DECOMPOSITIONS OF $M^{(1,2)}$-CONTINUITY AND COMPLETE $M^{(1,2)}$-CONTINUITY IN BIMINIMAL SPACES

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ABSTRACT
The purpose of this paper is to introduce the concepts of $M^{(1,2)}$-continuity and complete $M^{(1,2)}$-continuity in biminimal spaces, and study some properties of the generalizations of $m^{(1,2)}_x$-closed sets and $m^{(1,2)}_x$-open sets in biminimal spaces.

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1. INTRODUCTION:

Njastad [8] introduced the concepts of an $\alpha$-sets and Mashhour et al [7] introduced $\alpha$-continuous mappings in topological spaces. The topological notions of semi-open sets and semi-continuity, and preopen sets and precontinuity were introduced by Levine [5] and Mashhour et al [6], respectively. The concepts of minimal structures (briefly m-structures) were developed by Popa and Noiri [9] in 2000. Kelly [4] introduced the notions of bitopological spaces. Such spaces are equipped with two arbitrary topologies. Ravi and Lellis Thivagar [10] introduced weakly open sets called $\tau_2$-open sets in bitopological spaces. In this paper, we introduce $M^{(1,2)}$-continuity and complete $M^{(1,2)}$-continuity, and obtain their decompositions in biminimal spaces. At every places the new notions have been substantiated with suitable examples.

2. PRELIMINARIES:

Definition 2.1 [9] Let X be a nonempty set and $\wp(X)$ the power set of X. A subfamily $m_x$ of $\wp(X)$ is called a minimal structure (briefly m-structure) on X if $\emptyset \in m_x$ and $X \in m_x$.

Definition 2.2 [11] A set X together with two minimal structures $m^1_x$ and $m^2_x$ on X is called a biminimal space and is denoted by $(X, m^1_x, m^2_x)$. 
Throughout this paper, \((X, m_x^1, m_x^2)\) (or X) denote biminimal space.

**Definition 2.3** [11] Let \(S\) be a subset of \(X\). Then \(S\) is said to be \(m_x^{(1,2)}\)-open if \(S = A \cup B\) where \(A \in m_x^1\) and \(B \in m_x^2\). We call \(m_x^{(1,2)}\)-closed set is the complement of \(m_x^{(1,2)}\)-open.

The family of all \(m_x^{(1,2)}\)-open subsets of \((X, m_x^1, m_x^2)\) is denoted by \(m_x^{(1,2)}\)-O(X).

**Example 2.4** [11] Let \(X = \{a, b, c\}\), \(m_x^1 = \{\emptyset, X, \{a\}\}\) and \(m_x^2 = \{\emptyset, X, \{b\}\}\). Then the sets in \(\{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}\) are called \(m_x^{(1,2)}\)-open and the sets in \(\{\emptyset, X, \{b\}, \{a, c\}, \{a\}\}\) are called \(m_x^{(1,2)}\)-closed.

**Definition 2.5** [11] Let \(S\) be a subset of \(X\). Then

(i) the \(m_x^{(1,2)}\)-interior of \(S\), denoted by \(m_x^{(1,2)}\)-int(S), is defined by \(\cup \{F/F \subseteq S\) and \(F \in m_x^{(1,2)}\)-open\};

(ii) the \(m_x^{(1,2)}\)-closure of \(S\), denoted by \(m_x^{(1,2)}\)-cl(S), is defined by \(\cap \{F/S \subseteq F\) and \(F \in m_x^{(1,2)}\)-closed\}.

**Definition 2.6** [11] Let \(S\) be a subset of \(X\). Then \(S\) is said to be

(i) \(m_x^{(1,2)}\)-\(\alpha\)-open if \(S \subseteq m_x^{(1,2)}\)-int\(m_x^{(1,2)}\)-cl\(m_x^{(1,2)}\)-int(S));

(ii) \(m_x^{(1,2)}\)-semi-open if \(S \subseteq m_x^{(1,2)}\)-cl\(m_x^{(1,2)}\)-int(S));

(iii) \(m_x^{(1,2)}\)-preopen if \(S \subseteq m_x^{(1,2)}\)-int\(m_x^{(1,2)}\)-cl(S));

(iv) \(m_x^{(1,2)}\)-\(\alpha\)-closed if \(m_x^{(1,2)}\)-cl\(m_x^{(1,2)}\)-int\(m_x^{(1,2)}\)-cl(S)) \(\subseteq S\);

(v) \(m_x^{(1,2)}\)-preclosed if \(m_x^{(1,2)}\)-cl\(m_x^{(1,2)}\)-int(S)) \(\subseteq S\);

(vi) \(m_x^{(1,2)}\)-Semi-closed if \(m_x^{(1,2)}\)-int\(m_x^{(1,2)}\)-cl(S)) \(\subseteq S\).

The family of all \(m_x^{(1,2)}\)-\(\alpha\)-open [resp. \(m_x^{(1,2)}\)-Semi-open, \(m_x^{(1,2)}\)-preopen]

**Example 2.7** [11] Let \(Y = \{p, q, r\}\), \(m_y^1 = \{\emptyset, Y, \{p\}, \{p, q\}\}\) and \(m_y^2 = \{\emptyset, Y, \{q\}\}\). Then the sets in \(\{\emptyset, Y, \{p\}, \{q\}, \{p, q\}\}\) are \(m_y^{(1,2)}\)-open and the sets in \(\{\emptyset, Y, \{r\}, \{q, r\}, \{p, r\}\}\) are \(m_y^{(1,2)}\)-closed.

We have

\[m_y^{(1,2)}\)-O(Y) = \{\emptyset, Y, \{p\}, \{q\}, \{p, q\}\};\]

\[m_y^{(1,2)}\)-SO(Y) = \{\emptyset, Y, \{p\}, \{q\}, \{p, q\}, \{p, r\}, \{q, r\}\} and\]

\[m_y^{(1,2)}\)-PO(Y) = \{\emptyset, Y, \{p\}, \{q\}, \{p, q\}\}.

**Lemma 2.8** [11] Let \(X\) be a non-empty set and \(m_x^1, m_x^2\) minimal structures on \(X\). For subsets \(A\) and \(B\) of \(X\), the following properties hold:

(i) \(A \subseteq m_x^{(1,2)}\)-cl(A) and \(m_x^{(1,2)}\)-int(A) \(\subseteq A\);

(ii) If \(A\) is \(m_x^{(1,2)}\)-open then \(A = m_x^{(1,2)}\)-int(A);

(iii) If \(A\) is \(m_x^{(1,2)}\)-closed then \(A = m_x^{(1,2)}\)-cl(A);

(iv) If \(A \subseteq B\) then \(m_x^{(1,2)}\)-cl(A) \(\subseteq m_x^{(1,2)}\)-cl(B);

(v) If \(A \subseteq B\) then \(m_x^{(1,2)}\)-int(A) \(\subseteq m_x^{(1,2)}\)-int(B);

(vi) \(m_x^{(1,2)}\)-cl(X - A) = X - \(m_x^{(1,2)}\)-int(A) and \(m_x^{(1,2)}\)-int(X - A) = X - \(m_x^{(1,2)}\)-cl(A);

(vii) \(m_x^{(1,2)}\)-cl(\(\emptyset\)) = \(\emptyset\) and \(m_x^{(1,2)}\)-cl(X) = X = \(m_x^{(1,2)}\)-int(X);

(viii) \(m_x^{(1,2)}\)-cl\(m_x^{(1,2)}\)-cl(A)) = \(m_x^{(1,2)}\)-cl(A) and \(m_x^{(1,2)}\)-int\(m_x^{(1,2)}\)-int(A)) = \(m_x^{(1,2)}\)-int(A).
Definition: 2.9 [11] A biminimal space \((X, m^1_x, m^2_x)\) has the property \([u]\) if the arbitrary union of \(m^{(1,2)^r}_x\)-open sets is \(m^{(1,2)^r}_x\)-open.

A biminimal space \((X, m^1_x, m^2_x)\) has the property \([l]\) if the any finite intersection of \(m^{(1,2)^r}_x\)-open sets is \(m^{(1,2)^r}_x\)-open.

Lemma: 2.10 [11] The following are equivalent for the biminimal space \((X, m^1_x, m^2_x)\).

1. \((X, m^1_x, m^2_x)\) have property \([u]\);
2. If \(m^{(1,2)^r}_x\)-int(E) = E, then \(E \in m^{(1,2)^r}_x\)-O(X).
3. If \(m^{(1,2)^r}_x\)-cl(F) = F, then \(F^c \in m^{(1,2)^r}_x\)-O(X).

3. CHARACTERIZATIONS:

Definition: 3.1 Let \(S\) be a subset of \(X\). Then \(S\) is said to be

(i) regular \(m^{(1,2)^r}_x\)-open if \(S = m^{(1,2)^r}_x\)-int(m^{(1,2)^r}_x-cl(S)),

(ii) \(m^{(1,2)^r}_x\)-Semi-regular if it is both

The family of all \(m^{(1,2)^r}_x\)-semi-closed [resp. regular \(m^{(1,2)^r}_x\)-open] sets of \(X\) is denoted by \(m^{(1,2)^r}_x\)-SC(X) [resp. \(m^{(1,2)^r}_x\)-RO(X)].

The intersection of all \(m^{(1,2)^r}_x\)-semi-closed sets of \(X\) containing a subset \(S\) of \(X\) is called the \(m^{(1,2)^r}_x\)-semi-closure of \(S\) and is denoted by \(m^{(1,2)^r}_x\)-scl(S).

Remark: 3.2 A subset \(S\) of \(X\) is \(m^{(1,2)^r}_x\)-semi-closed if and only if \(m^{(1,2)^r}_x\)-scl(S) = \(S\).

Definition: 3.3 [12] A subset \(S\) of \(X\) is said to be \(m^{(1,2)^r}_x\)-semi-generalized closed (briefly \(m^{(1,2)^r}_x\)-sg-closed) if and only if \(m^{(1,2)^r}_x\)-scl(S) \(\subseteq\) \(F\) whenever \(S \subseteq F\) and \(F\) is \(m^{(1,2)^r}_x\)-semi-open set.

The complement of \(m^{(1,2)^r}_x\)-sg-closed set is \(m^{(1,2)^r}_x\)-sg-open.

Example: 3.4 Let \(X = \{a, b, c\}\), \(m^1_x = \{\phi, X, \{a\}\}\) and \(m^2_x = \{\phi, X\}\). Then the sets in \(\{\phi, X, \{a\}\}\) are called \(m^{(1,2)^r}_x\)-open and the sets in \(\{\phi, X, \{b, c\}\}\) are called \(m^{(1,2)^r}_x\)-closed. Also, the sets in \(\{\phi, X, \{b\}, \{c\}, \{b, c\}\}\) are called \(m^{(1,2)^r}_x\)-sg-closed.

Definition: 3.5 A subset \(S\) of \(X\) is said to be locally \(m^{(1,2)^r}_x\)-closed if \(S = M \cap N\), where \(M\) is \(m^{(1,2)^r}_x\)-open and \(N\) is \(m^{(1,2)^r}_x\)-closed.

Remark: 3.6 [11] Every \(m^{(1,2)^r}_x\)-closed set is \(m^{(1,2)^r}_x\)-\(\alpha\)-closed but not conversely.

Example: 3.7 Let \(X = \{a, b, c, d\}\), \(m^1_x = \{\phi, X, \{a\}, \{a, b\}\}\) and \(m^2_x = \{\phi, X, \{a, c\}\}\). Then the sets in \(\{\phi, X, \{a\}, \{a, b\}\}\) are called \(m^{(1,2)^r}_x\)-open and the sets in \(\{\phi, X, \{d\}, \{b, d\}, \{c, d\}, \{b, c, d\}\}\) are called \(m^{(1,2)^r}_x\)-closed. We have \(\{c\}\) is \(m^{(1,2)^r}_x\)-\(\alpha\)-closed set but not \(m^{(1,2)^r}_x\)-closed.

Also, the sets in \(\{\phi, X, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{b, c, d\}\}\) are called locally \(m^{(1,2)^r}_x\)-closed.

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Proposition: 3.8 Every $m_i(1,2)^r$-closed set is locally $m_i(1,2)^r$-closed.

Proof: S = X ∩ S where X is $m_i(1,2)^r$-open and S is $m_i(1,2)^r$-closed. Thus S is locally $m_i(1,2)^r$-closed.

Example: 3.9 The converse of Proposition 3.8 is not true in general.

Consider the Example 3.7. We have {a} is locally $m_i(1,2)^r$-closed but not $m_i(1,2)^r$-closed.

Proposition: 3.10 A subset S of X is $m_i(1,2)^r$-α-closed if and only if S is $m_i(1,2)^r$-semi-closed and $m_i(1,2)^r$-preclosed.

Proof: Let S be $m_i(1,2)^r$-α-closed. Then $m_i(1,2)^r$-$\int(m_i(1,2)^r$-$\text{int}(m_i(1,2)^r$-$\text{cl}(S))) \subseteq S$. Hence $m_i(1,2)^r$-$\text{int}(m_i(1,2)^r$-$\text{cl}(S)) \subseteq m_i(1,2)^r$-$\text{cl}(m_i(1,2)^r$-$\text{cl}(S))) \subseteq S$. Thus S is $m_i(1,2)^r$-semi-closed. Also $m_i(1,2)^r$-$\text{cl}(m_i(1,2)^r$-$\text{int}(S)) \subseteq m_i(1,2)^r$-$\text{cl}(m_i(1,2)^r$-$\text{cl}(S))) \subseteq S$. Thus S is $m_i(1,2)^r$-preclosed. Conversely, let S be $m_i(1,2)^r$-semi-closed and $m_i(1,2)^r$-preclosed. Since S is $m_i(1,2)^r$-semi-closed, $m_i(1,2)^r$-$\text{int}(m_i(1,2)^r$-$\text{cl}(S)) \subseteq S$. Thus S is $m_i(1,2)^r$-preclosed. Conversely, let S be $m_i(1,2)^r$-semi-closed and $m_i(1,2)^r$-preclosed. Since S is $m_i(1,2)^r$-semi-closed, $m_i(1,2)^r$-$\text{int}(m_i(1,2)^r$-$\text{cl}(S)) \subseteq S$. Thus S is $m_i(1,2)^r$-preclosed.

Example: 3.11 A $m_i(1,2)^r$-semi-closed or $m_i(1,2)^r$-preclosed set need not be $m_i(1,2)^r$-α-closed.

(i) Let X = {a, b, c}, $m_1^x = \{\phi, X, \{a\}\}$ and $m_2^x = \{\phi, X, \{b\}\}$. Then the sets in $\{\phi, X, \{a\}, \{b\}, \{a, b\}\}$ are called $m_i(1,2)^r$-open and the sets in $\{\phi, X, \{b, c\}, \{a, c\}, \{c\}\}$ are called $m_i(1,2)^r$-closed. We have

1. $m_i(1,2)^r$-$\alpha(O(X) = \{\phi, X, \{a\}, \{b\}, \{a, b\})$;
2. $m_i(1,2)^r$-$SO(X) = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}$.

Therefore {b} is $m_i(1,2)^r$-semi-closed but not $m_i(1,2)^r$-$\alpha$-closed.

(ii) Let X = {a, b, c}, $m_1^x = \{\phi, X, \{b, c\}\}$ and $m_2^x = \{\phi, X, \{a, b\}\}$. Then the sets in $\{\phi, X, \{b, c\}, \{a, b\}\}$ are called $m_i(1,2)^r$-open and the sets in $\{\phi, X, \{a\}, \{c\}\}$ are called $m_i(1,2)^r$-closed. We have

1. $m_i(1,2)^r$-$\alpha(O(X) = m_i(1,2)^r$-$SO(X) = \{\phi, X, \{a, b\}, \{b, c\}$;
2. $m_i(1,2)^r$-$PO(X) = \{\phi, X, \{b\}, \{a, b\}, \{b, c\}, \{a, c\}$.

Therefore {b} is $m_i(1,2)^r$-preclosed but not $m_i(1,2)^r$-$\alpha$-closed.

Proposition: 3.12 A $m_i(1,2)^r$-semi-closed set is $m_i(1,2)^r$-sg-closed.

Proof: Let S be $m_i(1,2)^r$-semi-closed. Then $m_i(1,2)^r$-$cl(S) = S \subseteq G$ where S \subseteq G and G is $m_i(1,2)^r$-semi-open. Thus S is $m_i(1,2)^r$-sg-closed.

Remark: 3.13 A $m_i(1,2)^r$-sg-closed set need not be $m_i(1,2)^r$-semi-closed.

Consider the Example 3.11 (ii). We have {a, c} is $m_i(1,2)^r$-sg-closed but not $m_i(1,2)^r$-semi-closed.

Proposition: 3.14 A $m_i(1,2)^r$-closed set is $m_i(1,2)^r$-sg-closed.
Proof: Let S be $m_1^{(1,2)^r}$-closed. Then S is $m_1^{(1,2)^r}$-$\alpha$-closed and also by Proposition 3.10., S is $m_1^{(1,2)^r}$-semi-closed. Moreover, by Proposition 3.12., S is $m_1^{(1,2)^r}$-sg-closed.

Remark: 3.15 The converse of Proposition 3.14 is not true in general.

Consider the Example 3.4., $\{c\}$ is $m_1^{(1,2)^r}$-sg-closed but not $m_1^{(1,2)^r}$-closed.

Proposition: 3.16 Let $(X, m_1^{1}, m_2^{1})$ have property [u]. Then any regular $m_1^{(1,2)^r}$-open set is $m_1^{(1,2)^r}$-open.

Proof: Let S be regular $m_1^{(1,2)^r}$-open. Then $S = m_1^{(1,2)^r}$-$\text{int}(m_1^{(1,2)^r}$-$\text{cl}(S))$. We have $S = m_1^{(1,2)^r}$-$\text{int}(S)$ . Thus, by Lemma 2.10, S is $m_1^{(1,2)^r}$-open.

Remark: 3.17 The converse of Proposition 3.16 is not true in general.

Consider the Example 3.4., $\{a\}$ is $m_1^{(1,2)^r}$-open but not regular $m_1^{(1,2)^r}$-open.

Proposition: 3.18 Every regular $m_1^{(1,2)^r}$-open set is $m_1^{(1,2)^r}$-semi-closed.

Proof: Let S be regular $m_1^{(1,2)^r}$-open. Then $S = m_1^{(1,2)^r}$-$\text{int}(m_1^{(1,2)^r}$-$\text{cl}(S))$. We have $m_1^{(1,2)^r}$-$\text{int}(m_1^{(1,2)^r}$-$\text{cl}(S)) \subseteq S$. Thus S is $m_1^{(1,2)^r}$-semi-closed.

Proposition: 3.19 Let $(X, m_1^{1}, m_2^{1})$ have property [u]. Then every regular $m_1^{(1,2)^r}$-open set is $m_1^{(1,2)^r}$-semi-regular.

Proof: Let S be regular $m_1^{(1,2)^r}$-open. Then by Proposition 3.18., S is $m_1^{(1,2)^r}$-semi-closed and also by Proposition 3.16., S is $m_1^{(1,2)^r}$-open (and so S is $m_1^{(1,2)^r}$-semi-open). Hence S is both $m_1^{(1,2)^r}$-semi-open and $m_1^{(1,2)^r}$-semi-closed. Thus S is $m_1^{(1,2)^r}$-semi-regular.

Remark: 3.20 Every $m_1^{(1,2)^r}$-semi-regular set is $m_1^{(1,2)^r}$-semi-closed but not conversely.

Example: 3.21 Let $X=\{a, b, c\}$, $m_1^{1} = \{\emptyset, X, \{a\}\}$ and $m_2^{1} = \{\emptyset, X, \{b\}\}$. Then the sets in $\{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ are called $m_1^{(1,2)^r}$-open and the sets in $\{\emptyset, X, \{b, c\}, \{a, c\}, \{c\}\}$ are called $m_1^{(1,2)^r}$-closed.

We have

$m_1^{(1,2)^r}$-$\alpha(\text{O}(X)) = m_1^{(1,2)^r}$-$\text{PO}(X) = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$;

$m_1^{(1,2)^r}$-$\alpha(\text{SO}(X)) = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$;

$m_1^{(1,2)^r}$-$\alpha(\text{SC}(X)) = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$ and

$m_1^{(1,2)^r}$-$\alpha(\text{RO}(X)) = \{\emptyset, X, \{a\}, \{b\}\}$.

We have $\{c\}$ is $m_1^{(1,2)^r}$-semi-closed but not $m_1^{(1,2)^r}$-semi-regular.

Remark: 3.22 $m_1^{(1,2)^r}$-$\alpha$-closed sets and regular $m_1^{(1,2)^r}$-open sets are independent of each other.

Consider the Example 3.4. We have $\{c\}$ is $m_1^{(1,2)^r}$-$\alpha$-closed but not regular $m_1^{(1,2)^r}$-open and Consider the Example 3.21. We have $\{a\}$ is regular $m_1^{(1,2)^r}$-open but not $m_1^{(1,2)^r}$-$\alpha$-closed.
Remark: 3.23 \( m_x^{(1,2)^r} \)-preclosed sets and \( m_x^{(1,2)^r} \)-open sets are independent of each other. Consider the Example 3.21. We have \( \{c\} \) is \( m_x^{(1,2)^r} \)-preclosed but not \( m_x^{(1,2)^r} \)-open and \( \{a\} \) is \( m_x^{(1,2)^r} \)-open but not \( m_x^{(1,2)^r} \)-preclosed.

Remark: 3.24 locally \( m_x^{(1,2)^r} \)-closed sets and \( m_x^{(1,2)^r} \)-preclosed sets are independent of each other. Consider the Example 3.21. We have \( \{a\} \) is locally \( m_x^{(1,2)^r} \)-closed but not \( m_x^{(1,2)^r} \)-open and \( \{b\} \) is \( m_x^{(1,2)^r} \)-open but not locally \( m_x^{(1,2)^r} \)-closed.

Remark: 3.25 By the previous Propositions, Examples and Remarks, we obtain the following diagram where \( A \rightarrow B \) means \( A \) implies \( B \) but \( B \) does not imply \( A \) and \( A \Leftrightarrow B \) means \( A \) and \( B \) are independent.

4. DECOMPOSITION OF \( M^{(1,2)^r} \)-CONTINUITY:

Proposition: 4.1 Let \((X, m_x^1, m_x^2)\) have property [I]. Let \( S \) be a subset of \( X \) such that \( m_x^{(1,2)^r} \)-cl(\( S \)) \( \in m_x^{(1,2)^r}\)-O(\( X \)). Then the following are equivalent.

(i) \( S \) is \( m_x^{(1,2)^r} \)-open.

(ii) \( S \) is an \( m_x^{(1,2)^r} \)-\( \alpha \)-open and locally \( m_x^{(1,2)^r} \)-closed.

Proof: (i) \( \Rightarrow \) (ii): Let \( S \) be an \( m_x^{(1,2)^r} \)-open. Then \( S \) is \( m_x^{(1,2)^r} \)-\( \alpha \)-open. Also \( S = X \cap S \) where \( S \) is \( m_x^{(1,2)^r} \)-open and \( X \) is \( m_x^{(1,2)^r} \)-closed. Thus \( S \) is locally \( m_x^{(1,2)^r} \)-closed.

(ii) \( \Rightarrow \) (i): Let \( S \) be \( m_x^{(1,2)^r} \)-\( \alpha \)-open and locally \( m_x^{(1,2)^r} \)-closed.

Since \( S \) is \( m_x^{(1,2)^r} \)-preopen, \( S \subseteq m_x^{(1,2)^r} \)-int(\( m_x^{(1,2)^r} \)-cl(\( S \))). Since \( S \) is locally \( m_x^{(1,2)^r} \)-closed, \( S = U \cap m_x^{(1,2)^r} \)-cl(\( S \)) where \( U \) is \( m_x^{(1,2)^r} \)-open.

Also \( S = U \cap m_x^{(1,2)^r} \)-cl(\( S \)) \( \cap U = U \cap S \subseteq U \cap m_x^{(1,2)^r} \)-int(\( m_x^{(1,2)^r} \)-cl(\( S \)) \( \subseteq U \cap m_x^{(1,2)^r} \)-cl(\( S \) = \( m_x^{(1,2)^r} \)-int(U \( \cap m_x^{(1,2)^r} \)-cl(\( S \)) \( = m_x^{(1,2)^r} \)-int(\( S \)). We have \( S \subseteq m_x^{(1,2)^r} \)-int(\( S \)). But \( m_x^{(1,2)^r} \)-int(\( S \) \( \subseteq S \). Hence \( S \) is \( m_x^{(1,2)^r} \)-open.

Definition: 4.2 [11] A mapping \( f: X \rightarrow Y \) is said to be

(i) \( M^{(1,2)^r} \)-continuous if \( f^{-1}(V) \) is \( m_x^{(1,2)^r} \)-open in \( X \) for every \( m_x^{(1,2)^r} \)-open subset \( V \) of \( Y \).

(ii) \( M^{(1,2)^r} \)-\( \alpha \)-continuous if \( f^{-1}(V) \) is an \( m_x^{(1,2)^r} \)-\( \alpha \)-open in \( X \) for every \( m_x^{(1,2)^r} \)-open subset \( V \) of \( Y \).
We introduce a new mapping as follows:

**Definition: 4.3** A mapping \( f : X \rightarrow Y \) is said to be \( M^{(1,2)}\)-LC continuous if \( f^{-1}(V) \) is a locally \( M^{(1,2)} \)-closed in \( X \) for every \( M^{(1,2)} \)-open subset \( V \) of \( Y \).

**Theorem: 4.4** Assume that the \( M^{(1,2)} \)-closure of any subset of \( X \) is \( M^{(1,2)} \)-open. Let \( f : X \rightarrow Y \) be a mapping where \( X \) has property \([I]\). Then the following are equivalent.

(i) \( f \) is \( M^{(1,2)} \)-continuous.

(ii) \( f \) is \( M^{(1,2)} \)-\( \alpha \)-continuous and \( M^{(1,2)} \)-LC continuous.

**Proof:** It is a decomposition of \( M^{(1,2)} \)-continuity from Proposition 4.1.

5. **DECOMPOSITION OF COMPLETE \( M^{(1,2)} \)-CONTINUITY:**

**Proposition: 5.1** Let \((X, M^{1}_{x}, M^{2}_{x})\) have property \([u]\) and \( S \subseteq X \). Then the following are equivalent.

(i) \( S \) is regular \( M^{(1,2)} \)-open.

(ii) \( S \) is \( M^{(1,2)} \)-open and \( M^{(1,2)} \)-semi-regular.

(iii) \( S \) is \( M^{(1,2)} \)-open and \( M^{(1,2)} \)-semi-closed.

(iv) \( S \) is \( M^{(1,2)} \)-open and \( M^{(1,2)} \)-sg-closed.

(v) \( S \) is \( M^{(1,2)} \)-\( \alpha \)-open and \( M^{(1,2)} \)-sg-closed.

**Proof:**

(i) \( \Rightarrow \) (ii): By Proposition 3.16 and 3.19.

(ii) \( \Rightarrow \) (iii): By Remark 3.20.

(iii) \( \Rightarrow \) (iv): By Proposition 3.12.

(iv) \( \Rightarrow \) (v): It is obvious.

(v) \( \Rightarrow \) (i): Since \( S \) is \( M^{(1,2)} \)-\( \alpha \)-open, \( S \) is \( M^{(1,2)} \)-preopen. Thus \( S \subseteq M^{(1,2)} \)-\( \text{int}(M^{(1,2)} \text{-cl}(S)) \). (1)

Also since \( S \) is \( M^{(1,2)} \)-sg-closed and \( M^{(1,2)} \)-semi-open, \( M^{(1,2)} \)-\( \text{scl}(S) \) \( \subseteq S \). But \( S \subseteq M^{(1,2)} \)-\( \text{scl}(S) \).

Therefore \( S = M^{(1,2)} \)-\( \text{scl}(S) \) and hence \( S \) is \( M^{(1,2)} \)-semi-closed. It implies \( M^{(1,2)} \)-\( \text{int}(M^{(1,2)} \text{-cl}(S)) \) \( \subseteq S \) – (2). From (1) and (2), \( S \) is regular \( M^{(1,2)} \)-open.

**Theorem: 5.2** For a biminimal space \( X \), the following holds.

\[
m^{(1,2)} \text{-RO}(X) = m^{(1,2)} \text{-PO}(X) \cap m^{(1,2)} \text{-SC}(X).
\]

**Proof:** Let \( S \in m^{(1,2)} \text{-RO}(X) \). Thus \( S = m^{(1,2)} \text{-\text{int}(M^{(1,2)} \text{-cl}(S))} \). Since \( S \subseteq M^{(1,2)} \text{-\text{int}(M^{(1,2)} \text{-cl}(S))} \) and \( m^{(1,2)} \text{-\text{int}(M^{(1,2)} \text{-cl}(S))} \subseteq S \), \( S \in M^{(1,2)} \text{-\text{PO}(X)} \) and \( S \in M^{(1,2)} \text{-\text{SC}(X)} \). Thus \( S \in m^{(1,2)} \text{-\text{RO}(X)} \) and \( S \in m^{(1,2)} \text{-\text{PO}(X)} \cap m^{(1,2)} \text{-\text{SC}(X)} \).

The converse part is obvious.

**Theorem: 5.3** For a subset \( S \) of \( X \), the following are equivalent.

(i) \( S \) is \( M^{(1,2)} \)-semi-open and \( M^{(1,2)} \)-sg-closed.

(ii) \( S \) is \( M^{(1,2)} \)-semi-regular.
Proof: (i) \implies (ii): Since $S$ is $m_x^{(1,2)*}$-semi-open and $m_x^{(1,2)*}$-sg-closed, $m_x^{(1,2)*}$-scl($S$) \subseteq S. Since $m_x^{(1,2)*}$-scl($S$) = $S \cup m_x^{(1,2)*}$-int($m_x^{(1,2)*}$-cl($S$)) [13], then $m_x^{(1,2)*}$-int($m_x^{(1,2)*}$-cl($S$)) \subseteq m_x^{(1,2)*}$-scl($S$) \subseteq S. That is, $m_x^{(1,2)*}$-int($m_x^{(1,2)*}$-cl($S$)) \subseteq S$. So $S$ is $m_x^{(1,2)*}$-semi-closed. This proves that $S$ is $m_x^{(1,2)*}$-semi-regular.

(ii) \implies (i): Since a $m_x^{(1,2)*}$-semi-regular set is both $m_x^{(1,2)*}$-semi-open and $m_x^{(1,2)*}$-semi-closed and every $m_x^{(1,2)*}$-semi-closed set is $m_x^{(1,2)*}$-sg-closed, hence $S$ is both $m_x^{(1,2)*}$-semi-open set and $m_x^{(1,2)*}$-sg-closed set.

Definition: 5.4 A mapping $f: X \to Y$ is said to be

(i) Completely $M^{(1,2)*}$-continuous if $f^{-1}(V)$ is regular $m_y^{(1,2)*}$-open set in $X$ for every $m_y^{(1,2)*}$-open subset $V$ of $Y$.

(ii) contra $M^{(1,2)*}$-sg-continuous if $f^{-1}(V)$ is $m_y^{(1,2)*}$-sg-closed set in $X$ for every $m_y^{(1,2)*}$-open subset $V$ of $Y$.

Theorem: 5.5 Let $f: X \to Y$ be a mapping. Then the following are equivalent.

(i) $f$ is completely $M^{(1,2)*}$-continuous.

(ii) $f$ is $M^{(1,2)*}$-continuous and contra $M^{(1,2)*}$-sg-continuous.

(iii) $f$ is $M^{(1,2)*}$-$\alpha$-continuous and contra $M^{(1,2)*}$-sg-continuous.

Proof: It is the decomposition of complete $M^{(1,2)*}$-continuity from Proposition 5.1.

REFERENCES:


[13] Ravi, O., Balamurugan, R. G and Krishnamoorthy, M. Decompositions of $M^{(1,2)^*}$-continuity and $M^{(1,2)^*}$-$\alpha$-continuity in biminimal spaces, (submitted).


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