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DECOMPOSITIONS OF $M^{(1,2)*}$-CONTINUITY AND COMPLETE $M^{(1,2)*}$-CONTINUITY IN BIMINIMAL SPACES

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ABSTRACT

The purpose of this paper is to introduce the concepts of $M^{(1,2)*}$-continuity and complete $M^{(1,2)*}$-continuity in biminimal spaces, and study some properties of the generalizations of $m_x^{(1,2)*}$-closed sets and $m_x^{(1,2)*}$-open sets in biminimal spaces.

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1. INTRODUCTION:

Njastad [8] introduced the concepts of an α-sets and Mashhour et al [7] introduced α-continuous mappings in topological spaces. The topological notions of semi-open sets and semi-continuity, and preopen sets and precontinuity were introduced by Levine [5] and Mashhour et al [6], respectively. The concepts of minimal structures (briefly m-structures) were developed by Popa and Noiri [9] in 2000. Kelly [4] introduced the notions of bitopological spaces. Such spaces are equipped with two arbitrary topologies. Ravi and Lellis Thivagar [10] introduced weakly open sets called $\tau_{1,2}$-open sets in bitopological spaces. In this paper, we introduce $M^{(1,2)*}$-continuity and complete $M^{(1,2)*}$-continuity, and obtain their decompositions in biminimal spaces. At every places the new notions have been substantiated with suitable examples.

2. PRELIMINARIES:

Definition: 2.1 [9] Let X be a nonempty set and $\wp$(X) the power set of X. A subfamily m_x of $\wp$(X) is called a minimal structure (briefly m-structure) on X if $\phi \in m_x$ and X $\in m_x$.

Definition: 2.2 [11] A set X together with two minimal structures m_x^1 and m_x^2 on X is called a biminimal space and is denoted by(X, m_x^1, m_x^2).

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Throughout this paper, (X, m_x^1, m_x^2) (or X) denote biminimal space.

Definition: 2.3 [11] Let S be a subset of X. Then S is said to be $m_x^{(1,2)^*}$-open if S = A ∪ B where A ∈ m_x^1 and B∈ m_x^2. We call $m_x^{(1,2)^*}$-closed set is the complement of $m_x^{(1,2)^*}$-open.

The family of all $m_x^{(1,2)^*}$-open subsets of (X, m_x^1, m_x^2) is denoted by $m_x^{(1,2)^*}$-O(X).

Example: 2.4 [11] Let X= {a, b, c}, m_x^1 = {ϕ, X, {a}} and m_x^2 = {ϕ, X, {b}}. Then the sets in {ϕ, X, {a}, {b}, {a, b}} are called $m_x^{(1,2)^*}$-open and the sets in {ϕ, X, {b, c}, {a, c}, {c}} are called $m_x^{(1,2)^*}$-closed.

Definition: 2.5 [11] Let S be a subset of X. Then
(i) the $m_x^{(1,2)^*}$-interior of S, denoted by $m_x^{(1,2)^*}$-int(S), is defined by ∪ {F/F ⊆ S and F is $m_x^{(1,2)^*}$-open};
(ii) the $m_x^{(1,2)^*}$-closure of S, denoted by $m_x^{(1,2)^*}$-cl(S), is defined by ∩ {F/S ⊆ F and F is $m_x^{(1,2)^*}$-closed}.

Definition: 2.6 [11] Let S be a subset of X. Then S is said to be
(i) $m_x^{(1,2)^*}$-α-open if S ⊆ $m_x^{(1,2)^*}$-int($m_x^{(1,2)^*}$-cl($m_x^{(1,2)^*}$-int(S)));
(ii) $m_x^{(1,2)^*}$-semi-open if S ⊆ $m_x^{(1,2)^*}$-cl($m_x^{(1,2)^*}$-int(S));
(iii) $m_x^{(1,2)^*}$-preopen if S ⊆ $m_x^{(1,2)^*}$-int($m_x^{(1,2)^*}$-cl(S));
(iv) $m_x^{(1,2)^*}$-α-closed if $m_x^{(1,2)^*}$-cl($m_x^{(1,2)^*}$-int($m_x^{(1,2)^*}$-cl(S))) ⊆ S;
(v) $m_x^{(1,2)^*}$-preclosed if $m_x^{(1,2)^*}$-cl($m_x^{(1,2)^*}$-int(S)) ⊆ S;
(vi) $m_x^{(1,2)^*}$-Semi-closed if $m_x^{(1,2)^*}$-int($m_x^{(1,2)^*}$-cl(S)) ⊆ S.

The family of all $m_x^{(1,2)^*}$-α-open [resp. $m_x^{(1,2)^*}$-Semi-open, $m_x^{(1,2)^*}$-preopen]

Example: 2.7 [11] Let Y = {p, q, r}, m_y^1 = {ϕ, Y, {p}, {p, q}} and m_y^2 = {ϕ, Y, {q}}.
Then the sets in {ϕ, Y, {p}, {q}, {p, q}} are called $m_y^{(1,2)^*}$-open and the sets in {ϕ, Y, {r}, {q, r}, {p, r}} are called $m_y^{(1,2)^*}$-closed. We have
$m_y^{(1,2)^*}$-αO(Y) = {ϕ, Y, {p}, {q}, {p, q}};
$m_y^{(1,2)^*}$-SO(Y) = {ϕ, Y,{p},{q},{p, q},{p, r},{q, r}} and
$m_y^{(1,2)^*}$-PO(Y) = {ϕ, Y, {p}, {q}, {p, q}}.

Lemma: 2.8 [11] Let X be a non-empty set and m_x^1 , m_x^2 minimal structures on X. For subsets A and B of X, the following properties hold:
(i) A ⊆ $m_x^{(1,2)^*}$-cl(A) and $m_x^{(1,2)^*}$-int(A) ⊆ A;
(ii) If A is $m_x^{(1,2)^*}$-open then A = $m_x^{(1,2)^*}$-int(A);
(iii) If A is $m_x^{(1,2)^*}$-closed then A = $m_x^{(1,2)^*}$-cl(A);
(iv) If A ⊆ B then $m_x^{(1,2)^*}$-cl(A) ⊆ $m_x^{(1,2)^*}$-cl(B);
(v) If A ⊆ B then $m_x^{(1,2)^*}$-int(A) ⊆ $m_x^{(1,2)^*}$-int(B);
(vi) $m_x^{(1,2)^*}$-cl(X – A) = X – $m_x^{(1,2)^*}$-int(A) and $m_x^{(1,2)^*}$-int(X – A) = X – $m_x^{(1,2)^*}$-cl(A);
(vii) $m_x^{(1,2)^*}$-cl(ϕ) = ϕ = $m_x^{(1,2)^*}$-int(ϕ) and $m_x^{(1,2)^*}$-cl(X) = X = $m_x^{(1,2)^*}$-int(X);
(viii) $m_x^{(1,2)^*}$-cl($m_x^{(1,2)^*}$-cl(A)) = $m_x^{(1,2)^*}$-cl(A) and $m_x^{(1,2)^*}$-int($m_x^{(1,2)^*}$-int(A)) = $m_x^{(1,2)^*}$-int(A).

Definition: 2.9 [11] A biminimal space (X, m_x^1, m_x^2) has the property [ᴜ] if the arbitrary union of $m_x^{(1,2)^*}$-open sets is $m_x^{(1,2)^*}$-open.

A biminimal space (X, m_x^1, m_x^2) has the property [I] if the any finite intersection of $m_x^{(1,2)^*}$-open sets is $m_x^{(1,2)^*}$-open.

Lemma: 2.10 [11] The following are equivalent for the biminimal space (X, m_x^1, m_x^2).

(1) (X, m_x^1, m_x^2) have property [ᴜ];

(2) If $m_x^{(1,2)^*}$-int(E) = E, then E ∈ $m_x^{(1,2)^*}$-O(X).

(3) If $m_x^{(1,2)^*}$-cl(F) = F, then F^c ∈ $m_x^{(1,2)^*}$-O(X).

3. CHARACTERIZATIONS:

Definition: 3.1 Let S be a subset of X. Then S is said to be

(i) regular $m_x^{(1,2)^*}$-open if S = $m_x^{(1,2)^*}$-int($m_x^{(1,2)^*}$-cl(S)),

(ii) $m_x^{(1,2)^*}$-Semi-regular if it is both

The family of all $m_x^{(1,2)^*}$-semi-closed [resp. regular $m_x^{(1,2)^*}$-open] sets of X is denoted by $m_x^{(1,2)^*}$-SC(X) [resp. $m_x^{(1,2)^*}$-RO(X)].

The intersection of all $m_x^{(1,2)^*}$-semi-closed sets of X containing a subset S of X is called the $m_x^{(1,2)^*}$-semi-closure of S and is denoted by $m_x^{(1,2)^*}$-scl(S).

Remark: 3.2 A subset S of X is $m_x^{(1,2)^*}$-semi-closed if and only if $m_x^{(1,2)^*}$-scl(S) = S.

Definition: 3.3 [12] A subset S of X is said to be $m_x^{(1,2)^*}$-semi-generalized closed (briefly $m_x^{(1,2)^*}$-sg-closed) if and only if $m_x^{(1,2)^*}$-scl(S) ⊆ F whenever S ⊆ F and F is $m_x^{(1,2)^*}$-semi-open set.

The complement of $m_x^{(1,2)^*}$-sg-closed set is $m_x^{(1,2)^*}$-sg-open.

Example: 3.4 Let X = {a, b, c}, m_x^1 = {ϕ, X, {a}} and m_x^2 = {ϕ, X}. Then the sets in {ϕ, X, {a}} are called $m_x^{(1,2)^*}$-open and the sets in {ϕ, X, {b, c}} are called $m_x^{(1,2)^*}$-closed. Also, the sets in {ϕ, X, {b}, {c}, {b, c}} are called $m_x^{(1,2)^*}$-sg-closed.

Definition: 3.5 A subset S of X is said to be locally $m_x^{(1,2)^*}$-closed if S = M ∩ N, where M is $m_x^{(1,2)^*}$-open and N is $m_x^{(1,2)^*}$-closed.

Remark: 3.6 [11] Every $m_x^{(1,2)^*}$-closed set is $m_x^{(1,2)^*}$-α-closed but not conversely.

Example: 3.7 Let X = {a, b, c, d}, m_x^1 = {ϕ, X, {a}, {a, b}} and m_x^2 = {ϕ, X, {a, c}}. Then the sets in {ϕ, X, {a}, {a, b}, {a, c}, {a, b, c}} are called $m_x^{(1,2)^*}$-open and the sets in {ϕ, X, {d}, {b, d}, {c, d}, {b, c, d}} are called $m_x^{(1,2)^*}$-closed. We have {c} is $m_x^{(1,2)^*}$-α-closed set but not $m_x^{(1,2)^*}$-closed.

Also, the sets in {ϕ, X, {a}, {b}, {c}, {d}, {a, b}, {a, c}, {b, c}, {b, d}, {c, d}, {a, b, c}, {b, c, d}} are called locally $m_x^{(1,2)^*}$-closed.

Proposition: 3.8 Every $m_x^{(1,2)^*}$-closed set is locally $m_x^{(1,2)^*}$-closed.

Proof: S = X ∩ S where X is $m_x^{(1,2)^*}$-open and S is $m_x^{(1,2)^*}$-closed. Thus S is locally $m_x^{(1,2)^*}$-closed.

Example: 3.9 The converse of Proposition 3.8 is not true in general.

Consider the Example 3.7. We have {a} is locally $m_x^{(1,2)^*}$-closed but not $m_x^{(1,2)^*}$-closed.

Proposition: 3.10 Asubset S of X is $m_x^{(1,2)^*}$-α-closed if and only if S is $m_x^{(1,2)^*}$-semi-closed and $m_x^{(1,2)^*}$-preclosed.

Proof: Let S be $m_x^{(1,2)^*}$-α-closed. Then $m_x^{(1,2)^*}$-cl($m_x^{(1,2)^*}$-int($m_x^{(1,2)^*}$-cl(S))) ⊆ S. Hence $m_x^{(1,2)^*}$-int($m_x^{(1,2)^*}$-cl(S))⊆ $m_x^{(1,2)^*}$-cl($m_x^{(1,2)^*}$-int($m_x^{(1,2)^*}$-cl(S)))⊆S. Thus S is $m_x^{(1,2)^*}$-semi-closed. Also $m_x^{(1,2)^*}$-cl($m_x^{(1,2)^*}$-int(S)) ⊆ $m_x^{(1,2)^*}$-cl($m_x^{(1,2)^*}$-int($m_x^{(1,2)^*}$-cl(S))) ⊆ S. Thus S is $m_x^{(1,2)^*}$-preclosed. Conversely, let S be $m_x^{(1,2)^*}$-semi-closed and $m_x^{(1,2)^*}$-preclosed. Since S is $m_x^{(1,2)^*}$-semi-closed, $m_x^{(1,2)^*}$-int($m_x^{(1,2)^*}$-cl(S)) ⊆ S implies $m_x^{(1,2)^*}$-int($m_x^{(1,2)^*}$-int($m_x^{(1,2)^*}$-cl(S))) ⊆ $m_x^{(1,2)^*}$-int(S). We have $m_x^{(1,2)^*}$-int($m_x^{(1,2)^*}$-cl(S)) ⊆ $m_x^{(1,2)^*}$-int(S). which implies $m_x^{(1,2)^*}$-cl($m_x^{(1,2)^*}$-int($m_x^{(1,2)^*}$-cl(S))) ⊆ $m_x^{(1,2)^*}$-cl($m_x^{(1,2)^*}$-int(S)) ⊆ S, as S is $m_x^{(1,2)^*}$-preclosed. Hence S is $m_x^{(1,2)^*}$-α-closed.

Example: 3.11 A $m_x^{(1,2)^*}$-semi-closed or $m_x^{(1,2)^*}$-preclosed set need not be $m_x^{(1,2)^*}$-α-closed.

(i) Let X = {a, b, c}, m_x^1 = {ϕ, X, {a}} and m_x^2 = {ϕ, X, {b}}. Then the sets in {ϕ, X, {a}, {b}, {a, b}} are called $m_x^{(1,2)^*}$-open and the sets in {ϕ, X, {b, c}, {a, c}, {c}} are called $m_x^{(1,2)^*}$-closed. We have

(1) $m_x^{(1,2)^*}$-αO(X) = {ϕ, X, {a}, {b}, {a, b}};

(2) $m_x^{(1,2)^*}$-SO(X) = {ϕ, X, {a}, {b}, {a, b}, {a, c}, {b, c}}.

Therefore {b} is $m_x^{(1,2)^*}$-semi-closed but not $m_x^{(1,2)^*}$-α-closed.

(ii) Let X = {a, b, c}, m_x^1 = {ϕ, X, {b, c}} and m_x^2 = {ϕ, X, {a, b}}. Then the sets in {ϕ, X, {b, c}, {a, b}} are called $m_x^{(1,2)^*}$-open and the sets in {ϕ, X, {a}, {c}} are called $m_x^{(1,2)^*}$-closed. We have

(1) $m_x^{(1,2)^*}$-αO(X) = $m_x^{(1,2)^*}$-SO(X) = {ϕ, X, {a, b}, {b, c}};

(2) $m_x^{(1,2)^*}$-PO(X) = {ϕ, X, {b}, {a, b},{b, c}, {a, c}}.

Therefore {b} is $m_x^{(1,2)^*}$-preclosed but not $m_x^{(1,2)^*}$-α-closed.

Proposition: 3.12 A $m_x^{(1,2)^*}$-semi-closed set is $m_x^{(1,2)^*}$-sg-closed.

Proof: Let **S** be $m_x^{(1,2)^*}$-semi-closed. Then $m_x^{(1,2)^*}$-scl(S) = S ⊆ G where S ⊆ G and G is $m_x^{(1,2)^*}$-semi-open. Thus S is $m_x^{(1,2)^*}$-sg-closed.

Remark: 3.13 A $m_x^{(1,2)^*}$-sg-closed set need not be $m_x^{(1,2)^*}$-semi-closed.

Consider the Example 3.11 (ii). We have {a, c} is $m_x^{(1,2)^*}$-sg-closed but not $m_x^{(1,2)^*}$-semi-closed.

Proposition: 3.14 A $m_x^{(1,2)^*}$-closed set is $m_x^{(1,2)^*}$-sg-closed.

Proof: Let S be $m_x^{(1,2)^*}$-closed. Then S is $m_x^{(1,2)^*}$-α-closed and also by Proposition 3.10., S is $m_x^{(1,2)^*}$-semi-closed. Moreover, by Proposition 3.12., S is $m_x^{(1,2)^*}$-sg-closed.

Remark: 3.15 The converse of Proposition 3.14 is not true in general.

Consider the Example 3.4., {c} is $m_x^{(1,2)^*}$-sg-closed but not $m_x^{(1,2)^*}$-closed.

Proposition: 3.16 Let (X, m_x^1, m_x^2) have property [ᵾ]. Then any regular $m_x^{(1,2)^*}$-open set is $m_x^{(1,2)^*}$-open.

Proof: Let S be regular $m_x^{(1,2)^*}$-open. Since S = $m_x^{(1,2)^*}$-int($m_x^{(1,2)^*}$-cl(S)), $m_x^{(1,2)^*}$-int(S) = $m_x^{(1,2)^*}$-int($m_x^{(1,2)^*}$-cl(S)). We have S = $m_x^{(1,2)^*}$-int(S). Thus, by Lemma 2.10, S is $m_x^{(1,2)^*}$-open.

Remark: 3.17 The converse of Proposition 3.16 is not true in general.

Consider the Example 3.4., {a} is $m_x^{(1,2)^*}$-open but not regular $m_x^{(1,2)^*}$-open.

Proposition: 3.18 Every regular $m_x^{(1,2)^*}$-open set is $m_x^{(1,2)^*}$-semi-closed.

Proof: Let S be regular $m_x^{(1,2)^*}$-open. Then S = $m_x^{(1,2)^*}$-int($m_x^{(1,2)^*}$-cl(S)). We have $m_x^{(1,2)^*}$-int($m_x^{(1,2)^*}$-cl(S)) $\subseteq$ S. Thus S is $m_x^{(1,2)^*}$-semi-closed.

Proposition: 3.19 Let (X, m_x^1, m_x^2) have property [ᵾ]. Then every regular $m_x^{(1,2)^*}$-open set is $m_x^{(1,2)^*}$-semi-regular.

Proof: Let S be regular $m_x^{(1,2)^*}$-open. Then by Proposition 3.18., S is $m_x^{(1,2)^*}$-semi-closed and also by Proposition 3.16., S is $m_x^{(1,2)^*}$-open (and so S is $m_x^{(1,2)^*}$-semi-open). Hence S is both $m_x^{(1,2)^*}$-semi-open and $m_x^{(1,2)^*}$-semi-closed. Thus S is $m_x^{(1,2)^*}$-semi-regular.

Remark: 3.20 Every $m_x^{(1,2)^*}$-semi-regular set is $m_x^{(1,2)^*}$-semi-closed but not conversely.

Example: 3.21 Let X = {a, b, c}, m_x^1 = {ϕ, X, {a}} and m_x^2 = {ϕ, X, {b}}. Then the sets in {ϕ, X, {a}, {b}, {a, b}} are called $m_x^{(1,2)^*}$-open and the sets in {ϕ, X, {b, c}, {a, c}, {c}} are called $m_x^{(1,2)^*}$-closed.

We have
$m_x^{(1,2)^*}$-αO(X) = $m_x^{(1,2)^*}$-PO(X) = {ϕ, X, {a}, {b}, {a, b}};
$m_x^{(1,2)^*}$-SO(X) = {ϕ, X, {a}, {b}, {a, b}, {a, c}, {b, c}};
$m_x^{(1,2)^*}$-SC(X) = {ϕ, X, {a}, {b}, {c}, {a, c}, {b, c}} and
$m_x^{(1,2)^*}$-RO(X) = {ϕ, X, {a}, {b}}.

We have {c} is $m_x^{(1,2)^*}$-semi-closed but not $m_x^{(1,2)^*}$-semi-regular.

Remark: 3.22 $m_x^{(1,2)^*}$-α-closed sets and regular $m_x^{(1,2)^*}$-open sets are independent of each other.

Consider the Example 3.4. We have {c} is $m_x^{(1,2)^*}$-α-closed but not regular $m_x^{(1,2)^*}$-open and Consider the Example 3.21. We have {a} is regular $m_x^{(1,2)^*}$-open but not $m_x^{(1,2)^*}$-α-closed.

Remark: 3.23 $m_x^{(1,2)*}$-preclosed sets and $m_x^{(1,2)*}$-open sets are independent of each other.Consider the Example 3.21. We have {c} is $m_x^{(1,2)*}$-preclosed but not $m_x^{(1,2)*}$-open and {a} is $m_x^{(1,2)*}$-open but not $m_x^{(1,2)*}$-preclosed.

Remark: 3.24 locally $m_x^{(1,2)*}$-closed sets and $m_x^{(1,2)*}$-preclosed sets are independent of each other.

Consider the Example 3.21. We have {a} is locally $m_x^{(1,2)*}$-closed but not $m_x^{(1,2)*}$-preclosed and Consider the Example 3.11 (ii). We have {b} is $m_x^{(1,2)*}$-preclosed but not locally $m_x^{(1,2)*}$-closed.

Remark: 3.25 By the previous Propositions, Examples and Remarks, we obtain the following diagram where $A \rightarrow B$ means A implies B but B does not imply A and $A \nleftrightarrow B$ means A and B are independent

$$\begin{tikzcd}
m_x^{(1,2)*}\text{-sg-closed} \arrow[r, shift right, "/" marking] \arrow[d, shift right, "/" marking] & m_x^{(1,2)*}\text{-semi-closed} \arrow[l, shift right] \arrow[r, shift right, "/" marking] \arrow[d, shift right, "/" marking] & m_x^{(1,2)*}\text{-semi-regular} \arrow[l, shift right] \\
m_x^{(1,2)*}\text{-closed} \arrow[u, shift right] \arrow[r, shift left] \arrow[d, shift right] & m_x^{(1,2)*}\text{-}\alpha\text{-closed} \arrow[u, shift right] \arrow[l, shift left, "/" marking] \arrow[r, shift left, "/" marking] \arrow[d, shift right] & \text{regular } m_x^{(1,2)*}\text{-open} \arrow[l, shift left, "/" marking] \\
\text{locally } m_x^{(1,2)*}\text{–closed} \arrow[u, shift right, "/" marking] \arrow[r, shift left, "/" marking] & m_x^{(1,2)*}\text{-preclosed} \arrow[u, shift right, "/" marking] \arrow[l, shift left, "/" marking] \arrow[r, shift left, "/" marking] & m_x^{(1,2)*}\text{-open} \arrow[l, shift left, "/" marking]
\end{tikzcd}$$

4. DECOMPOSITION OF $M^{(1,2)*}$-CONTINUITY:

Proposition: 4.1 Let (X, m_x^1, m_x^2) have property [I]. Let S be a subset of X such that $m_x^{(1,2)*}$-cl(S) ∈ $m_x^{(1,2)*}$-O(X). Then the following are equivalent.

(i) S is $m_x^{(1,2)*}$-open.

(ii) S is an $m_x^{(1,2)*}$-α-open and locally $m_x^{(1,2)*}$-closed.

Proof: (i) ⟹ (ii): Let S be an $m_x^{(1,2)*}$-open. Then S is $m_x^{(1,2)*}$-α-open. Also S = X ∩ S where S is $m_x^{(1,2)*}$-open and X is $m_x^{(1,2)*}$-closed. Thus S is locally $m_x^{(1,2)*}$-closed.

(ii) ⟹ (i): Let S be $m_x^{(1,2)*}$-α-open and locally $m_x^{(1,2)*}$-closed.
Since S is $m_x^{(1,2)*}$-preopen, $S \subseteq m_x^{(1,2)*}$-int($m_x^{(1,2)*}$-cl(S)). Since S is locally $m_x^{(1,2)*}$-closed, S = U ∩ $m_x^{(1,2)*}$-cl(S) where U is $m_x^{(1,2)*}$-open.
Also S = U ∩ $m_x^{(1,2)*}$-cl(S) ∩ U = U ∩ S ⊂ U ∩ $m_x^{(1,2)*}$-int($m_x^{(1,2)*}$-cl(S)) ⊆ U ∩ $m_x^{(1,2)*}$-cl(S) = $m_x^{(1,2)*}$-int(U ∩ $m_x^{(1,2)*}$-cl(S)) = $m_x^{(1,2)*}$-int(S). We have S ⊆ $m_x^{(1,2)*}$-int(S). But $m_x^{(1,2)*}$-int(S) ⊆ S. Hence S is $m_x^{(1,2)*}$-open.

Definition: 4.2 [11] A mapping f: X ⟶ Y is said to be

(i) $M^{(1,2)*}$-continuous if f^{-1}(V) is $m_x^{(1,2)*}$-open in X for every $m_y^{(1,2)*}$-open subset V of Y.

(ii) $M^{(1,2)*}$-α-continuous if f^{-1}(V) is an $m_x^{(1,2)*}$-α-open in X for every $m_y^{(1,2)*}$-open subset V of Y.

We introduce a new mapping as follows:

Definition: 4.3 A mapping $f : X \longrightarrow Y$ is said to be $M^{(1,2)^*}$-LC continuous if $f^{-1}(V)$ is a locally $m_x^{(1,2)^*}$-closed in X for every $m_y^{(1,2)^*}$-open subset V of Y.

Theorem: 4.4 Assume that the $m_x^{(1,2)^*}$-closure of any subset of X is $m_x^{(1,2)^*}$-open. Let f: X ⟶ Y be a mapping where X has property [I]. Then the following are equivalent.

(i) f is $M^{(1,2)^*}$-continuous.

(ii) f is $M^{(1,2)^*}$-α-continuous and $M^{(1,2)^*}$-LC continuous.

Proof: It is a decomposition of $M^{(1,2)^*}$-continuity from Proposition 4.1.

5. DECOMPOSITION OF COMPLETE $M^{(1,2)^*}$-CONTINUITY:

Proposition: 5.1 Let (X, m_x^1, m_x^2) have property [u] and $S \subset X$. Then the following are equivalent.

(i) S is regular $m_x^{(1,2)^*}$-open.

(ii) S is $m_x^{(1,2)^*}$-open and $m_x^{(1,2)^*}$-semi-regular.

(iii) S is $m_x^{(1,2)^*}$-open and $m_x^{(1,2)^*}$-semi-closed.

(iv) S is $m_x^{(1,2)^*}$-open and $m_x^{(1,2)^*}$-sg-closed.

(v) S is $m_x^{(1,2)^*}$-α-open and $m_x^{(1,2)^*}$-sg-closed.

Proof:
(i) ⟹ (ii): By Proposition 3.16 and 3.19.
(ii) ⟹ (iii): By Remark 3.20.
(iii) ⟹ (IV): By Proposition 3.12.
(iv) ⟹ (v): It is obvious.
(v) ⟹ (i): Since S is $m_x^{(1,2)^*}$-α-open, S is $m_x^{(1,2)^*}$-preopen. Thus $S \subseteq m_x^{(1,2)^*}$-int($m_x^{(1,2)^*}$-cl(S))-(1).

Also since S is $m_x^{(1,2)^*}$-sg-closed and $m_x^{(1,2)^*}$-semi-open, $m_x^{(1,2)^*}$-scl(S) ⊆ S. But S ⊆ $m_x^{(1,2)^*}$-scl(S).

Therefore S = $m_x^{(1,2)^*}$-scl(S) and hence S is $m_x^{(1,2)^*}$-semi-closed. It implies $m_x^{(1,2)^*}$-int($m_x^{(1,2)^*}$-cl(S)) ⊆ S – (2). From (1) and (2), S is regular $m_x^{(1,2)^*}$-open.

Theorem: 5.2 For a biminimal space X, the following holds.

$m_x^{(1,2)^*}$-RO(X) = $m_x^{(1,2)^*}$-PO(X) ∩ $m_x^{(1,2)^*}$-SC(X).

Proof: Let S ∈ $m_x^{(1,2)^*}$-RO(X). Thus S = $m_x^{(1,2)^*}$-int($m_x^{(1,2)^*}$-cl(S)). Since S ⊆ $m_x^{(1,2)^*}$-int($m_x^{(1,2)^*}$-cl(S)) and $m_x^{(1,2)^*}$-int($m_x^{(1,2)^*}$-cl(S)) ⊆ S, S ∈ $m_x^{(1,2)^*}$-PO(X) and S ∈ $m_x^{(1,2)^*}$-SC(X). Thus
S ∈ $m_x^{(1,2)^*}$-PO(X) ∩ $m_x^{(1,2)^*}$-SC(X).

The converse part is obvious.

Theorem: 5.3 For a subset S of X, the following are equivalent.

(i) S is $m_x^{(1,2)^*}$-semi-open and $m_x^{(1,2)^*}$-sg-closed.

(ii) S is $m_x^{(1,2)^*}$-semi-regular.

Proof: (i) ⟹ (ii): Since S is $m_x^{(1,2)*}$-semi-open and $m_x^{(1,2)*}$-sg-closed, $m_x^{(1,2)*}$-scl(S) ⊆ S. Since $m_x^{(1,2)*}$-scl(S) = S ∪ $m_x^{(1,2)*}$-int($m_x^{(1,2)*}$-cl(S)) [13], then $m_x^{(1,2)*}$-int($m_x^{(1,2)*}$-cl(S)) ⊆ $m_x^{(1,2)*}$-scl(S) ⊆ S. That is, $m_x^{(1,2)*}$-int($m_x^{(1,2)*}$-cl(S)) ⊆ S. So S is $m_x^{(1,2)*}$-semi-closed. This proves that S is $m_x^{(1,2)*}$-semi-regular.
(ii) ⟹ (i): Since a $m_x^{(1,2)*}$-semi-regular set is both $m_x^{(1,2)*}$-semi-open and $m_x^{(1,2)*}$-semi-closed and every $m_x^{(1,2)*}$-semi-closed set is $m_x^{(1,2)*}$-sg-closed, hence S is both $m_x^{(1,2)*}$-semi-open set and $m_x^{(1,2)*}$-sg-closed set.

Definition: 5.4 A mapping f: X ⟶ Y is said to be
(i) Completely $M^{(1,2)*}$-continuous if f^{-1}(V) is regular $m_x^{(1,2)*}$-open set in X for every $m_y^{(1,2)*}$-open subset V of Y.
(ii) contra $M^{(1,2)*}$-sg-continuous if f^{-1}(V) is $m_x^{(1,2)*}$-sg-closed set in X for every $m_y^{(1,2)*}$-open subset V of Y.

Theorem: 5.5 Let f: X ⟶ Y be a mapping. Then the following are equivalent.
(i) f is completely $M^{(1,2)*}$-continuous.
(ii) f is $M^{(1,2)*}$-continuous and contra $M^{(1,2)*}$-sg-continuous.
(iii) f is $M^{(1,2)*}$-α-continuous and contra $M^{(1,2)*}$-sg-continuous.

Proof: It is the decomposition of complete $M^{(1,2)*}$-continuity from Proposition 5.1.

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