International Journal of Mathematical Archive-2(12), 2011, Page: 2598-2600
Available online through www.ijma.info ISSN 2229 – 5046

FIXED POINT THEOREM IN PSUEDOCOMPACT TYCHONOV'S SPACE

Rajesh Shrivastava and Manavi Kohli*

Govt. Science and Commerce College Benazir, Bhopal, India

Corresponding author mail address: manavi.kohli@rediffmail.com

(Received on: 07-12-11; Accepted on: 23-12-11)

ABSTRACT

In this paper we have established fixed point theorem for contractive type maps on Psuedocompact Tychonov space. Our results generalize the corresponding results of Edelstein [6], Fisher [3], Leader [7], Harinath [1], Jain and Dixit [2], Naidu [5].

INTRODUCTION:

Harinath [1] earlier established some fixed point theorems in Psuedocompact Tychonov space. Jain and Dixit [2] also established some fixed point theorems in Psuedocompact Tychonov space, which generalizes the result of Fisher [3], Harinath [1] and Liu Zeqing [4] and also established some coincidences point theorems in Psuedocompact Tychonov space. The notion of Psuedocompact Tychonov space is defined as follows:

A topological space X is said to be Psuedocompact if every real- valued continuous function of X is bounded. It is known that every compact space is Psuedocompact, but the converse need not be true. We define a non – negative function F on $X \times X$ satisfying the following properties –

(i) $F(x, y) = 0$, if $x = y$; $x, y \in X$.
(ii) $F(x, y) = F(y, x)$; $x, y \in X$
(iii) $F(x, y) \leq F(x, z) + F(z, y)$; $x, y, z \in X$
(iv) F is lower semi- continuous.

It is well known that if X is psuedocompact then its product $X \times X$ need not be psuedocompact and therefore, any continuous real valued function defined on $X \times X$ need not be bounded. But we can construct a function on X which is equivalent to a function on $X \times X$, so that it is bounded.

Theorem: Let P be a Psuedocompact Tychonov's space and d be a non – negative real valued continuous function over $P \times P$ ($P \times P$ is a Tychonov's space but need not be Psuedocompact). Suppose d also satisfies the condition:

$$d(STx, Sy) < \alpha_1 \frac{d(Tx, STx)\, d(y, Sy)}{d(Tx, Sy) + d(Tx, y)} + \alpha_2 \frac{d(Tx, Sy)\, d(y, STx)}{d(y, STx) + d(Tx, y)} + \alpha_3\, [d(Tx, STx) + d(y, Sy)]$$
$$+ \alpha_4\, [d(Tx, Sy) + d(Tx, y)] + \alpha_5\, [d(Tx, Sy) + d(y, STx)] + \alpha_6\, [d(y, STx) + d(Tx,y)] \quad (1)$$

for all $x, y \in P$ with $Tx \neq y$, $Tx \neq Sy$, $STx \neq y$
where $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6 > 0$, $\alpha_1 + 2\alpha_3 + \alpha_4 + 2\alpha_5 + \alpha_6 < 1$ and $\alpha_1 + \alpha_2 + 4\alpha_4 + 4\alpha_5 + 4\alpha_6 < 1$

Then S and T have unique common fixed point.

Proof: Let $\varphi: P \to R$ by $\varphi(p) = d(STp, Tp)$ fpr all $p \in P$ where R is the set of real numbers. Clearly φ is continuous functions, since P is Psuedo Compact Tychonov's space, every real bounded continuous function over P is bounded and attain its bounds, thus there exists a point $v \in P$ such that

***Corresponding author: Manavi Kohli*,*E-mail: manavi.kohli@rediffmail.com**

$\varphi(v) = \inf\{\varphi(p): p \in P\}$, where the inf denotes the infimum or the greatest lower bound in R.

We now affirm that v is a fixed point for S. If not, let us suppose that

$Sv \neq v$, then using (1), we have,

$$\varphi(Sv) = d(STSv, TSv)$$

$$= d(STSv, STv)$$

$$< \alpha_1 \frac{d(TSv, STSv)\, d(Tv, STv)}{d(TSv, STv) + d(TSv, Tv)} + \alpha_2 \frac{d(TSv, STv)\, d(Tv, STSv)}{d(Tv, STSv) + d(TSv, Tv)} + \alpha_3 [d(TSv, STSv) + d(Tv, STv)]$$
$$+ \alpha_4 [d(TSv, STv) + d(TSv, Tv)] + \alpha_5 [d(TSv, STv) + d(Tv, STSv)] + \alpha_6 [d(Tv, STSv) + d(TSv, Tv)]$$

$$< \alpha_1 d(STSv, STv) + \alpha_2 \times 0 + \alpha_3 [d(TSv, STSv) + d(Tv, STv)] + \alpha_4 d(STv, Tv) + \alpha_5 d(Tv, STSv)$$
$$+ \alpha_6 [d(Tv, STSv) + d(STv, Tv)]$$

$$= \alpha_1 d(STSv, STv) + \alpha_3 [d(STSv, STv) + d(STv, Tv)] + \alpha_4 d(STv, Tv) + \alpha_5 [d(STSv, STv) + d(STv, Tv)]$$
$$+ \alpha_6 d(STSv, STv)$$

$$= (\alpha_1 + \alpha_3 + \alpha_5 + \alpha_6) d(STSv, STv) + (\alpha_3 + \alpha_4 + \alpha_5) d(STv, Tv)$$

$$d(STSv, STv) < \frac{\alpha_3 + \alpha_4 + \alpha_5}{1 - \alpha_1 - \alpha_3 - \alpha_5 - \alpha_6} d(STv, Tv)$$

since $\alpha_1 + 2\alpha_3 + \alpha_4 + 2\alpha_5 + \alpha_6 < 1$

from above $\varphi(Sv) < \varphi(v)$ which is a contradiction and hence Sv = v i.e. $v \in P$ is a fixed point for S. Using (2), STv = TSv = Tv

Now, we shall prove that Tv = v, if possible, let $Tv \neq v$, then,

$$d(STv, Sv) < \alpha_1 \frac{d(Tv, STv)\, d(v, Sv)}{d(Tv, Sv) + d(Tv, v)} + \alpha_2 \frac{d(Tv, Sv)\, d(v, STv)}{d(v, STv) + d(Tv, v)} + \alpha_3 [d(Tv, STv) + d(v, Sv)]$$
$$+ \alpha_4 [d(Tv, Sv) + d(Tv, v)] + \alpha_5 [d(Tv, Sv) + d(v, STv)] + \alpha_6 [d(v, STv) + d(Tv, v)]$$

Therefore,

$$d(Tv, v) < \alpha_1 \frac{d(Tv, v)\, d(v, Tv)}{d(v, Tv) + d(Tv, v)} + \alpha_2 \frac{d(Tv, v)\, d(v, Tv)}{d(v, Tv) + d(Tv, v)} + \alpha_3 [d(Tv, Tv) + d(v, v)]$$
$$+ \alpha_4 [d(Tv, v) + d(Tv, v)] + \alpha_5 [d(Tv, v) + d(v, Tv)] + \alpha_6 [d(v, Tv) + d(Tv, v)]$$

$$= (\alpha_1/2 + \alpha_2/2 + 2\alpha_4 + 2\alpha_5 + 2\alpha_6) d(Tv, v)$$

$$= \alpha_1 + \alpha_2 + 4\alpha_4 + 4\alpha_5 + 4\alpha_6 < 1$$

Thus, from above,

$d(Tv, v) < d(Tv, v)$

leading to a contradiction and hence Tv = v, i.e. v is a fixed point of T

UNIQUENESS: Claim: Let w be another fixed point of S. Then using (3), we get

$$d(STv, Sw) < \alpha_1 \frac{d(Tv, STv)\, d(w, Sw)}{d(Tv, Sw) + d(Tv, w)} + \alpha_2 \frac{d(Tv, Sw)\, d(w, STv)}{d(w, STv) + d(Tv, w)} + \alpha_3 [d(Tv, STv) + d(w, Sw)]$$
$$+ \alpha_4 [d(Tv, Sw) + d(Tv, w)] + \alpha_5 [d(Tv, Sw) + d(w, STv)] + \alpha_6 [d(w, STv) + d(Tv, w)]$$

$$= \alpha_1 \frac{d(v, v)\, d(w, w)}{d(v, w) + d(v, w)} + \alpha_2 \frac{d(v, w)\, d(w, v)}{d(w, v) + d(v, w)} + \alpha_3 [d(v, v) + d(w, w)] + \alpha_4 [d(v, w) + d(v, w)]$$
$$+ \alpha_5 [d(v, w) + d(w, v)] + \alpha_6 [d(w, v) + d(v, w)]$$

$$d(v, w) < \alpha_2 /2 + 2\alpha_4 + 2\alpha_5 + 2\alpha_6 \; d(v, w)$$

This implies d(v, w) < d(v, w) since $\alpha_2 + 4\alpha_4 + 4\alpha_5 + 4\alpha_6 < 1$ is a contradiction.

Hence proved that $v \in P$ is unique common fixed point of S and T.

REFERENCES:

[1] Harinath, K.S.: "A Chain of Result Fixed Points "Indian J. Pure. Math 10 (12) 1979.

[2] Jain, R.K and Dixit, S.P.: "Some Resilts on Fixed Point in Psuedocomact Tychonov Spaces" Indian J. Pure Appl. Math 15 (5) (1984) 455.

[3] Fisher, B.: "On Three Fixed Point Mapping for Compact Metric Spaces, Indian J. Pure Appl. Math. 8(4) (1977), 479.

[4] Zeqing, Liu.: "On Coincidence Point Theorems in Psuedocompact Tychonov Spaces" Bull. Cal. Math. Soc. 84 (1992), 573-574.

[5] Naidu, S.V.R. and Rao, K.P.R.: "Fixed Point and Coincidence Point Theorem" Indian J. Pure. Appl. Math. 19 (3), (March 1988) 255-268.

[6] Edelstein, M.: On fixed and periodic points under contractive mappings, J. London Math. Soc. 37 (1962), 74-79.

[7] Leader, S.: Uniformly contractive fixed points in compact metric spaces, Proc. Amer. Math. Soc. 86 (1982), 153-158.

****