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N-CLASS A COMPOSITION OPERATORS

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ABSTRACT

In this paper we introduce generalization of concept of class A operator, N-class A operators. We will characterize N-class A composition operators and we will show that N-class A and $(M,2)_N$ operators coincides in the case of composition operators.

Key words and phrases: *Class A operator, paranormal operator and composition operators.*

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1. INTRODUCTION AND PRELIMINARIES:

Let (X,Σ,λ) be a sigma finite measure space. Composition operator C on $L^2(X,\Sigma,\lambda)$ induced by a non singular measurable transformation T from X into itself with the condition that the measure λT^{-1} is absolutely continuous with respect to the measure λ and the Radon-Nikodym derivative $\left(\frac{d\lambda T^{-1}}{d\lambda}\right)=f_0$ is essentially bounded is given by

$$Cf=(f\circ T) \text{ For every } f\in L^2(X,\Sigma,\lambda)$$

The Radon-Nikodym derivative of the measure $\lambda(T^k)^{-1}$ with respect to λ is denoted by $f_0^{(k)}$, where T^k is obtained by composing $T-k$ times. Every essentially bounded complex valued measurable function f_0 induces the bounded operator M_{f_0} on $L^2(\lambda)$, which is defined by

$$M_{f_0}f=f_0f \text{ for every } f\in L^2(\lambda).$$

Let $B(H)$ denote the algebra of all bounded linear operator acting on a Hilbert space H . Generalization of concepts of paranormal, hyponormal, N-paranormal, N-quasi-hyponormal are studied in [1] by Braha et al. For an operator T in $B(H)$ we say that N-quasi-hyponormal, if $N\|T^2x\|\geq\|T^*Tx\|$, T is N-paranormal, if $N\|T^2x\|\geq\|Tx\|^2$ for a fixed $N>0$.An operator $T\in B(H)$ is said to be class A if $|T^2|\geq|T|^2$ [7], class $(M,2)_N$ if $NT^{*2}T^2\geq(T^*T)^2$.

2. N-CLASS A OPERATORS:

Let us define N-class A operators as follows:

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Definition: 2.1. An operator $T \in B(H)$ is N-class A if $N\left|T^2\right| \geq |T|^2$ for a fixed $N > 0$.

Theorem: 2.2. Let $T \in B(H)$. If T is N-class A, then $\|Tx\| \leq N\left\|\left(T^{*2}T^2\right)^{\frac{1}{2}} x\right\|\|x\|$ for all $x \in H$.

Proof: Let T is N-class A. Then $N\left|T^2\right| \geq |T|^2$ holds for a fixed $N > 0$. Respectively

$$N\left(T^{*2}T^2\right)^{\frac{1}{2}} \geq T^*T \Leftrightarrow N\left(T^{*2}T^2\right)^{\frac{1}{2}} - T^*T \geq 0$$

$$\Leftrightarrow \left(\left(N\left(T^{*2}T^2\right)^{\frac{1}{2}} - T^*T\right)x, x\right) \geq 0 \text{ for all } x \in H$$

$$\Leftrightarrow \left(N\left(T^{*2}T^2\right)^{\frac{1}{2}} x, x\right) \geq (Tx, Tx) \quad \text{for all } x \in H$$

Thus,

$$\Leftrightarrow N\left\|\left(T^{*2}T^2\right)^{\frac{1}{2}}\right\|\|x\| \geq \|Tx\| \quad \text{for all } x \in H$$

Theorem: 2.3. Let $T \in B(H)$. Then T is N-class A unilateral weighted shift operator with weighted sequence α_n, then $\alpha_n{}^2\alpha_{n+2}{}^2 \geq \frac{1}{\sqrt{N}}\alpha_n{}^4, n = 1, 2, 3...$

Proof: Suppose T is N-class A, then we have $N\left|T^2\right| \geq |T|^2$ holds for a fixed $N > 0$.

Thus,

$$\left(N\left(T^{*2}T^2\right)^{\frac{1}{2}} - T^*T\right)e_n \geq 0$$

Since $NT^{*2}T^2 e_n = N\alpha_n{}^2\alpha_{n+2}{}^2 e_n$ and since $T^*Te_n = \alpha_n{}^2 e_n$, we have

$$N\left(\alpha_n{}^2\alpha_{n+2}{}^2\right)^{\frac{1}{2}} \geq \alpha_n{}^2$$

And so,

$$\alpha_n{}^2\alpha_{n+2}{}^2 \geq \frac{1}{\sqrt{N}}\alpha_n{}^4$$

By definition it is clear that every N-class A operator is N-paranormal.

3. N-CLASS COMPOSITION OPERATORS:

Composition operator on class A is studied in [9] by Panayappan and Senthilkumar. Now we characterize N-class A composition operators as follows.

The relation of being equal almost everywhere, denoted by a.e , is an equivalence relation in $L^2(X, \Sigma, \lambda)$ and this equivalence relation splits $L^2(X, \Sigma, \lambda)$ into equivalence classes.

Theorem: 3.1. Let $C \in B\left(L^2(\lambda)\right)$. Then C is N-class A if and only if $f_0 \leq N\left(f_0{}^2\right)^{\frac{1}{2}}$ a.e.

Proof:. By definition 2.1, C is N-class A if and only if $N\left|C^2\right| \geq |C|^2$.Thus

$$\left(\left(N\left|C^2\right| - |C|^2\right)\chi_E, \chi_E\right) \geq 0$$

For every characteristic function χ_E of E in Σ. Since $C^{*2}C^2 = f_0^{(2)}$ and since $C^*C = f_0$, we have

$$\int_E \left(Nf\left(f_0{}^2\right)^{\frac{1}{2}} - f_0 \right) d\lambda \geq 0$$

Thus C is N-class A if and only if $f_0 \leq N\left(f_0{}^2\right)^{\frac{1}{2}}$ a.e.

Example: 3.2 Let X=N, the set of all natural numbers and λ be the counting measure on it. Define $T : N \to N$ by $T(1) = T(2) = T(3) = T(4) = 1$, $T(5n+m) = n+1$ for $m = 0,1,2,3,4$ and $n \in N$. Then $f_0 \leq N\left(f_0{}^2\right)^{\frac{1}{2}}$ a.e. for every $n \in N$.

Theorem: 3.3. Let $C \in B\left(L^2(\lambda)\right)$. Then C is $(M,2)_N$ if and only if $N\left(f_0{}^{(2)}\right)^{\frac{1}{2}} \geq f_0{}^2$ a.e.

Proof: The proof of the theorem is similar to that o theorem 3.1.

Theorem: 3.4. Let $C \in B\left(L^2(\lambda)\right)$. Then C is $(M,2)_N$ if and only if C is N-class A.

Proof of the above theorem is clear from 3.1 and theorem 3.3.

Theorem: 3.5. Let $C \in B\left(L^2(\lambda)\right)$. If C is N-class A, then C is $\sqrt{N}-$paranormal.

Proof: If C is N-class A, then C is $(M,2)_N$ by theorem 3.4. Then by applying [1] [Proposition 4.3], C is $\sqrt{N}-$ paranormal.

In [2] Sh. Lohaj introduced N-quasi-normal operators. An operator is N-quasi-normal if $T\left(T^*T\right) = N\left(\left(T^*T\right)T\right)$. Now we characterize N-quasi-normal operators as follows.

Theorem: 3.6. Let $C \in B\left(L^2(\lambda)\right)$. Then C is N-quasinormal if and only if $f_0 = N\, f_0 \circ T$ a.e.

Proof: Since C is N-quasinormal, $C\left(C^*C\right) = N\left(\left(C^*C\right)C\right)$

Thus,

$$\begin{aligned}
& CM_{f_0} = NM_{f_0}C \\
& \Leftrightarrow M_{f_0 \circ T}C = NM_{f_0}C \\
& \Leftrightarrow \left(M_{f_0 \circ T} - NM_{f_0}\right)C = 0 \\
& \Leftrightarrow f_0 = N\, f_0 \circ T \text{ a.e.}
\end{aligned}$$

4. WEIGHTED N-CLASS A COMPOSITION OPERATORS:

If ω is a non negative complex valued $\sum$ measurable function, then the weighted composition operator W on $L^2(X, \Sigma, \lambda)$ induced by a non singular measurable transformation T from X into given by

$$Wf = \omega(f \circ T), f \in L^2(X, \Sigma, \lambda)$$

In case that $\omega = 1$, we say that W is a composition operator denoted by C. To examine the weighted composition operators effectively Alan Lambert [8] associated conditional expectation operator E with T as $E\left(./T^{-1}\Sigma\right) = E(.).E(f)$ is defined for each non-negative measurable function $f \in L^p\, (1 \leq p)$ and is uniquely determined by the condition

(i) $E(f)$ is $T^{-1}\sum$ measurable.

(ii) If B is any $T^{-1}\sum$ measurable set for which $\int_B f\, d\lambda$ converges we have $\int_B f\, d\lambda = \int_B E(f)\, d\lambda.$

As an operator on L^p, E is the projection on to the closure of range of C. E is the identity on L^p if and only if $T^{-1}\sum=\sum$. Detailed discussion of E is found in [4, 5, 6].

The following theorem due to Campbell and Jamison [4] is well known.

Theorem: 4.1 For $\omega \geq 0$

$$i) W^{*}Wf = f_0 E\left(\omega^2\right) \circ T^{-1} f$$

$$ii) WW^{*} f = \omega\left(f_0 \circ T\right) E\left(\omega f\right)$$

Following theorem characterizes weighted N-class A composition operators.

Theorem: 4.2 Let $W \in B\left(L^2(\lambda)\right)$.Then W is a N-class A if and only if

$N\left(f_0{}^2 E\left(\omega_2{}^2 \circ T^{-2}\right)\right) \geq \left(f_0 E\left(\omega^2\right) \circ T^{-1}\right)$ a.e.

Proof: By definition C is N-class A if and only if $N\left|W^2\right| \geq \left|W\right|^2$. Thus

$$\left(\left(N\left|W^2\right| - \left|W\right|^2\right)\chi_E, \chi_E\right) \geq 0$$

For every characteristic function χ_E of E in $\sum$. Since $W^{*2}W^2 f = f_0{}^2 E\left(\omega_2^2 \circ T^{-2}\right) f$ and since $W^{*}Wf = f_0 E\left(\omega^2\right) \circ T^{-1} f$, we have

$$\int_E \left(N\left(f_0{}^2 E\left(\omega_2^2 \circ T^{-2}\right)\right)^{\frac{1}{2}} - \left(f_0 E\left(\omega^2\right) \circ T^{-1}\right) \right) d\lambda \geq 0$$

Thus,

W is a N-class A if and only if $N\left(f_0{}^2 E\left(\omega_2^2 \circ T^{-2}\right)\right)^{\frac{1}{2}} \geq \left(f_0 E\left(\omega^2\right) \circ T^{-1}\right)$ a.e.

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