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STRONGLY ALMOST DOUBLE DIFFERENCE SEQUENCE SPACES

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ABSTRACT

In the present paper we introduce strongly almost double difference analytic and entire sequence spaces. We also study some topological properties and inclusion relations between these spaces.

Keywords: *entire sequence, analytic sequence, double sequence.*

AMS Classification: *40A05, 40C05, 40D05.*

INTRODUCTION AND PRELIMINARIES:

The initial works on double sequences is found in Bromwich [4]. Later on , it was studied by Hardy [7], Moricz [9], Moricz and Rhoades [10], Tripathy ([16], [17]), Basarir and Sonalcan [3] and many others. Hardy [7] introduced the notion of regular convergence for double sequences. Recently, Zeltser [19] in her Ph.D thesis has essentially studied both the theory of topological double sequence spaces and the theory of summability of double sequences. Mursaleen and Edely [12] have recently introduced the statistical convergence and Cauchy convergence for double sequences and given the relation between statistical convergent and strongly Cesaro summable double sequences. Nextly, Mursaleen [11], Mursaleen and Edely [13] have defined the almost strong regularity of matrices for double sequences and apply these matrices to establish a core theorem and introduced the M-core for double sequences and determined those four dimensional matrices transforming every bounded double sequences x = (x_{mn}) into one whose core is a subset of the M-core of x. More recently, Altay and Basarir [1] have defined the spaces $BS, BS(t), CS_p, CS_{bp}, CS_r$ and BV of double sequences consisting of all double series whose sequence of partial sums are in the spaces $M_u, M_u(t), C_p, C_{bp}, C_r$ and L_u respectively and also examined some properties of these sequence spaces and determined the α- duals of the spaces BS, BV, CS_{bp} and the $\beta(v)$-duals of the spaces CS_{bp} and CS_r of double series. Now, recently Basar and Sever [2] have introduced the Banach space L_q of double sequences corresponding to the well known space l_q of single sequences and examined some properties of the space L_q. The class of sequences which are strongly Cesaro summable with respect to a modulus was introduced by Maddox [8] as an extension of the definition of strongly Cesaro summable sequences. Connor [5] further extended this notion to strong A-summability with respect to a modulus where $A = (a_{n,k})$ is a non-negative regular matrix. Using the definition Connor established connections between strong A-summability, strong A-summability with respect to a modulus and A-statistical convergence. In 1900, Pringsheim [14] presented a definition for convergence of double sequences. Following Pringsheim work, Hamilton and Robison [6] and [15], respectively presented a series of necessary and sufficient conditions on the entries of $A = (a_{m,n,k,l})$ that ensure the preservation of Pringsheim type convergence on the following transformation of double sequences $(Ax)_{m,n} = \sum_{k,l=0,0}^{\infty,\infty} a_{m,n,k,l}x_{k,l}$.

Let w^2 denote the set of all complex double sequences. A sequence x = (x_{mn}) is said to be prime sense double analytic if $sup_{mn}\, |x_{mn}|^{1/m+n} < \infty$. The vector space of all prime sense double analytic sequences is denoted by Λ^2. A sequence x = (x_{mn}) is said to be prime sense double entire sequence if $|x_{mn}|^{1/m+n} \to 0 \; as \; m,n \to \infty$. The vector space of all prime sense double entire sequences is denoted by Γ^2. The spaces Λ^2 and Γ^2 are metric spaces with the metric

$d(x,y) = sup_{mn}\{|x_{mn} - y_{mn}|^{1/m+n} : m,n = 1,2,3,\dots\}$ for all x = (x_{mn}) and y = (y_{mn}) in Γ^2.

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The double difference sequence spaces defined by

$Z(\Delta) = \{x = x_{mn} \in w^2 : (\Delta x_{mn}) \in Z\}$ where $Z = \Lambda^2, \Gamma^2$ and

$\Delta x_{mn} = (x_{mn} - x_{m,n+1}) - (x_{m+1,n} - x_{m+1,n+1}) = (x_{mn} - x_{m,n+1} - x_{m+1,n} + x_{m+1,n+1})$.

Let X be a linear metric space. A function $p: X \to \mathbb{R}$ is called paranorm, if

(1) $p(x) \geq 0$, for all $x \in X$,
(2) $p(-x) = p(x)$, for all $x \in X$,
(3) $p(x+y) \leq p(x) + p(y)$, for all $x, y \in X$,
(4) if (λ_n) is a sequence of scalars with $\lambda_n \to \lambda_n$ as $n \to \infty$ and (x_n) is a sequence of vectors with $p(x_n - x) \to 0$ as $n \to \infty$, then $p(\lambda_n x_n - \lambda x) \to 0$ as $n \to \infty$.

A paranorm p for which $p(x) = 0$ implies $x = 0$ is called total paranorm and the pair (X, p) is called a total paranormd space. It is well known that the metric of any linear metric space is given by some total paranorm (see [18], Theorem 10.4.2, p-183).

Let A = (a_{mn}^{jk}) be an four-dimensional infinite regular matrix of non-negative complex numbers and p = (p_{mn}) be a sequence of positive real numbers. Let $\mu = (\lambda_{mn})$ be a multiplier sequence, then we define the following sequence spaces in the present paper:

$\Gamma^2[\mathrm{A}, \Delta_s^r, \mu, p] = \left\{x = (x_{mn}) \in w^2 : lim_{j,k \to \infty} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left\{a_{mn}^{jk} \left(|\lambda_{mn} \Delta_s^r(x_{mn})|^{1/m+n}\right)^{p_{mn}}\right\} = 0\right\}$

$\Lambda^2[\mathrm{A}, \Delta_s^r, \mu, p] = \left\{x = (x_{mn}) \in w^2 : sup_{j,k} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left\{a_{mn}^{jk} \left(|\lambda_{mn} \Delta_s^r(x_{mn})|^{1/m+n}\right)^{p_{mn}}\right\} < \infty\right\}$.

The following inequality will be used throughout the paper. Let p = (p_{mn}) be a bounded sequence of positive real numbers. Let H = $\sup_{mn}$ and

D = max $(1, 2^{H-1})$. Then we have

$$|a_{mn} + b_{mn}|^{p_{mn}} \leq D(|a_{mn}|^{p_{mn}} + |b_{mn}|^{p_{mn}}) \quad (1)$$

The main purpose of the paper is to introduce some double analytic and entire sequence spaces. We also make an effort to study some topological properties and inclusion relations between above defined spaces.

MAIN RESULTS:

Theorem 2.1: Let A = (a_{mn}^{jk}) be a non-negative matrix and p = (p_{mn}) be a bounded sequence of positive real numbers. Then

(1) $\Gamma^2[\mathrm{A}, \Delta_s^r, \mu, p]$ and $\Lambda^2[\mathrm{A}, \Delta_s^r, \mu, p]$ are linear spaces over the field $\mathbb{C}$.
(2) $\Gamma^2[\mathrm{A}, \Delta_s^r, \mu, p] \subset \Lambda^2[\mathrm{A}, \Delta_s^r, \mu, p]$.

Proof: It is easy to prove so we omit the details.

Theorem 2.2: Let $N_1 = \min\{n_0 : sup_{m,n \geq n_0} \{a_{mn}^{jk}(|\lambda_{mn} \Delta_s^r x_{mn}|^{1/m+n})^{p_{mn}} < \infty\}\}$ and

$N_2 = \min\{n_0 : sup_{m,n \geq n_0} \; p_{mn} < \infty\}$ where $N = \max(N_1, N_2)$

(a) $\Gamma^2[\mathrm{A}, \Delta_s^r, \mu, p]$ is a paranormed space with

$$g(x) = lim_{N \to \infty} sup_{m,n \geq N} \{a_{mn}^{jk}(|\lambda_{mn} \Delta_s^r x_{mn}|^{1/m+n})^{p_{mn}}\}^{1/M} \quad (2)$$

if and only if $\gamma > 0$, where $\gamma = lim_{N \to \infty} inf_{m,n \geq N} p_{mn}$ and M = max $(1, sup_{m,n \geq N} p_{mn})$.

(b) $\Gamma^2[\mathrm{A}, \Delta_s^r, \mu, p]$ is complete with the paranorm (2).

Proof: (a) Let $\Gamma^2[A, \Delta_s^r, \mu, p]$ is a paranormed space with (2) and suppose that $\gamma = 0$. Then $\alpha = inf_{m,n\geq N} p_{mn} = 0$ for all N ∈ ℕ and hence we obtain

g(ξ x) $= lim_{N\to\infty} sup_{m,n\geq N} |\xi|^{p_{mn}/M} = 1$ for all $\xi \in (0,1]$, where x = (α) ∈ $\Gamma^2[A, \Delta_s^r, \mu, p]$. When $\xi \to 0$ does not imply that $\xi x \to 0$, when x is fixed. This is a contradiction that (2) to be a paranorm.

Conversely, let $\gamma > 0$. It is trivial that g(0) = 0, g(-x) = g(x) and g(x + y) ≤ g(x) + g(y). Since $\gamma > 0$, there exists a positive number β such that $p_{mn} > \beta$ for sufficiently large positive integer m, n. Hence for any $\xi \in \mathbb{C}$, we may write $|\xi|^{p_{mn}} \leq \max(|\xi|^M, |\xi|^\beta)$ for sufficiently large positive integers m, n > N. Therefore, we obtain that $g(\xi x) \leq \max(|\xi|, |\xi|^{\beta/M})\, g(x)$, using this one can prove that $\xi x \to 0$ whenever x is fixed and $\xi \to 0$ or $\xi \to 0$ and $x \to 0$, or ξ is fixed and $x \to 0$. This completes the proof of the theorem.

Let (x^{il}) be a Cauchy sequence in $\Gamma^2[A, \Delta_s^r, \mu, p]$, where $x^{il} = (x_{mn}^{il})_{mn\in\mathbb{N}}$. Then for every $\varepsilon > 0$ $(0 < \varepsilon < 1)$ there exists a positive integer s_0 such that

$$g(x^{il} - x^{rt}) = lim_{N\to\infty} sup_{m,n\geq N} \left\{a_{mn}^{jk}\left(\left|\lambda_{mn}\Delta_s^r(x_{mn}^{il} - x_{mn}^{rt})\right|^{1/m+n}\right)^{p_{mn}}\right\}^{1/M} < \varepsilon/2 \tag{3}$$

$or\ all\ i, l, r, t > s_0$.

From (3) there exists a positive integer n_0 such that

$$sup_{m,n\geq N} \left\{a_{mn}^{jk}\left(\left|\lambda_{mn}\Delta_s^r(x_{mn}^{il} - x_{mn}^{rt})\right|^{1/m+n}\right)^{p_{mn}}\right\}^{1/M} < \varepsilon/2\ for\ all\ i, l, r, t > s_0\ and\ for\ N > n_0.$$

Hence

$$\left\{a_{mn}^{jk}\left(\left|\lambda_{mn}\Delta_s^r(x_{mn}^{il} - x_{mn}^{rt})\right|^{1/m+n}\right)^{p_{mn}}\right\}^{1/M} < \varepsilon/2 < 1 \tag{4}$$

so that

$$\left\{a_{mn}^{jk}\left(\left|\lambda_{mn}\Delta_s^r(x_{mn}^{il} - x_{mn}^{rt})\right|^{1/m+n}\right)^{p_{mn}}\right\} < \left\{a_{mn}^{jk}\left(\left|\lambda_{mn}\Delta_s^r(x_{mn}^{il} - x_{mn}^{rt})\right|^{1/m+n}\right)^{p_{mn}}\right\}^{1/M} < \varepsilon/2 \tag{5}$$

$for\ all\ i, l, r, t > s_0\ and\ m, n, j, k > n_0$.

This implies that $(x_{mn}^{il})_{mn\in\mathbb{N}}$ is a Cauchy sequence in ℂ for each fixed $m, n, j, k > n_0$. Hence the sequence $(x_{mn}^{il})_{mn\in\mathbb{N}}$ convergent to x_{mn} say,

$$lim_{i,l\to\infty} x_{mn}^{il} = x_{mn}\ for\ each\ fixed\ m, n > n_0 \tag{6}$$

Getting x_{mn}, we define $x = (x_{mn})$. From (3), we get $s_1 > 0$ such that

$$g(x^{il} - x) = lim_{N\to\infty} sup_{m,n\geq N} \left\{a_{mn}^{jk}\left(\left|\lambda_{mn}\Delta_s^r(x_{mn}^{il} - x_{mn}^{rt})\right|^{1/m+n}\right)^{p_{mn}}\right\}^{1/M} < \varepsilon/2 \tag{7}$$

as r, t → ∞ for all i, l > s_1. Thus $lim_{i,l\to\infty} x^{il} = x$.

Now we show that x = (x_{mn}) ∈ $\Gamma^2[A, \Delta_s^r, \mu, p]$. Since $x^{il} \in \Gamma^2[A, \Delta_s^r, \mu, p]$ for each $(k, l) \in \mathbb{N} \times \mathbb{N}$, for each $\varepsilon > 0$ $(0 < \varepsilon < 1)$ there exists a positive integer $n_1 \in \mathbb{N}$ such that

$$\left\{a_{mn}^{jk}\left(\left|\lambda_{mn}\Delta_s^r(x_{mn}^{il})\right|^{1/m+n}\right)^{p_{mn}}\right\}^{1/M} < \varepsilon\ for\ every\ m, n > n_1 \tag{8}$$

By (7), (8) and using inequality (1), we get

$$\begin{aligned}\left\{a_{mn}^{jk}\left(|\lambda_{mn}\Delta_s^r(x_{mn})|^{1/m+n}\right)^{p_{mn}}\right\}^{1/M} &\le \left\{a_{mn}^{jk}\left(\left|\lambda_{mn}\Delta_s^r(x_{mn}^{il})\right|^{1/m+n}\right)^{p_{mn}}\right\}^{1/M} + \left\{a_{mn}^{jk}\left(\left|\lambda_{mn}\Delta_s^r(x_{mn}^{il} - x_{mn})\right|^{1/m+n}\right)^{p_{mn}}\right\}^{1/M} \\ &\le \varepsilon/2 + \varepsilon/2 = \varepsilon\end{aligned}$$

for all i, l > max (s_0, s_1) and m, n > max (n_0, n_1). Therefore x $\in$ Γ^2[A, Δ_s^r, μ, p]. This completes the proof of the theorem.

Theorem 2.3: Let $0 \le p_{mn} \le q_{mn}$ and let $\left\{\frac{q_{mn}}{p_{mn}}\right\}$ be bounded. Then Γ^2[A, Δ_s^r, μ, q] $\subset$ Γ^2[A, Δ_s^r, μ, p].

Proof: Let x $\in$ Γ^2[A, Δ_s^r, μ, q]. Then

$$\left\{a_{mn}^{jk}\left(|\lambda_{mn}\Delta_s^r(x_{mn})|^{1/m+n}\right)^{q_{mn}}\right\} \to 0, as\ m,n \to \infty \tag{9}$$

Let $t_{mn} = \left\{a_{mn}^{jk}\left(|\lambda_{mn}\Delta_s^r(x_{mn})|^{1/m+n}\right)^{q_{mn}}\right\}$ and $\gamma_{mn} = p_{mn}/q_{mn}$. Since $p_{mn} \le q_{mn}$, we have $0 \le \gamma_{mn} \le 1$. Take $0 < \gamma < \gamma_{mn}$. Now we define two sequences u_{mn} and v_{mn} as

$$u_{mn} = \begin{Bmatrix} t_{mn}, & if\ (t_{mn} \ge 1) \\ 0, & if\ (t_{mn} < 1)\end{Bmatrix}$$

and

$$v_{mn} = \begin{Bmatrix} 0, & if\ (t_{mn} \ge 1) \\ t_{mn}, & if\ (t_{mn} < 1)\end{Bmatrix}. \tag{10}$$

Then

$$t_{mn} = u_{mn} + v_{mn}, \qquad t_{mn}^{\gamma_{mn}} = u_{mn}^{\gamma_{mn}} + v_{mn}^{\gamma_{mn}}.$$

It follows that

$$u_{mn}^{\gamma_{mn}} \le u_{mn} \le t_{mn}, \quad v_{mn}^{\gamma_{mn}} \le v_{mn}^{\gamma} \tag{11}$$

Since $t_{mn}^{\gamma_{mn}} = u_{mn}^{\gamma_{mn}} + v_{mn}^{\gamma_{mn}},\ then\ t_{mn}^{\gamma_{mn}} \le t_{mn} + v_{mn}^{\gamma}$.

$$\left\{a_{mn}^{jk}\left(|\lambda_{mn}\Delta_s^r(x_{mn})|^{1/m+n}\right)^{q_{mn}}\right\}^{\gamma_{mn}} \le \left\{a_{mn}^{jk}\left(|\lambda_{mn}\Delta_s^r(x_{mn})|^{1/m+n}\right)\right\}^{q_{mn}}$$

$$\left\{a_{mn}^{jk}\left(|\lambda_{mn}\Delta_s^r(x_{mn})|^{1/m+n}\right)^{q_{mn}}\right\}^{p_{mn}/q_{mn}} \le \left\{a_{mn}^{jk}\left(|\lambda_{mn}\Delta_s^r(x_{mn})|^{1/m+n}\right)\right\}^{q_{mn}} \tag{12}$$

$$\left\{a_{mn}^{jk}\left(|\lambda_{mn}\Delta_s^r(x_{mn})|^{1/m+n}\right)\right\}^{p_{mn}} \le \left\{a_{mn}^{jk}\left(|\lambda_{mn}\Delta_s^r(x_{mn})|^{1/m+n}\right)\right\}^{q_{mn}}$$

But

$$\left\{a_{mn}^{jk}\left(|\lambda_{mn}\Delta_s^r(x_{mn})|^{1/m+n}\right)\right\}^{q_{mn}} \to 0, as\ m,n \to \infty \qquad (by\ \ 9)$$

Therefore

$$\left\{a_{mn}^{jk}\left(|\lambda_{mn}\Delta_s^r(x_{mn})|^{1/m+n}\right)\right\}^{p_{mn}} \to 0, as\ m,n \to \infty$$

Hence x $\in$ Γ^2[A, Δ_s^r, μ, p].

Thus Γ^2[A, Δ_s^r, μ, q] $\subset$ Γ^2[A, Δ_s^r, μ, p]. This completes the proof of the theorem.

Theorem 2.4: Let A be a non-negative regular matrix and p = (p_{mn}) be such that $0 < \inf p_{mn} \le p_{mn} \le H = \sup p_{mn}$. Then $X(\Delta_s^r,\ \mu) \subseteq \Lambda^2[\mathrm{A}, \Delta_s^r, \mu, p]$ for $X = l_\infty^2$, where

$$X(\Delta_s^r, \mu) = \{x = (x_{mn}) : \lambda_{mn}\Delta_s^r(x_{mn}) \in X\}.$$

Proof: Let $\in l_\infty^2(\Delta_s^r,\ \mu)$, then there exists K > 0, such that $|\lambda_{mn}\Delta_s^r(x_{mn})| \le K$, for all $m, n \in \mathbb{N}$, we get

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} a_{mn}^{jk}\ |\lambda_{mn}\Delta_s^r(x_{mn})|^{p_{mn}} \le \max\{K^h, K^H\} \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} a_{mn}^{jk} < \infty.$$

Thus $x \in \Lambda^2[\mathrm{A}, \Delta_s^r, \mu, p]$. Hence $l_\infty^2(\Delta_s^r,\ \mu) \subset \Lambda^2[\mathrm{A}, \Delta_s^r, \mu, p]$.

Theorem 2.5:

(1) Let $0 < \inf p_{mn} \le p_{mn} \le 1$. Then $\Gamma^2[\mathrm{A}, \Delta_s^r, \mu, p] \subset \Gamma^2[\mathrm{A}, \Delta_s^r, \mu]$.
(2) Let $1 \le p_{mn} \le \sup p_{mn} < \infty$. Then $\Gamma^2[\mathrm{A}, \Delta_s^r, \mu] \subset \Gamma^2[\mathrm{A}, \Delta_s^r, \mu, p]$.

Proof: (1) Let $x \in \Gamma^2[\mathrm{A}, \Delta_s^r, \mu, p]$. Then

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left\{a_{mn}^{jk}\left(|\lambda_{mn}\Delta_s^r(x_{mn})|^{1/m+n}\right)\right\}^{p_{mn}} \to 0, as\ j, k \to \infty$$

This implies that

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} a_{mn}^{jk}\ |\lambda_{mn}\Delta_s^r(x_{mn})|^{1/m+n} \to 0, as\ j, k \to \infty$$

Thus $x \in \Gamma^2[\mathrm{A}, \Delta_s^r, \mu]$. Hence $\Gamma^2[\mathrm{A}, \Delta_s^r, \mu, p] \subset \Gamma^2[\mathrm{A}, \Delta_s^r, \mu]$. This completes the proof.

(2)Let $p_{mn} \ge 1$ for each m, n and $\sup p_{mn} < \infty$. suppose $x \in \Gamma^2[\mathrm{A}, \Delta_s^r, \mu]$, then

$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} a_{mn}^{jk}\ |\lambda_{mn}\Delta_s^r(x_{mn})|^{1/m+n} \to 0, as\ j, k \to \infty$ (13)

Since $1 \le p_{mn} \le \sup p_{mn} < \infty$, we have

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left\{a_{mn}^{jk}\left(|\lambda_{mn}\Delta_s^r(x_{mn})|^{1/m+n}\right)\right\}^{p_{mn}} \le \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} a_{mn}^{jk}\ |\lambda_{mn}\Delta_s^r(x_{mn})|^{1/m+n}$$

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left\{a_{mn}^{jk}\left(|\lambda_{mn}\Delta_s^r(x_{mn})|^{1/m+n}\right)\right\}^{p_{mn}} \to 0, as\ j, k \to \infty \quad (using\ 13)$$

Thus $x \in \Gamma^2[\mathrm{A}, \Delta_s^r, \mu, p]$. This completes the proof of the theorem.

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