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ωI -NORMAL AND ωI -REGULAR SPACES

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ABSTRACT

I_g -normal and I_g –regular space are introduced and investigated by M. Navaneethakrishnan et.al [2]. In this paper we introduce and investigate some properties of ωI -normal and ωI -regular spaces via ideal.

Key words: *ωI -closed sets and ωI –open sets, completely condense ideals, ω -closed sets, ω -normal and ω -regular spaces.*

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1. Introduction and Preliminaries:

An ideal *I* on a topological space is a collection of subsets of X which satisfies (i) $A \in I$ and $B \subset A$ implies $B \in I$ and (ii) $A \in I$ and $B \in I$ implies $A \cup B \in I$. Given a topological space (X, τ) with an ideal *I* on X and if $\wp(X)$ is the set of all subsets of X, a set operator (.)*: $\wp(X) \to \wp(X)$, called a local function [2] of A with respect to τ and I is defined as follows: for $A \subset X$, $A^*(I,\tau) = \{x \in X / U \cap A \notin I for every U \in \tau(x)\}$ where $\tau(x) = U \in \tau / x \in U$ .We will make use of the basic facts about the local functions without mentioning it explicitly. A Kuratowski closure operator cl*(.) for a topology $\tau^*(I,\tau)$ called the * -topology, finer than τ and is defined by $cl^*(A) = A \cup A^*(I,\tau)$.when there is no chance for confusion we simply write A^* instead of $A^*(I,\tau)$ and τ^* or $\tau^*(I)$ .X* is often a proper subset of X. The hypothesis X=X* is equivalent to the hypothesis $\tau \cap I = \phi$ .For every ideal topological space (X,τ,I) there exist a topology, $\tau^*(I)$ finer than τ and β generated by $(I,\tau) = (U \setminus I : U \in \tau and I \in I)$ but in general $\beta(I,\tau)$ is not always a topology [2].If *I* is an ideal on X ,then (X,τ,I) is called ideal space. By an ideal space we always mean an ideal topological space (X,τ,I) with no separation properties assumed. If $A \subset X$, cl(A) and int(A) will respectively denote the closure and interior of A in $(X,\tau,)$ and $cl^*(A)$ and $\operatorname{int}^*(A)$ will respectively denote the closure and interior of A in (X,τ,I)

Definition: 1.1 [7] A subset *A* of a topological space (*X, τ*) is said to be αgs -closed, if $\alpha cl(A) \subset U$ whenever $A \subset U$ and $U \in$ SO(*X, τ*).

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Definition: 1.2 [1] A subset A of an ideal topological space (X,τ,I) is called ωI – closed if $cl^*(A)\subseteq U$ whenever A ⊆ U and U is semi- I- open in (X,τ,I) .

Lemma: 1.1 [6] Let (X,τ,I) be an ideal space. If I is completely condense, then $\tau^*\subset\tau^\alpha$.

Lemma: 1.2 [1, Theorem 2.10] In an ideal topological space (X,τ,I) the following are equivalent:

(i) A is ωI -closed

(ii) $cl^*(A)\subseteq U$ for some semi-I-open set U containing A.

Lemma: 1.3 Let (X,τ,I) be an ideal space. Then every subset of X is ωI –closed if and only if every semi-I-open set is *-closed.

Proof: Suppose that every subset of X is ωI - closed. Let U be a semi -I-open set then U is ωI -closed and $U^*\subset U$ implies $cl^*(U)\subset U$. Hence U is *-closed.

Conversely, suppose that every semi-I-open set is *-closed. Let A be non empty subset of X contained in a semi-I-open set U. Then $cl^*(A)\subset U^*$ implies $cl^*(A)\subset U$. This proves that A is ωI -closed.

Lemma: 1.4 A set A is ωI –open if and only if $F\subseteq \text{int}^*(A)$ whenever F is semi –I-closed and $F\subseteq A$.

Proof: Suppose that $F\subseteq\text{int}^*(A)$, where F is semi-I-closed and $F\subseteq A$ .Let $A^c\subseteq U$, where U is semi-I-open .Then $U^c\subseteq A$ and U^c is semi-I-closed. Therefore, $U^c\subseteq\text{int}^*(A)$ .Since $U^c\subseteq\text{int}^*(A)$, we have $(\text{int}^*(A))^c\subseteq U$, i.e. $cl^*(A^c)\subseteq U$, since $cl^*(A^c)=(\text{int}^*(A))^c$ .Thus A^c is ωI –closed, i.e. A is ωI –open.

2. ωI -Normal spaces:

Definition: 2.1 An ideal space (X,τ,I) is said to be an ωI -normal if for every pair of disjoint closed sets A and B, there exist ωI – open sets U and V such that $A\subseteq U$ and $B\subseteq V$.

Remark 2.1 Since every open set is ωI -open, every normal space is ωI - normal. But a normal space need not be ωI -normal as the following example shows.

Example: 2.1 Let $X=\{a,b,c\}$, $\tau=\{X,\phi,\{b\},\{a,b\},\{b,c\}\}$, $I=\{\phi,\{b\}\}$. Then $\varphi^*=\varphi$, $(\{a,b\})^*=\{a\},(\{b,c\})^*=\{c\},\{b\}^*=\varphi,$ and $X^*=(\{a,c\})$ .Since every semi-I-open set is *-closed and so by lemma 1.3, every subset of X is ωI - closed and hence every subset of X is ωI -open. This implies that (X,τ,I) is ωI - normal. Now, $\{a\}$ and $\{c\}$ are disjoint closed subsets of X which are not separated by disjoint open sets and so (X,τ) is not normal.

Theorem: 2.1 Let (X,τ,I) an ideal space where I is completely condense. Then the following are equivalent.

(i) (X,τ,I) is normal.

(ii) For every pair of disjoint closed sets A and B containing A, there exists disjoint ωI - open set U and V of X such that $A\subset U$ and $B\subset V$.

(iii) For every closed set A and an open set V containing A, there exists an ωI - -open set U such that $A\subset U\subset Cl^*(U)\subset V$

Proof: $(i) \Rightarrow (ii)$ It is clear, since every open set is ωI -open.

$(ii) \Rightarrow (iii)$ Let A be a closed set and V be an open set containing A. Since A and X-V are disjoint closed sets, there exist disjoint ωI - open sets U and W such that $A \subset U$ and $X - V \subset W$. Again $U \cap W = \phi$ implies that $U \cap \text{int}^*(W) = \phi$ and so $Cl^*(U) \subset X - \text{int}^*(W)$. Since X-V is closed and W is ωI -open, $X - V \subset W$ implies that $X - V \subset \text{int}^*(W)$ and so $X - \text{int}^*(W) \subset V$. Thus, we have, $A \subset U \subset Cl^*(U) \subset X - \text{int}*(W) \subset V$ which proves (c).

$(iii) \Rightarrow (i)$ Suppose A and B are disjoint closed subsets of X. By hypothesis, there exists a ωI - open set U of X such that $A \subset U \subset Cl^*(U) \subset X - B$. Since U is ωI - open $A \subset \text{int}^*(U)$ .Since , $PO(X) \cap I = \{\varphi\}$ by Lemma1.1 $\tau^* \subset \tau^\alpha$ and so $\text{int}^*(U), X \setminus cl^*(U) \in \tau^\alpha$ .Thus, $A \subseteq \text{int}^*(U) \subseteq \text{int}(cl(\text{int}(\text{int}^*(U)))) = G(say)$ and $B \subset X \setminus cl*(U) \subseteq \text{int}(cl(\text{int}(X \setminus cl^*(U)))) = H(say)$ .To show that (X, τ) is normal, it suffices to show that $G \cap H = \varphi$ .In fact $x \in G \cap H \Rightarrow \text{int}(cl(\text{int}(\text{int}^*(U))))$ and $x \in H \Rightarrow x \in cl(\text{int}(\text{int}^*(U)))$ and $x \in H \in \tau \Rightarrow$ there exists $y \in \text{int}(\text{int}^*(U))$ and $y \in H \subset cl(\text{int}(X \setminus cl^*(U))) \Rightarrow \text{int}(\text{int}^*(U)) \cap \text{int}(X \setminus cl^*(U)) \neq \varphi \Rightarrow$ G and H are disjoint required disjoint open sets containing A and B respectively, which proves (i).

Theorem: 2.2 In an ideal space the following are equivalent.

(i) (X, τ, I) is ωI – normal.

(ii) For every pair of disjoint closed sets A and B containing A, there exists disjoint ωI open set U and V of X such that $A \subset U$ and $B \subset V$.

(iii) For every closed set A and an open set V containing A, there exists an ωI -open set U such that $A \subset U \subset Cl^*(U) \subset V$

Proof: $(i) \Rightarrow (ii)$ The proof follows from the definition of ωI -normal space.

$(ii) \Rightarrow (iii)$ Let A be a closed set and V be an open set containing A. Since A and X-V are disjoint closed sets, there exist disjoint ωI -open sets U and W such that $A \subset U$ and $X - V \subset W$. Again $U \cap W = \phi$ implies that $U \cap \text{int}^*(W) = \phi$ and so $Cl^*(U) \subset X - \text{int}^*(W)$. Since X-V is closed and W is ωI –open, $X - V \subset W$ implies that $X - V \subset \text{int}^*(W)$ and so $X - \text{int}^*(W) \subset V$. Thus, we have, $A \subset U \subset Cl^*(U) \subset X - \text{int}*(W) \subset V$ which proves (c).

$(iii) \Rightarrow (i)$ Suppose A and B are disjoint closed sub sets of X. By hypothesis, there exists a ωI - open set U of X such that $A \subset U \subset Cl^*(U) \subset X - B$. If $W = X - cl^*(U)$ then U and W are disjoint ωI -open sets containing A and B respectively. Therefore, (X, τ, I) is ωI –normal.

Theorem: 2.3 Let (X, τ, I) an ideal space where *I* is completely condense. If (X, τ, I) is ωI -normal then it is a normal space.

Proof: Suppose that I is completely condense. By Theorem 4.3.1, (X, τ, I) is ωI normal if and only if each pair of disjoint ωI –open sets U and V such that $A \subset U$ and $B \subset V$ if and only if X is normal, by theorem 2.1.

Theorem: 2.4 Let (X, τ, I) be an ωI – normal space. If F is closed and A is ω - closed set such that $A \cap F = \varphi$, there exist disjoint ωI -open sets U and V such that $A \subset U$ and $F \subset V$.

Proof: Since $A \cap F = \varphi$, $A \subset X - F$. where $X - F$ is semi open .Therefore, by hypothesis $cl(A) \subset X - F$. Since $cl(A) \cap F = \varphi$ and X is ωI – normal, there exist disjoint ωI -open sets U and V such that $A \subset U$ and $F \subset V$.

The following corollary 2.1 gives properties of normal spaces. If $I=\{\varphi\}$ in theorem 2.4 then we have the following corollary 2.1 the proof of which follows from theorem 2.3 and Remark 2.2, since $\{\varphi\}$ is a completely condense ideal. If $I = \mathcal{N}$ in theorem 2.4, we have the corollary 2.2, since $\tau^*(\mathcal{N})=\tau^\alpha$ and ωI -open sets coincide with αgs –open sets.

Corollary: 2.1 Let (X,τ) be a normal space. If F is semi closed and A is ω - closed set disjoint from F, then there exist disjoint ω-open sets U and V such that $A\subset U$ and $F\subset V$.

Corollary: 2.2 Let (X,τ,I) be an ideal space where $I = \mathcal{N}$. If F is a closed set and A is an ω –closed set disjoint from F, there exist disjoint αgs –open sets U and V such that $A\subset U$ and $F\subset V$.

Theorem: 2.5 Let (X,τ,I) be an ideal space which is ωI – normal .Then the following hold.

(i) For every closed set A and an ω -open set B containing A, there exists an ωI -open set U such that $A\subset \operatorname{int}^*(U)\subset U\subset B$

(ii) For every ω -closed set A and every open set B containing A, there exists an ωI -open set U such that $A\subset U\subset Cl^*(U)\subset B.$

Proof: (i) Let A be a closed set and B be an ω-open set containing A. Then $A\cap(X-B)=\phi$ where A is closed and X-B is ω -closed sets. By theorem 4.4.4 there exist disjoint ωI -open sets U and V such that $A\subset U$ and $X-B\subset V$. Since $U\cap V=\phi$, we have $U\subset X-V$ by Theorem 2.3.4 $A\subset \operatorname{int}^*(U)$. Therefore, $A\subset \operatorname{int}^*(U)\subset U\subset X-V\subset B$ This proves (i).

(ii) Let A be a ω closed set and B be a semi open set containing A. Then $X-B$ is a semi closed set contained in the ω-open set X-A. By (a), there exists an ωI -open set V such that $X-B\subset \operatorname{int}^*(V)\subset V\subset X-A.$ Therefore, $A\subset X-V\subset cl^*(X-V)\subset B.$ If $U=X-V,$ then $A\subset U\subset Cl^*(U)\subset B$ and so U is the required ωI -closed set.

The following Corollaries 2.3 and 2.4 give some properties of normal spaces. If $I=\{\varphi\}$ in theorem 2.5 then we have the following corollary 2.2. If $I = \mathcal{N}$ is theorem 2.5, then we have the following corollary 2.3.

Corollary: 2.3 Let (X,τ,I) be a normal space. Then the following hold.

(i) For every closed set A and an ω-open set B containing A, there exists an ω-open set U such that $A\subset \operatorname{int}(U)\subset U\subset B$

(ii) For every ω -closed set A and every open set B containing A, there exists an ωI -open set U such that $A\subset U\subset Cl(U)\subset B$

Corollary: 2.4 Let (X,τ,I) be a normal space. Then the following hold.

(i) For every closed set A and an ω -open set B containing A, there exists an αgs -open set U such that $A\subset \operatorname{int}_\alpha(U)\subset U\subset B$

(ii) For every ω -closed set A and every open set B containing A, there exists an αgs -closed set U such that $A\subset U\subset Cl_\alpha(U)\subset B.$

Definition: 2.2 An ideal space (X,τ,I) is said to be an I_ω – normal if for every pair of disjoint – ωI closed sets A and B , there exist open sets U and V such that $A\subseteq U$ and $B\subseteq V$ .Since every closed set is ω I-closed, every I_ω -normal space is normal. But a normal space need not be I_ω-normal as the following example shows.

Example: 2.2 Let $X=\{a,b,c,d\}$, $\tau=\{X,\phi,\{a\}\}$, $I=\{\phi,\{a\}\}$. Since $\{a\}^*=\varphi$, $X^*=(\{a,c\})$ Since every semi-I-open set is *-closed and so by Theorem 2.3.8, every subset of X is ωI - closed. Now $A=\{a,b\}$ and $B=\{c,d\}$ are disjoint ωI -closed sets but they are not separated by disjoint open sets and so (X,τ,I) is not I_ω-normal. Since there is no pair of disjoint closed sets, (X,τ,I) is normal.

Theorem: 2.6 In an ideal space the following are equivalent.

(i) (X,τ,I) is I_ω – normal.

(ii) For every ωI closed set A and every ωI -open set B containing A, there exists an open set U of X such that $A\subset U\subset Cl(U)\subset V$

Proof: $(i)\Rightarrow(ii)$ Let A be a ωI closed set and V be an ωI -open set containing A. Since A and X-B are disjoint ωI closed sets, there exist disjoint open sets U and V such that $A\subset U$ and $X-B\subset V$. Again $U\cap V=\phi$ implies that $cl(U)\subset X-V$ Therefore, $A\subset U\subset Cl(U)\subset X-V\subset B.$ This proves (ii)

$(ii)\Rightarrow(i)$ Suppose A and B are disjoint ωI - closed sub sets of X then the ωI - closed set A is contained in the ωI - open set X-B. By hypothesis, there exists an open set U of X such that $A\subset U\subset Cl(U)\subset X-B.$ .If $V=X-Cl(U)$,then U and V are disjoint open sets containing A and B respectively. Therefore, (X,τ,I) is I_ω – normal. This proves (i).

If $I=\{\varphi\}$, then I_ω –normal spaces coincide with ω–normal spaces and so if we take $I=\{\varphi\}$ in theorem 2.6, then we have the following characterization for ω–normal spaces.

Corollary: 2.5 In a space (X,τ) the following are equivalent.

(i) X is ω–normal.

(ii) For every ω closed set A and every ω-open set B containing A, there exists an open set U of X such that $A\subset U\subset Cl(U)\subset V$

Theorem: 2.7 In an ideal space (X,τ,I) the following are equivalent.

(i) (X,τ,I) is I_ω – normal.

(ii) For every pair of disjoint ωI - closed sets A and B, there exists disjoint an open set U of X containing A such that $cl(U)\cap B=\varphi$

(iii) For every pair of disjoint ωI - closed sets A and B, there exists an open set U containing A and an open set V containing B such that $cl(U)\cap cl(V)=\varphi$.

Proof: $(i)\Rightarrow(ii)$ suppose that A and B are disjoint ωI – closed subsets of X. Then the ωI closed set A is contained in the ωI – open set X-B. By theorem 2.5 there exists an open set U such that $A\subset U\subset Cl(U)\subset X-B$ .Therefore, U is the required open set containing A such that $Cl(U)\cap B=\varphi$.

$(ii)\Rightarrow(iii)$ .Let A and B be two disjoint ω I-open sets of X. By hypothesis, there exists an open set U containing A such that $Cl(U)\cap B=\varphi$.Also, $Cl(U)$ and B are disjoint ωI – closed sets of X. By hypothesis, there exists an open set V containing B such that $cl(U)\cap cl(V)=\varphi$

$(iii)\Rightarrow(i)$ .The proof is clear.

If $I=\{\varphi\}$, in theorem 2.7 then we have the following characterizations of ω- normal spaces.

Corollary: 2.6 In a space (X,τ), the following are equivalent.

(i) (X,τ) is ω – normal.

(ii) For every pair of disjoint ω - closed sets A and B, there exists disjoint semi-open set U of X containing A such that $cl(U)\cap B=\varphi$

(iii) For every pair of disjoint ω - closed sets A and B, there exists disjoint semi-open set U of X containing A and a open set V containing B such that $cl(U)\cap cl(V)=\varphi$.

Theorem: 2.8 Let (X,τ,I) be I_{ω}– normal. If A and B are two disjoint ωI -closed subsets of X, then there exists disjoint open sets U and V such that $Cl^{*}(A)\subset U$ and $Cl^{*}(B)\subset V$.

Proof: Suppose that A and B are two disjoint ωI -closed sets. By theorem 2.7 (iii) there exists an open set V containing B such that $cl(U)\cap cl(V)=\varphi$. Since A is ωI -closed set, $A\subset U$ implies that $Cl^{*}(A)\subset U$. Similarly, $Cl^{*}(B)\subset V$.

If $I=\{\varphi\}$, in theorem 2.8 then we have the following property of disjoint ω-closed sets in ω - normal spaces.

Corollary: 2.7 Let (X,τ) be a ω – normal. If A and B are two disjoint ω-closed subsets of X, then there exists disjoint open sets U and V such that $Cl(A)\subset U$ and $Cl(B)\subset V$.

Theorem: 2.9 Let (X,τ,I) is I_{ω} – normal. If A is ωI -closed set and B is an ωI -open set containing A, then there exists an open set U such that $A\subset Cl^{*}(A)\subset U\subset \operatorname{int}^{*}(B)\subset B$.

Proof: Suppose A is an ωI -closed set and B is an ωI-open set containing A. Since A and X-B are disjoint ωI -closed sets, by theorem 2.7, there exist disjoint open sets U and V such that $Cl^{*}(A)\subset U$ and $Cl^{*}(X-B)\subset V$ .Now $X-\operatorname{int}^{*}(B)=Cl^{*}(X-B)\subset V$ implies that $X-V\subset \operatorname{int}^{*}(B)$. Again $U\cap V=\varphi$ implies that $U\subset X-V$ and so $A\subset Cl^{*}(A)\subset U\subset X-V\subset \operatorname{int}^{*}(B)\subset B.$

If $I=\{\varphi\}$, in theorem 2.9, then we have the following corollary 2.8

Corollary: 2.8 Let (X,τ) is ω – normal. If A is ω-closed set and B is a ω-open set containing A, then there exists an open set U such that $A\subset Cl(A)\subset U\subset \operatorname{int}(B)\subset B$.

The following Theorem 14 gives a characterization of normal spaces in terms of ω–open sets which follows from lemma 2.1 if $I=\{\varphi\}$.

Theorem: 2.9 Let (X,τ) be a space. Then the following are equivalent.

(a) X is normal.

(b) For every pair of disjoint closed sets A and B containing A, there exists disjoint ω- open sets U and V of X such that $A\subset U$ and $B\subset V$.

(c) For every closed set A and an open set V containing A, there exists a ω -open set U such that $A\subset U\subset Cl(U)\subset V$

3. ωI -Regular spaces:

Definition: 3.1 An ideal space (X,τ,I) is said to be ωI -regular if for each pair consisting of a point x and a closed set B not containing x, there exist disjoint ωI - open sets U and V such that $x\in U$ and $B\subset V$. .

Remarks: 3.1 It is obvious that every regular space is ωI -regular, since every open set is ωI -open. The following example 3.1 shows that an ωI –regular space need not be regular.

Example: 3.1 Consider the ideal space of Example 2.1.Then $\varphi^* = \varphi$, $(\{a,b\})^* = \{a\}, (\{b,c\})^* = \{c\}, \{b\}^* = \varphi,$ and $X^* = (\{a,c\})$ Since every semi-I-open set is *-closed and so by lemma 1.3, every subset of X is ωI - closed and hence every subset of X is ωI -open. This implies that (X,τ,I) is ωI - normal. Now, $\{c\}$ is a closed set not containing $a \in X$. $\{c\}$ and $\{a\}$ are not separated by disjoint open sets and so (X,τ) is not regular.

Theorem: 3.1 Let (X,τ,I) an ideal space. Then the following are equivalent.

(i) (X,τ,I) is ωI –regular

(ii) For every closed set B not containing $x \in U$, there exists disjoint ωI open set U and V of X such that $x \in U$ and $B \subset V.$

(iii) For every open V containing $x \in X$, there exists an ωI -open set U such that $x \in U \subset Cl^*(U) \subset V$

Proof: $(i) \Rightarrow (ii)$ It is clear, since every open set is ωI -open.

$(ii) \Rightarrow (iii)$ Let V be an open subset such that such that $x \in V$. Then $X - V$ is a closed set not containing x. Therefore, there exist disjoint ωI –open sets U and W such that $x \in U$ and $X - V \subset W$. Now, $X - V \subset W$ implies that $X - V \subset \text{int}^*(W)$ and so $X - \text{int}^*(W) \subset V.$ Again $U \cap W = \phi$ implies that $U \cap \text{int}^*(W) = \phi$ and so, $Cl^*(U) \subset X - \text{int}*(W)$ Therefore $x \in U \subset Cl^*(U) \subset V.$ This proves (iii)

$(iii) \Rightarrow (i)$ Let B be a closed set not containing x.By hypothesis, there exists an ωI - open set U of X such that $x \in U \subset Cl^*(U) \subset X - B.$ If $W = X - cl^*(U)$ then U and W are disjoint ωI - open sets such that $x \in U$ and $B \subset W$. This proves (i).

Theorem: 3.2 If (X,τ,I) is an ωI –regular, T_1-space where I is completely condense, then X is regular.

Proof: Let B be a closed set not containing $x \in X$. By Theorem 3.1 there exists an ωI - open set U of X such that $x \in U \subset Cl^*(U) \subset X - B.$ Since X is a T_1-space, $\{x\}$ is closed and so $\{x\} \subset \text{int}^*(U)$ by lemma 1.4 .Since I is completely condense, $\tau^* \subset \tau^\alpha$ and so, $\text{int}^*(U) = X - cl^*(U)$ are $\tau^\alpha -$open sets. Now,
$x \in \text{int}^*(U) \subset \text{int}(cl(\text{int}^*(U)))) = G$ and $B \subset X - cl^*(U) \subset \text{int}(cl(\text{int}(X - cl^*(U))) = H$ .Then G and H are disjoint open sets containing x and B respectively. Therefore, X is regular.

If $I = \{\varphi\}$ in Theorem 3.2, then we have the following corollary 3.1 which gives characterizations of regular spaces.

Corollary: 3.1 If (X,τ,I) is a T_1-space , then the following are equivalent.

(a) X is regular .

(b) For every closed set B not containing $x \in U$, there exists disjoint ω open set U and V of X such that $x \in U$ and $B \subset V.$

(c) For every open set V containing $x \in X$, there exists a ω-open set U such that $x \in U \subset Cl(U) \subset V$

Theorem: 3.2 If every semi-I- open subset of an ideal space (X,τ,I) is $*$-closed, then (X,τ,I) is ωI –regular**.**

Proof: Suppose every semi-I- open subset of an ideal space (X,τ,I) is $*$-closed .Then by lemma 1.4,every subset of X is ωI –closed and hence every subset of X is ωI -open. If B is a closed set not containing x, then $\{x\}$ and B are the required disjoint ωI –open sets containing x and B respectively. Therefore, (X,τ,I) is ωI –regular.

The following example 3.2 shows that the reverse direction of the above theorem 3.2 is not true.

Example: 3.2 Consider the real line R with usual topology. Let $I = \{\varphi\}$. Then R is regular and hence ωI –regular. But semi-I-open sets are not semi-I-closed and hence semi-I-open sets are not $*$-closed.

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