AN APPLICATION OF FUZZY SOFT SETS IN DECISION MAKING PROBLEMS USING FUZZY SOFT MATRICES

1Tridiv Jyoti Neog*

1Research Scholar, Department of Mathematics, CMJ University, Shillong, Meghalaya,
E-mail: tridivjyoti@gmail.com

2Dusmanta Kumar Sut

2Assistant Professor, Department of Mathematics, N.N.Saikia College, Jorhat, Assam,
E-mail: sutdk001@yahoo.com

(Received on: 10-11-11; Accepted on: 24-11-11)

ABSTRACT
Application of fuzzy soft sets in decision making problems is one of the most recent topics developed while trying to solve real life problems in an imprecise environment. It has been accepted that for the complement of a fuzzy soft set as initiated by Maji, the set theoretic axioms of contradiction and exclusion are not valid. In this context, Neog and Sut put forward a new definition of complement of a fuzzy soft set and showed that all the properties of complement of a set in classical sense are satisfied by fuzzy soft sets also according to the proposed definition of complement. In this paper, we have applied our new notion of complement of a fuzzy soft set to obtain the solution of a decision problem in an imprecise environment. Our work is an attempt to apply our notions of fuzzy soft complement and fuzzy soft matrix together with a new fuzzy matrix operation “addition” in a decision making problem.

Keywords: Soft set, fuzzy soft set, fuzzy matrix, fuzzy soft matrix, fuzzy soft complement, membership value matrix.

1. INTRODUCTION:
In the Zadehian theory of fuzzy sets [11], the classical set theoretic axioms of contradiction and exclusion are not satisfied. H. K. Baruah [1] reintroduced the notion of fuzzy sets in a different way and put forward an extended definition of fuzzy sets. According to him, two functions, namely fuzzy membership function and fuzzy reference function are necessary to represent a fuzzy set completely. The fuzzy membership value is the difference between the fuzzy membership function and the fuzzy reference function. Accordingly, the definitions of union and intersection of fuzzy sets have been proposed. The notion of complement of a fuzzy set has been given a new look and using this new definition of complement, the classical set theoretic axioms of contradiction and exclusion are found to be valid for fuzzy sets also [1]. Neog and Sut [6,7] further modified the notions initiated by Baruah [1] to avoid degenerate cases and generalized the notion of union and intersection of fuzzy sets taken over different universes [10].

In 1999, Molodstov [5] initiated the notion of soft sets and later in 2003, Maji [3] put forward a hybrid model known as fuzzy soft set, which is a combination of soft set and fuzzy set. Neog and Sut [7,8] showed with counter examples that the axioms of contradiction and exclusion are not valid in case of soft sets and fuzzy soft sets if we use the notion of complement initiated by Maji [3,4]. Accordingly, in [7,8], Neog and Sut put forward new definitions of complement of soft sets and fuzzy soft sets and showed that all the properties of complement of a set are satisfied by soft sets and fuzzy soft sets according to the proposed definition of complement.

Matrices play an important role in the broad area of science and engineering. However, the classical matrix theory sometimes fail to solve the problems involving uncertainties, occurring in an imprecise environment. In [9], Neog and Sut proposed a matrix representation of a fuzzy soft set and successfully applied the proposed notion of fuzzy soft matrix in medical diagnosis using fuzzy soft complement initiated in [7]. In this paper, we have defined the “addition operation” for fuzzy matrices and an attempt has been made to apply our notion in solving a decision problem.

2. PRELIMINARIES:
In this section we furnish below some earlier works on soft sets, fuzzy soft sets and fuzzy soft matrices in brief.

*Corresponding author: 1Tridiv Jyoti Neog*, *E-mail: tridivjyoti@gmail.com*
Definition: 2.1[5] A pair \((F, E)\) is called a soft set (over \(U\)) if and only if \(F\) is a mapping of \(E\) into the set of all subsets of the set \(U\). In other words, the soft set is a parameterized family of subsets of the set \(U\). Every set \(F(\varepsilon), \varepsilon \in E\), from this family may be considered as the set of \(\varepsilon\) - elements of the soft set \((F, E)\), or as the set of \(\varepsilon\) - approximate elements of the soft set.

Definition: 2.2[3] A pair \((F, A)\) is called a fuzzy soft set over \(U\) where \(F : A \rightarrow \tilde{P}(U)\) is a mapping from \(A\) into \(\tilde{P}(U)\). 

Definition: 2.3[7] A fuzzy soft set \((F, A)\) over \(U\) is said to be null fuzzy soft set (with respect to the parameter set \(A\)), denoted by \(\tilde{\varnothing}\) if \(\forall \varepsilon \in A, F(\varepsilon)\) is the null fuzzy set \(\varnothing\).

Definition: 2.4[7] A fuzzy soft set \((F, A)\) over \(U\) is said to be absolute fuzzy soft set (with respect to the parameter set \(A\)), denoted by \(\tilde{A}\) if \(\forall \varepsilon \in A, F(\varepsilon)\) is the absolute fuzzy set \(U\).

Definition: 2.5[7] The complement of a fuzzy soft set \((F, A)\) is denoted by \((F, A)^\prime\) and is defined by \((F, A)^\prime = (F^\prime, A)\) where \(F^\prime : A \rightarrow \tilde{P}(U)\) is a mapping given by \(F^\prime(\alpha) = [F(\alpha)]^\prime\), \(\forall \alpha \in A\).

Definition: 2.6[9] (Matrix Representation of a Fuzzy Soft Set) Let \(U = \{c_1, c_2, c_3, \ldots, c_m\}\) be the universal set and \(E\) be the set of parameters given by \(E = \{e_1, e_2, e_3, \ldots, e_n\}\). Then the fuzzy soft set \((F, E)\) can be expressed in matrix form as \(A = [a_{ij}]_{m \times n}\) or simply by \([a_{ij}]\), \(i = 1, 2, 3, \ldots, m; j = 1, 2, 3, \ldots, n\) and \(a_{ij} = ([\mu_{ij}(c_1), \mu_{ij}(c_2)]); \) where \(\mu_{ij}(c_1)\) and \(\mu_{ij}(c_2)\) represent the fuzzy membership function and fuzzy reference function respectively of \(c_i\) in the fuzzy set \(F(e_j)\). We shall identify a fuzzy soft set with its fuzzy soft matrix and use these two concepts interchangeably. The set of all \(m \times n\) fuzzy soft matrices over \(U\) will be denoted by \(FSM_{m \times n}\).

Example: 2.1[9] Let \(U = \{c_1, c_2, c_3, c_4\}\) be the universal set and \(E\) be the set of parameters given by \(E = \{e_1, e_2, e_3\}\). We consider a fuzzy soft set

\[
(F, E) = \{F(e_1) = \{(c_1, 0,7,0), (c_2, 0,1,0), (c_3, 0,2,0), (c_4, 0,6,0)\}, F(e_2) = \{(c_1, 0,8,0), (c_2, 0,6,0), (c_3, 0,1,0), (c_4, 0,5,0)\}, F(e_3) = \{(c_1, 0,1,0), (c_2, 0,2,0), (c_3, 0,7,0), (c_4, 0,3,0)\}\}
\]

We would represent this fuzzy soft set in matrix form as

\[
[a_{ij}]_{3 \times 3} = \begin{bmatrix}
(0,7,0) & (0,8,0) & (0,1,0) \\
(0,1,0) & (0,6,0) & (0,2,0) \\
(0,2,0) & (0,1,0) & (0,7,0) \\
(0,6,0) & (0,5,0) & (0,3,0)
\end{bmatrix}_{3 \times 3}
\]

Definition: 2.7[9] We define the membership value matrix corresponding to the matrix \(A\) as \(MV(A) = [\delta_{(a_{ij})}]_{m \times n}\), where \(\delta_{(a_{ij})} = \mu_{ij}(c_1) - \mu_{ij}(c_2)\) \(\forall i = 1, 2, 3, \ldots, m\) and \(j = 1, 2, 3, \ldots, n\), where \(\mu_{ij}(c_1)\) and \(\mu_{ij}(c_2)\) represent the fuzzy membership function and fuzzy reference function respectively of \(c_i\) in the fuzzy set \(F(e_j)\).

3. FUZZY SOFT SETS IN DECISION MAKING:

Definition: 3.1 Let \(U = \{c_1, c_2, c_3, \ldots, c_m\}\) be the universal set and \(E\) be the set of parameters given by \(E = \{e_1, e_2, e_3, \ldots, e_n\}\). Let the set of all \(m \times n\) fuzzy soft matrices over \(U\) be \(FSM_{m \times n}\). Let \(A, B \in FSM_{m \times n}\), where \(A = [a_{ij}]_{m \times n}, a_{ij} = ([\mu_{ij}(c_1), \mu_{ij}(c_2)]\) and \(B = [b_{ij}]_{m \times n}, b_{ij} = ([\chi_{ij}(c_1), \chi_{ij}(c_2)]\). To avoid degenerate cases we assume that \(\min(\mu_{ij}(c_1), \chi_{ij}(c_1)) \geq \max(\mu_{ij}(c_2), \chi_{ij}(c_2))\) for all \(i\) and \(j\). We define the operation ‘addition (+)’ between \(A\) and \(B\) as
\[ A + B = C, \text{ where } C = \left[ c_{ij} \right]_{nm}, c_{ij} = \left[ \max(\mu_{j1}(c_i), \chi_{j1}(c_i)) \right] \left[ \min(\mu_{j2}(c_i), \chi_{j2}(c_i)) \right] \]

If \( \mu_{j2}(c_i) = \chi_{j2}(c_i) = 0 \) \( \forall i, j \), then our definition reduces to

\[ A + B = C, \text{ where } C = \left[ c_{ij} \right]_{nm}, c_{ij} = \left[ \max(\mu_{j1}(c_i), \chi_{j1}(c_i)) \right] \min(0, 0) \]

\[ = \left[ \max(\mu_{j1}(c_i), \chi_{j1}(c_i)) \right] 0 \]

\[ = \max(\mu_{j1}(c_i), \chi_{j1}(c_i)), \text{ which is the definition of addition (max) of two fuzzy matrices} \] [2] \text{ in the usual sense where fuzzy reference function is 0.}

**Example: 3.1** Let \( U = \{c_1, c_2, c_3, c_4\} \) be the universal set and \( E \) be the set of parameters given by \( E = \{e_1, e_2, e_3\} \). We consider the fuzzy soft sets

\[
(F, E) = \{(F(e_1)) = \{(c_1, 0.300), (c_2, 0.510), (c_3, 0.603), (c_4, 0.500)\},
(F(e_2)) = \{(c_1, 0.701), (c_2, 0.905), (c_3, 0.701), (c_4, 0.802)\},
(F(e_3)) = \{(c_1, 0.602), (c_2, 0.700), (c_3, 0.702), (c_4, 0.300)\} \}

\[
(G, E) = \{(G(e_1)) = \{(c_1, 0.801), (c_2, 0.702), (c_3, 0.502), (c_4, 0.401)\},
(G(e_2)) = \{(c_1, 0.900), (c_2, 0.900), (c_3, 0.800), (c_4, 0.700)\},
(G(e_3)) = \{(c_1, 0.503), (c_2, 0.901), (c_3, 0.601), (c_4, 0.803)\} \}

The fuzzy soft matrices representing these two fuzzy soft sets are respectively

\[
A = \begin{bmatrix}
0.300 & 0.701 & 0.602 \\
0.501 & 0.905 & 0.700 \\
0.603 & 0.701 & 0.702 \\
0.500 & 0.802 & 0.300
\end{bmatrix}_{4 \times 3} \quad \text{and} \quad B = \begin{bmatrix}
0.801 & 0.900 & 0.503 \\
0.702 & 0.900 & 0.901 \\
0.502 & 0.800 & 0.601 \\
0.401 & 0.700 & 0.803
\end{bmatrix}_{4 \times 3}
\]

Here \( A + B = \begin{bmatrix}
0.800 & 0.900 & 0.602 \\
0.701 & 0.900 & 0.900 \\
0.602 & 0.800 & 0.701 \\
0.500 & 0.800 & 0.800
\end{bmatrix}_{4 \times 3} \)

**Proposition: 3.1** Let \( A, B, C \in FSM_{m \times n} \). Then the following results hold.

(i) \( A + B = B + A \) \quad (\text{Commutative Law})

(ii) \( (A + B) + C = A + (B + C) \) \quad (\text{Associative Law})

**Proof:** The proof is straightforward and follows from definition.

**Definition: 3.2** Let \( A, B \in FSM_{m \times n} \). Let the corresponding membership value matrices be \( MV(A) = \left[ \delta_{ij} \right]_{m \times n} \) and \( MV(B) = \left[ \delta_{ij} \right]_{m \times n} \), \( i = 1, 2, 3, \ldots, m; \ j = 1, 2, 3, \ldots, n \). Then the score matrix \( S_{(A, B)} \) would be defined as \( S_{(A, B)} = \left[ \rho_{ij} \right]_{m \times n} \) where \( \rho_{ij} = \delta_{ij} - \delta_{ij} \).

**Definition: 3.3** Let \( A, B \in FSM_{m \times n} \). Let the corresponding membership value matrices be \( MV(A) = \left[ \delta_{ij} \right]_{m \times n} \), \( MV(B) = \left[ \delta_{ij} \right]_{m \times n} \) respectively and the score matrix be \( S_{(A, B)} = \left[ \rho_{ij} \right]_{m \times n}, \ i = 1, 2, 3, \ldots, m; \ j = 1, 2, 3, \ldots, n \). Then the total score for each \( c_i \) in \( U \) would be calculated by the formula \( S_i = \sum_{j=1}^{n} \rho_{ij} \).

Suppose \( U \) is a set of certain number of cities, \( E \) is a set of parameters related to healthy environment of a city which vary with time. We construct a fuzzy soft set \( (F, E) \) over \( U \) representing the healthy environment of the cities at an instant \( t \), where \( F \) is a mapping \( F : E \rightarrow \widetilde{F}(U) \), \( \widetilde{F}(U) \) is the set of all fuzzy subsets of \( U \). We further construct another
fuzzy soft set \((G,E)\) over \(U\) representing the healthy environment of the cities at an instant \(t'\). The matrices \(A\) and \(B\) corresponding to the fuzzy soft sets \((F,E)\) and \((G,E)\) are constructed. We compute the complements \((F,E)^{\complement}\) and \((G,E)^{\complement}\) and write the matrices \(\overline{A}\) and \(\overline{B}\) corresponding to \((F,E)^{\complement}\) and \((G,E)^{\complement}\) respectively. Using definition 3.1 we compute \(A + B\), which represents the maximum membership of healthy environment of the cities between the time instances \(t\) and \(t'\) and then compute \(\overline{A} + \overline{B}\), which represents the maximum membership of non-healthy environment of the cities between the time instances \(t\) and \(t'\). Using definition 2.7, we compute \(MV(A + B)\) and \(MV(\overline{A} + \overline{B})\). The score matrix \(S_{(A+B)(\overline{A}+\overline{B})}\) is constructed using definition 3.2 and the total score \(S_t\) for each \(c_i\) in \(U\) is calculated using definition 3.3. Finally we would find \(S_k = \max_i(S_t)\), then we conclude that the city \(c_k\) has the maximum healthy environment between the time instances \(t\) and \(t'\) respectively. If \(S_k\) has more than one value, the process is repeated by reassessing the parameters for healthy environment.

4. ALGORITHM:

1. Input the fuzzy soft matrices \((F,E)\) and \((G,E)\). Also write the fuzzy soft matrices \(A\) and \(B\) corresponding to \((F,E)\) and \((G,E)\) respectively.

2. Write the fuzzy soft matrices \((F,E)^{\complement}\) and \((G,E)^{\complement}\). Also write the fuzzy soft matrices \(\overline{A}\) and \(\overline{B}\) corresponding to \((F,E)^{\complement}\) and \((G,E)^{\complement}\) respectively.

3. Compute \(A + B\) and \(MV(A + B)\).

4. Compute \(\overline{A} + \overline{B}\) and \(MV(\overline{A} + \overline{B})\).

5. Compute the score matrix \(S_{(A+B)(\overline{A}+\overline{B})}\):

6. Compute the total score \(S_t\) for each \(c_i\) in \(U\).

7. Find \(S_k = \max_i(S_t)\), then we conclude that the city \(c_k\) has the maximum healthy environment between the time instances \(t\) and \(t'\) respectively.

8. If \(S_k\) has more than one value, then go to step (1) and repeat the process by reassessing the parameters for healthy environment.

5. APPLICATION IN A DECISION MAKING PROBLEM:

Let \((F,E)\) and \((G,E)\) be two fuzzy soft sets representing the healthy environment of four cities \(U = \{c_1,c_2,c_3,c_4\}\) at instants \(t\) and \(t'\) respectively. Let \(E = \{ e_1(less-Crowdness), e_2(Noise-free), e_3(non-Pollution) \}\) be the set of parameters which would vary with time.

\[
(F,E) = \{ F(e_1) = \{(c_1,0.7,0),(c_2,0.5,0),(c_3,0.1,0),(c_4,0.6,0)\},
F(e_2) = \{(c_1,0.9,0),(c_2,0.6,0),(c_3,0.5,0),(c_4,0.7,0)\},
F(e_3) = \{(c_1,0.5,0),(c_2,0.4,0),(c_3,0.4,0),(c_4,0.9,0)\}\}
\]

\[
(G,E) = \{ G(e_1) = \{(c_1,0.6,0),(c_2,0.4,0),(c_3,0.3,0),(c_4,0.5,0)\},
G(e_2) = \{(c_1,0.8,0),(c_2,0.9,0),(c_3,0.7,0),(c_4,0.3,0)\},
G(e_3) = \{(c_1,0.5,0),(c_2,0.9,0),(c_3,0.1,0),(c_4,0.6,0)\}\}
\]

These two fuzzy soft sets are represented by the following fuzzy matrices respectively.

\[
A = \begin{bmatrix} c_1 & e_1 & e_2 & e_3 \\ c_2 & (0.7,0) & (0.9,0) & (0.5,0) \\ c_3 & (0.5,0) & (0.6,0) & (0.4,0) \\ c_4 & (0.1,0) & (0.5,0) & (0.4,0) \end{bmatrix}
\]

\[
B = \begin{bmatrix} c_1 & e_1 & e_2 & e_3 \\ c_2 & (0.6,0) & (0.8,0) & (0.5,0) \\ c_3 & (0.4,0) & (0.9,0) & (0.9,0) \\ c_4 & (0.3,0) & (0.7,0) & (0.1,0) \end{bmatrix}
\]

The fuzzy soft sets representing the non-healthy environment of the four cities \(U = \{c_1,c_2,c_3,c_4\}\) at instants \(t\) and \(t'\) respectively are given by
\( (F, E)^\star = \{F^\star (e_1) = \{(c_1, 1.0.7), (c_2, 1.0.5), (c_3, 1.0.1), (c_4, 1.0.6)\}, \\
F^\star (e_2) = \{(c_1, 1.0.9), (c_2, 1.0.6), (c_3, 1.0.5), (c_4, 1.0.7)\}, \\
F^\star (e_3) = \{(c_1, 1.0.5), (c_2, 1.0.4), (c_3, 1.0.4), (c_4, 1.0.9)\}\}\)

\( (G, E)^\star = \{G^\star (e_1) = \{(c_1, 1.0.6), (c_2, 1.0.4), (c_3, 1.0.3), (c_4, 1.0.5)\}, \\
G^\star (e_2) = \{(c_1, 1.0.8), (c_2, 1.0.9), (c_3, 1.0.7), (c_4, 1.0.3)\}, \\
G^\star (e_3) = \{(c_1, 1.0.5), (c_2, 1.0.9), (c_3, 1.0.1), (c_4, 1.0.6)\}\}\)

These two fuzzy soft sets are represented by the following fuzzy matrices in order.

\[
\begin{array}{ccc}
\text{c}_1 & \text{c}_2 & \text{c}_3 \\
(1.0.7) & (1.0.9) & (1.0.5) \\
(1.0.5) & (1.0.6) & (1.0.4) \\
(1.0.1) & (1.0.5) & (1.0.4) \\
(1.0.6) & (1.0.7) & (1.0.9) \\
\end{array}
\text{and} \quad 
\begin{array}{ccc}
\text{c}_1 & \text{c}_2 & \text{c}_3 \\
(1.0.6) & (1.0.8) & (1.0.5) \\
(1.0.4) & (1.0.9) & (1.0.9) \\
(1.0.3) & (1.0.7) & (1.0.1) \\
(1.0.5) & (1.0.3) & (1.0.6) \\
\end{array}
\]

Then the fuzzy soft matrix \( A + B \) represents the maximum membership function of healthy environment of the cities between the time instances \( t \) and \( t' \).

The membership value matrix \( MV(A + B) \) gives the respective membership values for healthy environment of the cities.

\[
\begin{array}{ccc}
\text{c}_1 & \text{c}_2 & \text{c}_3 \\
(0.7, 0) & (0.9, 0) & (0.5, 0) \\
(0.5, 0) & (0.9, 0) & (0.9, 0) \\
(0.3, 0) & (0.7, 0) & (0.4, 0) \\
(0.6, 0) & (0.7, 0) & (0.9, 0) \\
\end{array}
\text{and} \quad 
\begin{array}{ccc}
\text{c}_1 & \text{c}_2 & \text{c}_3 \\
(0.7, 0.9) & (0.5, 0.9) & (0.3, 0.7) \\
(0.6, 0.7) & (0.6, 0.7) & (0.9, 0.9) \\
\end{array}
\]

Again the fuzzy soft matrix \( \overline{A + B} \) represents the maximum membership function of non - healthy environment of the cities between the time instances \( t \) and \( t' \).

The membership value matrix \( MV(\overline{A + B}) \) gives the respective membership values for non - healthy environment of the cities between the time instances \( t \) and \( t' \).

\[
\begin{array}{ccc}
\text{c}_1 & \text{c}_2 & \text{c}_3 \\
(1.0, 6) & (1.0, 8) & (1.0, 5) \\
(1.0, 4) & (1.0, 6) & (1.0, 4) \\
(1.0, 1) & (1.0, 5) & (1.0, 1) \\
(1.0, 5) & (1.0, 3) & (1.0, 6) \\
\end{array}
\text{and} \quad 
\begin{array}{ccc}
\text{c}_1 & \text{c}_2 & \text{c}_3 \\
(0.4, 0.2) & (0.6, 0.4) & (0.9, 0.5) \\
(0.5, 0.7) & (0.5, 0.7) & (0.4, 0.6) \\
\end{array}
\]

We now calculate the score matrix \( S_{\overline{(A + B)|\overline{A + B}}} \) and total score for healthy environment of each city.

\[
\begin{array}{ccc}
\text{c}_1 & \text{c}_2 & \text{c}_3 \\
0.3 & 0.7 & 0.4 \\
-0.1 & 0.5 & 0.3 \\
-0.6 & 0.2 & -0.5 \\
0.1 & 0. & 0.5 \\
\end{array}
\]

\[
\begin{array}{c}
S_1 \\
S_2 \\
S_3 \\
S_4 \\
\end{array}
\begin{array}{c}
1.4 \\
0.7 \\
-0.9 \\
0.6 \\
\end{array}
\]

Total score for healthy environment:

\[
\begin{array}{c}
S_1 \\
S_2 \\
S_3 \\
S_4 \\
\end{array}
\begin{array}{c}
1.4 \\
0.7 \\
-0.9 \\
0.6 \\
\end{array}
\]
We see that $S_1$ has the maximum value and thus conclude that the city $c_1$ has got the highest total score and hence the city having the most healthy environment among all the cities between the instances $t$ and $t'$. 

6. CONCLUSION:

We have applied the notion fuzzy soft matrices and complement of fuzzy soft sets in a decision making problem. We hope that our work would enhance this study on fuzzy soft sets as well as matrices.

7. REFERENCES: