}{4t})+t^2\int_{t_0}^{t} s\sqrt{\ } x(s-\frac{3}{4s})\sin x(s-\frac{3}{4s}),\ p=\frac{1}{2} \tag{4.1}$$

for $0<t<1$ The corresponding second order linear differential equation is,

$$((1-t^2)x')'+2x=0 \tag{4.2}$$

where ,

$$f(t,x(\alpha(t)),\int_{t_0}^{t}k(t,s,x(\alpha(s))))=t^2x(t-\frac{3}{4t})+t^2\int_{t_0}^{t}s\sqrt{\ } x(s-\frac{3}{4s})\sin x(s-\frac{3}{4s})$$

$f(t,x,y)$ is continuous on the set $I\times R\times R$

$$|\,f(t,x(\alpha(t)),\int_{t_0}^{t}k(t,s,x(\alpha(s))))\,|=t^2[\,|\,x(t-\frac{3}{4t})\,|+|\int_{t_0}^{t}s\sqrt{\ } x(s-\frac{3}{4s})\sin x(s-\frac{3}{4s})\,|]$$

$$k(t,s,x(\alpha(s)))=s\sqrt{\ } x(s-\frac{3}{4s})\sin x(s-\frac{3}{4s})$$

k is continuous on $I\times I\times R$.

$$|k(t,s,x(\alpha(s)))| = s\,|\sqrt{x(s-\frac{3}{4s})}|$$

$p = \frac{1}{2}$

$$\alpha(t) = t - \frac{3}{4t}$$

$\alpha(t) \le t$, $g(t) = t^2$, $h(s) = s$, clearly g and h are continuous on $I = (0,1)$. The solutions of equation (4.2) are

$$x_1(t) = t$$

$$x_2(t) = \frac{t}{2}\log|\frac{1+t}{1-t}| - 1$$

which are bounded on I , $I = (0,1)$,

$$(1-x^2)W(x_1(t), x_1(t)) = 1$$

Observe that all the conditions of Theorem 3.1 are satisfied.Therefore by theorem 3.1 the solution x(t) of equation (1.1) is bounded on $I = (0,1)$

B: Consider the second order integrodifferential equation with retarded argument,

$$(exp(2t)x')' + 2exp(2t)x = \frac{1}{1+t^2}x(t-\frac{3}{4t}) + \frac{1}{1+t}\int_{t_0}^{t}\frac{1}{\sqrt{s}}exp(-t+\frac{3}{4t})\sqrt{x(s-\frac{3}{4s})}\cos x(s-\frac{3}{4s})ds] \quad (4.3)$$

for $t > 1$ and the corresponding second order linear differential equation ,

$$(exp(2t)x')' + 2exp(2t)x = 0 \quad (4.4)$$

$$f(t, x(\alpha(t))\int_{t_0}^{t}k(t,s,x(\alpha(s)))) = \frac{1}{1+t^2}x(t-\frac{3}{4t}) + \frac{1}{1+t}\int_{t_0}^{t}\frac{1}{\sqrt{s}}exp(-t+\frac{3}{4t})\sqrt{x(s-\frac{3}{4s})}\cos x(s-\frac{3}{4s})ds$$

$f(t,x,y)$ is continuous on the set $I \times R \times R$, $\beta = 1$

$$|f(t, x(\alpha(t)), \int_{t_0}^{t}k(t,s,x(\alpha(s)))| \le \frac{1}{1+t}[x(t-\frac{3}{4t}) + exp(-t+\frac{3}{4t})\int_{t_0}^{t}\frac{1}{\sqrt{s}}\sqrt{x(s-\frac{3}{4s})}\cos x(s-\frac{3}{4s})ds]$$

$$k(t,s,x(\alpha(t))) = \frac{1}{\sqrt{s}}\sqrt{x(s-\frac{3}{4s})}\cos x(s-\frac{3}{4s})$$

k is continuous on $I \times I \times R$.

$$|k(t,s,x(\alpha(t)))| \le \frac{1}{\sqrt{s}}|\sqrt{x(s-\frac{3}{4s})}|$$

Note that the functions ,

$$g(t) = \frac{1}{t+1}$$

and

$$h(t) = \frac{1}{\sqrt{s}}$$

. g and h are continuous functions on I. The equation (4.4) has the solutions

$$x_1(t) = exp(-t)\cos(t) \ , \ x_2(t) = exp(-t)\sin(t), t \in I$$

$$|x_1(t)| \le exp(-(t))$$

,

$$| x_2(t) | \leq exp(-(t))$$

$$W(x_1, x_2) = exp(-2(t))$$

$$exp(2(t))W(x_1, x_2) = 1$$

Observe that all the conditions of Theorem 3.2 are satisfied.Hence by applying Theorem 3.2 to solution x(t)of (1.1) $| x(t) | \to 0$ as $t \to \infty$ and the conclusion of theorem is true .

REFERENCES:

[1] J. S. Beadley,*Comparison Theorem for square integrability of solutions of (r(t)y')' +a(t)y=f(t,y),* Glasgow Math. J. 13(1), (1972),75-79

[2] R. Bellman, *Stability Thoery Of Differential Equaations* ,Mc-Graw-Hill, 1953.

[3] P. Hartman,*Ordinary Differential Equaations*,John Wiley, New York. 1964

[4] B. G. Pachpatte,*Boundedness And Asymptotic Behavior Of Second Order Integrodifferential Equations*, J. Math. Phy. Sci.92, (1975),171-175

[5] B. G.Pachpatte,*Inequalities For Differential and Integral equations,*Academic Press, 1998

[6] B. G Pachpatte,*On Some Retarded Integral Inequalities And Applications*,J.Inequal.Pure And Appl. Math.3(2) Art. 18, 2002

[7] B.G Pachpatte,Integral And Finite Differerce Inequalities And Applications, North Holland, Oxford,2006

[8] P. Waltman,*On Asymptotic Behavior Of Solutions Of A Nonlinear Equation*, Proc.Amer.Math. Soc.15,6,(1964), 918-923.

[9] P. K. Wong,*Bounds For Solutions To A Class Of Non Linear Second Order Differential Equations*, J. Differential Equation 7,1, (1970),139-146.

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