



GENERALIZED SEMI CLOSED SETS IN BIGENERALIZED TOPOLOGICAL SPACES

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ABSTRACT

The aim of the paper is to introduce the concept of $\mu_{(m,n)}$ -generalized semi closed sets in bigeneralized topological spaces and study some of their properties.

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Keywords: *generalized topological spaces, generalized semi topological spaces, bigeneralized topological spaces, $\mu_{(m,n)}$ -generalized semi closed set.*

1. INTRODUCTION:

Generalized closed sets in a topological space were introduced by Levine [6] in order to extend many of the important properties of closed sets to a larger family. For instance, it was shown that compactness; normality and completeness in a uniform space are inherited by generalized closed subsets. The study of bitopological spaces was first initiated by Kelly [5] and thereafter a large number of papers have been done to generalize the topological concepts to bitopological setting. Fukutake [6] introduced generalized closed sets and pairwise generalized closure operator in bitopological spaces. Csaszar [2] introduced the concepts of generalized neighborhood systems and generalized topological spaces. Boonpok [1] introduced the concept of bigeneralized topological spaces and studied (m, n) -closed sets and (m, n) -open sets in bigeneralized topological spaces.

In this paper, we introduce the notions of $\mu_{(m,n)}$ -gs closed sets in bigeneralized topological spaces and study some of their properties.

2. PRELIMINARIES:

We recall some basic definitions and notations. Let X be a set and denote $\exp X$ the power set of X . A subset μ of $\exp X$ is said to be a *generalized topology* (briefly GT) on X if $\emptyset \in \mu$ and an arbitrary union of elements of μ belongs to μ [2]. Let μ be a GT on X , the elements of μ are called μ -open sets and the complements of μ -open sets are called μ -closed sets. If $A \subseteq X$, then $i_\mu(A)$ denotes the union of all μ -open sets contained in A and $c_\mu(A)$ is the intersection of all μ -closed sets containing A [3].

Proposition: 2.1 [8] Let (X, μ) be a generalized topological space. For sub sets A and B of X , the following properties hold:

- (1) $c_\mu(X - A) = X - i_\mu(A)$ and $i_\mu(X - A) = X - c_\mu(A)$;
- (2) If $(X - A) \in \mu$ then $c_\mu(A) = A$ and if $A \in \mu$ then $i_\mu(A) = A$;
- (3) If $A \subseteq B$, then $c_\mu(A) \subseteq c_\mu(B)$ and $i_\mu(A) \subseteq i_\mu(B)$;
- (4) $A \subseteq c_\mu(A)$ and $i_\mu(A) \subseteq A$;
- (5) $c_\mu(c_\mu(A)) = c_\mu(A)$ and $i_\mu(i_\mu(A)) = i_\mu(A)$.

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Definition: 2.2 [1] Let X be a nonempty set and let μ_1, μ_2 be generalized topologies on X . The triple (X, μ_1, μ_2) is said to be a *bigeneralized topological space* (briefly BGTS).

Let (X, μ_1, μ_2) be a bigeneralized topological space and A be a subset of X .

The closure of A and the interior of A with respect to μ_m are denoted by $c_{\mu_m}(A)$ and $i_{\mu_m}(A)$ respectively, for $m = 1, 2$.

Definition: 2.3 [1] A subset A of a bigeneralized topological space (X, μ_1, μ_2) is called *(m, n)-closed* if $c_{\mu_m}(c_{\mu_n}(A)) = A$, where $m, n = 1, 2$ and $m \neq n$. The complement of a (m, n) -closed set is called *(m, n)-open*.

Proposition: 2.4 [1] Let (X, μ_1, μ_2) be a bigeneralized topological space and A be a subset of X . Then A is (m, n) -closed if and only if A is both μ -closed in (X, μ_m) and (X, μ_n) .

Proposition: 2.5 [1] Let (X, μ_1, μ_2) be a bigeneralized topological space. Then A is (m, n) -open if and only if $i_{\mu_m}(i_{\mu_n}(A)) = A$.

Definition: 2.6 [7] A subset A of a bigeneralized topological space (X, μ_1, μ_2) is said to be *(m, n) generalized closed* (briefly $\mu_{(m,n)}$ -closed) if $c_{\mu_n}(A) \subseteq U$ whenever $A \subseteq U$ and U is a μ_m -open set in X , where $m, n = 1, 2$ and $m \neq n$. The complement of a $\mu_{(m,n)}$ -closed set is said to be *(m, n) generalized open* (briefly $\mu_{(m,n)}$ -open).

Definition: 2.7 A subset A of a bigeneralized topological space (X, μ_1, μ_2) is said to be *(m, n) semi closed* if $i_{\mu_m}(c_{\mu_n}(A)) \subseteq A$, where $m, n = 1, 2$ and $m \neq n$. The complement of (m, n) semi closed set is called *(m, n) semi open*.

3. GENERALIZED SEMI CLOSED SETS:

In this section, we introduce $\mu_{(m,n)}$ -gs closed sets in bigeneralized topological spaces and study some of their properties.

Definition: 3.1 A subset A of a bigeneralized topological space (X, μ_1, μ_2) is said to be *(m, n) generalized semi closed* (briefly $\mu_{(m,n)}$ -gs closed) if $sc_{\mu_n}(A) \subseteq U$ whenever $A \subseteq U$ and U is a μ_m -open set in X , where $m, n = 1, 2$ and $m \neq n$. The complement of a $\mu_{(m,n)}$ -gs closed set is said to be *(m, n) generalized semi open* (briefly $\mu_{(m,n)}$ -gs open).

The family of all $\mu_{(m,n)}$ -gs closed (resp. $\mu_{(m,n)}$ -gs open) sets of (X, μ_1, μ_2) is denoted by $\mu_{(m,n)}$ -GSC(X) (resp. $\mu_{(m,n)}$ -GSO(X)), where $m, n = 1, 2$ and $m \neq n$.

A subset A of a bigeneralized topological space (X, μ_1, μ_2) is called *pairwise μ -gs closed* if A is $\mu_{(1,2)}$ -gs closed and $\mu_{(2,1)}$ -gs closed. The complement of a pairwise μ -gs closed set is called *pairwise μ -gs open*.

Lemma: 3.2 Every (m, n) -closed set is $\mu_{(m,n)}$ -gs closed. The converse is not true as can be seen from the following example.

Example: 3.3 Let $X = \{a, b, c\}$. Consider two generalized topologies $\mu_1 = \{\emptyset, \{a\}, \{a, b\}, X\}$ and $\mu_2 = \{\emptyset, \{c\}, \{b, c\}\}$ on X . Then $\{a\}$ is $\mu_{(1,2)}$ -gs closed but not $(1, 2)$ -closed.

Proposition: 3.4 Let (X, μ_1, μ_2) be a bigeneralized topological space and A is a subset of X . If A is μ_n -gs closed, then A is $\mu_{(m,n)}$ -gs closed, where $m, n = 1, 2$ and $m \neq n$.

Lemma: 3.5 The union of two $\mu_{(m,n)}$ -gs closed sets is not a $\mu_{(m,n)}$ -gs closed set in general as can be seen from the following example.

Example: 3.6 Let $X = \{a, b, c, d\}$. Consider two generalized topologies $\mu_1 = \{\emptyset, \{a, b\}, \{b, c\}, \{a, b, c\}, X\}$ and $\mu_2 = \{\emptyset, \{a, b, d\}, \{b, c, d\}, X\}$ on X . Then $\{a\}$ and $\{c\}$ are $\mu_{(1,2)}$ -gs closed but $\{a\} \cup \{c\} = \{a, c\}$ is not $\mu_{(1,2)}$ -gs closed.

Proposition: 3.7 Let (X, μ_1, μ_2) be a bigeneralized topological space. If A is $\mu_{(m,n)}$ -gs closed and F is (m, n) closed, then $A \cap F$ is $\mu_{(m,n)}$ -gs closed, where $m, n = 1, 2$ and $m \neq n$.

Proof: Let $A \cap F \subseteq U$ and U be μ_m -open. Then $A \subseteq U \cup (X - F)$. By Proposition 2.4, F is μ_m -closed.

Hence $sc_{\mu_n}(A) \subseteq U \cup (X - F)$. Therefore, $sc_{\mu_n}(A) \cap F \subseteq U$. Since F is (m, n) closed, by Proposition 2.4, F is μ_n -closed which in turn implies that F is μ_n -semi closed. Hence $sc_{\mu_n}(F) = F$.

We obtain $sc_{\mu_n}(A \cap F) \subseteq sc_{\mu_n}(A) \cap sc_{\mu_n}(F) = sc_{\mu_n}(A) \cap F \subseteq U$. Hence, $A \cap F$ is $\mu_{(m, n)}$ -gs closed.

Proposition: 3.8 Let μ_1 and μ_2 be generalized topologies on X . If $\mu_1 \subseteq \mu_2$ then $\mu_{(2, 1)}\text{-GSC}(X) \subseteq \mu_{(1, 2)}\text{-GSC}(X)$.

Proposition: 3.9 For each element x of a bigeneralized topological space (X, μ_1, μ_2) , $\{x\}$ is μ_m -closed or $X - \{x\}$ is $\mu_{(m, n)}$ -gs closed, where $m, n = 1, 2$ and $m \neq n$.

Proof: Let $x \in X$ and the singleton $\{x\}$ be not μ_m -closed. Then $X - \{x\}$ is not μ_m -open. If $X \in \mu_m$, then X is the only μ_m -open set which contains $X - \{x\}$. Hence $X - \{x\}$ is $\mu_{(m, n)}$ -gs closed and if $X \notin \mu_m$, then $X - \{x\}$ is $\mu_{(m, n)}$ -gs closed as there is no μ_m -open set which contains $X - \{x\}$ and hence the condition is satisfied vacuously.

Proposition: 3.10 Let A be a subset of a bigeneralized topological space (X, μ_1, μ_2) . If A is $\mu_{(m, n)}$ -gs closed, then $sc_{\mu_n}(A) - A$ contains no non-empty μ_m -closed set, where $m, n = 1, 2$ and $m \neq n$.

Proof: Let A be a $\mu_{(m, n)}$ -gs closed set and $F \neq \emptyset$ is a μ_m -closed set such that $F \subseteq sc_{\mu_n}(A) - A$. Then $A \subset X - F$, $X - F$ is μ_m -open and since A is $\mu_{(m, n)}$ -gs closed, we have $sc_{\mu_n}(A) \subseteq X - F$. Therefore, $F \subseteq X - sc_{\mu_n}(A)$. Thus $F \subseteq sc_{\mu_n}(A) \cap (X - sc_{\mu_n}(A)) = \emptyset$. This is a contradiction. Thus $sc_{\mu_n}(A) - A$ contains no non-empty μ_m -closed set.

Proposition: 3.11 Let μ_1 and μ_2 be generalized topologies on X . If A is $\mu_{(m, n)}$ -gs closed, then $c_{\mu_m}(\{x\}) \cap A \neq \emptyset$ holds for each $x \in sc_{\mu_n}(A)$, where $m, n = 1, 2$ and $m \neq n$.

Proof: Let $x \in sc_{\mu_n}(A)$. Suppose that $c_{\mu_m}(\{x\}) \cap A = \emptyset$. Then $A \subseteq X - c_{\mu_m}(\{x\})$. Since A is $\mu_{(m, n)}$ -gs closed and $X - c_{\mu_m}(\{x\})$ is μ_m -open, we get $sc_{\mu_n}(A) \subseteq X - c_{\mu_m}(\{x\})$. Hence, $sc_{\mu_n}(A) \cap c_{\mu_m}(\{x\}) = \emptyset$. This is a contradiction.

Proposition: 3.12 If A is a $\mu_{(m, n)}$ -gs closed set of (X, μ_1, μ_2) such that $A \subseteq B \subseteq sc_{\mu_n}(A)$, then B is a $\mu_{(m, n)}$ -gs closed set, where $m, n = 1, 2$ and $m \neq n$.

Proof: Let A be a $\mu_{(m, n)}$ -gs closed set and $A \subseteq B \subseteq sc_{\mu_n}(A)$. Let $B \subseteq U$ and U is μ_m -open. Then $A \subseteq U$. Since A is $\mu_{(m, n)}$ -gs closed, we have $sc_{\mu_n}(A) \subseteq U$. Since $B \subseteq sc_{\mu_n}(A)$, then $sc_{\mu_n}(B) \subseteq sc_{\mu_n}(A) \subseteq U$. Hence B is $\mu_{(m, n)}$ -gs closed.

Theorem: 3.13 A subset A of a bigeneralized topological space (X, μ_1, μ_2) is $\mu_{(m, n)}$ -gs open iff for every subset F of X , $F \subseteq si_{\mu_n}(A)$ whenever F is μ_m -closed and $F \subseteq A$, where $m, n = 1, 2$ and $m \neq n$.

Proof: Suppose that A is $\mu_{(m, n)}$ -gs open. Let $F \subseteq A$ and F be μ_m -closed. Then $X - A \subseteq X - F$ and $X - F$ is μ_m -open. Since $X - A$ is $\mu_{(m, n)}$ -gs closed, $sc_{\mu_n}(X - A) \subseteq X - F$. Thus $X - si_{\mu_n}(A) \subseteq X - F$ and hence $F \subseteq si_{\mu_n}(A)$.

Conversely, suppose that $F \subseteq si_{\mu_n}(A)$ whenever F is μ_m -closed and $F \subseteq A$. Let $X - A \subseteq U$ and U is μ_m -open. Then $X - U \subseteq A$ and $X - U$ is μ_m -closed. By assumption, we have $X - U \subseteq si_{\mu_n}(A)$. Then $X - si_{\mu_n}(A) \subseteq U$. Therefore, $sc_{\mu_n}(X - A) \subseteq U$. Thus, $X - A$ is $\mu_{(m, n)}$ -gs closed. Hence A is $\mu_{(m, n)}$ -gs open.

Theorem: 3.14 Let A and B be subsets of a bigeneralized topological space (X, μ_1, μ_2) such that $si_{\mu_n}(A) \subseteq B \subseteq A$. If A is $\mu_{(m, n)}$ -gs open then B is $\mu_{(m, n)}$ -gs open, where $m, n = 1, 2$ and $m \neq n$.

Proof: Suppose that $si_{\mu_n}(A) \subseteq B \subseteq A$. Let F be μ_m -closed such that $F \subseteq B$. Then $F \subseteq A$ also. Since A is $\mu_{(m,n)}$ -gs open, $F \subseteq si_{\mu_n}(A)$. Since $si_{\mu_n}(A) \subseteq B$, we have $si_{\mu_n}(si_{\mu_n}(A)) \subseteq si_{\mu_n}(B)$.

Consequently, $si_{\mu_n}(A) \subseteq si_{\mu_n}(B)$.

Hence $F \subseteq si_{\mu_n}(B)$. Therefore, B is $\mu_{(m,n)}$ -gs open.

Proposition: 3.15 If a subset A of a bigeneralized topological space (X, μ_1, μ_2) is $\mu_{(m,n)}$ -gs closed, then $sc_{\mu_n}(A) - A$ is $\mu_{(m,n)}$ -gs open, where $m, n = 1, 2$ and $m \neq n$.

Proof: Suppose that A is $\mu_{(m,n)}$ -gs closed. Let $X - (sc_{\mu_n}(A) - A) \subseteq U$ and U be μ_m -open. Then $X - U \subseteq sc_{\mu_n}(A) - A$ and $X - U$ is μ_m -closed. By proposition 3.10, $sc_{\mu_n}(A) - A$ cannot contain a non-empty μ_m -closed set. Consequently, $X - U = \emptyset$ and hence $U = X$. Therefore, $sc_{\mu_n}(X - (sc_{\mu_n}(A) - A)) \subseteq U$ so we obtain $X - (sc_{\mu_n}(A) - A)$ is $\mu_{(m,n)}$ -gs closed. Hence, $sc_{\mu_n}(A) - A$ is $\mu_{(m,n)}$ -gs open.

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