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RADIATION AND CHEMICAL REACTION EFFECT ON FREE CONVECTION MHD FLOW THROUGH A POROUS MEDIUM BOUNDED BY VERTICAL SURFACE

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ABSTRACT

Effect of radiation and chemical reaction on free convection MHD flow through a porous medium bounded by vertical surface is studied here. The fluid considered is gray, absorbing-emitting radiation but a non-scattering medium. The governing equations involved in the present analysis are solved by the two-term perturbation method. The velocity, temperature, concentration, skin friction and Nusselt number are studied for different parameters like thermal Grashof number, mass Grashof number, Schmidt number, magnetic field parameter, permeability parameter, Prandtl number, radiation parameter, Eckert number and chemical reaction parameter.

Keywords: *MHD, radiation effect, chemical effect, free convection.*

2000 MS Classification: *76W05, 78A40, 74F25, 76R10.*

INTRODUCTION:

Study of MHD flow with heat and mass transfer plays an important role in chemical, mechanical and biological Sciences. Some important applications are cooling of nuclear reactors, liquid metals fluid, power generation system and aero dynamics. The response of laminar skin friction and heat transfer to fluctuations in the stream velocity was studied by Lighthill [5]. Free convection effects on the oscillating flow past an infinite vertical porous plate with constant suction - I, was studied by Soundalgekar [15] which was further improved by Vajravelu et al. [17]. Further researches in these areas were done by Gupta et al. [1], Jaiswal et al. [3] and Soundalgekar et al. [16] by taking different models. Some effects like radiation and mass transfer on MHD flow were studied by Muthucumaraswamy et al. [7, 8] and Prasad et al. [9]. Radiation effects on mixed convection along a vertical plate with uniform surface temperature were studied by Hossain and Takhar [2]. Mass transfer effects on the flow past an exponentially accelerated vertical plate with constant heat flux was studied by Jha, Prasad and Rai [4]. Earlier we have studied some MHD flow models [10, 11, 12] and [13] considering variable temperature alongwith mass diffusion [11] and rotation effects [12].

Radiation and free convection flow past a moving plate was considered by Raptis and Perdikis [14]. In this paper, we are considering the rotation and chemical reaction effects on MHD flow past an impulsively started vertical plate with variable mass diffusion. We are considering effect of chemical reaction on free convection MHD flow through a porous medium bounded by vertical surface. The results are shown with the help of graphs (Fig-1 to Fig-9) and table-1.

MATHEMATICAL ANALYSIS:

We consider a steady flow of an incompressible viscous fluid through a porous medium occupying a semi-infinite region of the space bounded by a vertical infinite surface in the presence of an imposed uniform magnetic field B_0, normal to the plate. The y' axis is taken along the surface in an upward direction. The fluid properties are assumed to be constant except for the density in the body force term. A chemically reactive species is emitted from the vertical surface into a hydrodynamic flow field. It diffuses into the fluid, where it undergoes a homogeneous chemical reaction. The reaction is assumed to take place entirely in the stream. Under the above assumptions, the flow is governed by the following set of equations:

$$\frac{\partial v'}{\partial y'} = 0, \tag{1}$$

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$$v'\frac{\partial u'}{\partial y'} = g\beta\left(T'-T'_\infty\right) + g\beta^*\left(C'-C'_\infty\right) + \upsilon\frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^2 u'}{\rho} - \frac{\upsilon u'}{K_p}, \tag{2}$$

$$v'\frac{\partial T'}{\partial y'} = \frac{\alpha}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} + \frac{\upsilon}{C_p}\left(\frac{\partial u'}{\partial y'}\right)^2 - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y'}, \tag{3}$$

$$v'\frac{\partial C'}{\partial y'} = D\frac{\partial^2 C'}{\partial y'^2} - K_c\left(C'-C'_\infty\right). \tag{4}$$

Equation (1) gives:

$$v' = -v_0, \tag{5}$$

where $v_0 > 0$ and v' is the steady normal velocity of suction on the surface.

The boundary conditions are as follows:

$$\left.\begin{array}{l} u'=0,\ T'=T'_w,\ C'=C'_w \ \text{ at } \ y'=0\ ; \\ u'\to 0,\ T'\to T'_\infty,\ C'\to C'_\infty \ \text{ as } \ y'\to\infty\ . \end{array}\right\} \tag{6}$$

The local radiant for the case of an optically thin gray gas is expressed by

$$\frac{\partial q_r}{\partial y'} = -4a^*\sigma\left(T'^4_\infty - T'^4\right), \tag{7}$$

where a^* is absorption constant.

Considering the temperature difference within the flow sufficiently small, T'^4 can be expressed as the linear function of temperature. This is accomplished by expanding T'^4 in a Taylor series about T'_∞ and neglecting higher-order terms

$$T'^4 \cong 4T'^3_\infty T' - 3T'^4_\infty. \tag{8}$$

Using equations (7) and (8), equation (3) becomes

$$v'\frac{\partial T'}{\partial y'} = \frac{\alpha}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} + \frac{\upsilon}{C_p}\left(\frac{\partial u'}{\partial y'}\right)^2 + \frac{16a^*\sigma}{\rho C_p}\left(T'_\infty - T'\right), \tag{9}$$

Introducing the following non - dimensional quantities:

$$\left.\begin{array}{l} u=\dfrac{u'}{v_0},\ y=\dfrac{v_0 y'}{\upsilon},\ \theta=\dfrac{\left(T'-T'_\infty\right)}{\left(T'_w-T'_\infty\right)},\ G_r=\dfrac{g\beta\upsilon\left(T'_w-T'_\infty\right)}{v_0^3}, S_c=\dfrac{\upsilon}{D}, \\ R=\dfrac{16a^*\sigma T'^3_\infty\upsilon}{\alpha v_0^2}, C=\dfrac{\left(C'-C'_\infty\right)}{\left(C'_w-C'_\infty\right)}, M=\dfrac{\sigma B_0^2\upsilon}{\rho v_0^2}, K=\dfrac{\upsilon K_p}{v_0^2}, k_0=\dfrac{\upsilon K_c}{v_0^2}, \\ G_m=\dfrac{g\beta^*\upsilon\left(C'_w-C'_\infty\right)}{v_0^3}, E=\dfrac{v_0^2}{C_p\left(T'_w-T'_\infty\right)} \ \text{ and } \ P_r=\dfrac{\rho\upsilon C_p}{\alpha}. \end{array}\right\} \tag{10}$$

Using (10), equations (1), (2), (4) and (9) reduce to:

$$\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y} - \left(M + \frac{1}{K}\right)u = -G_r\theta - G_m C, \tag{11}$$

$$\frac{\partial^2\theta}{\partial y^2}+P_r\frac{\partial\theta}{\partial y}-R\theta=-P_rE\left(\frac{\partial u}{\partial y}\right)^2, \tag{12}$$

$$\frac{\partial^2 C}{\partial y^2}+S_c\frac{\partial C}{\partial y}-S_ck_0C=0. \tag{13}$$

Also, the boundary condition (6) reduces to:

$$\left.\begin{array}{l} u=0,\ \theta=1,\ C=1 \text{ at } y=0\ ; \\ u\to 0,\ \theta\to 0,\ C\to 0 \text{ as } y\to\infty\ . \end{array}\right\} \tag{14}$$

The dimensionless governing equations (11), (12) and (13), subject to the boundary conditions (14), are solved by the perturbation method. Expanding u,θ and C in the power of the Eckert number E (assuming that E is very small). We can write:

$$u=u_0+Eu_1+O\left(E^2\right),\quad \theta=\theta_0+E\theta_1+O\left(E^2\right),\quad C=C_0+EC_1+O\left(E^2\right). \tag{15}$$

Substituting the equation (15) into equations (11)-(14), equating the coefficients at the terms with the same power of E, and neglecting terms of E^2 and higher orders, we get the following equations:

Zero order;

$$\frac{\partial^2 u_0}{\partial y^2}+\frac{\partial u_0}{\partial y}-mu_0=-G_r\theta_0-G_mC_0, \tag{16}$$

$$\frac{\partial^2\theta_0}{\partial y^2}+P_r\frac{\partial\theta_0}{\partial y}-R\theta_0=0, \tag{17}$$

$$\frac{\partial^2 C_0}{\partial y^2}+S_c\frac{\partial C_0}{\partial y}-S_ck_0C_0=0. \tag{19}$$

First order;

$$\frac{\partial^2 u_1}{\partial y^2}+\frac{\partial u_1}{\partial y}-mu_1=-G_r\theta_1-G_mC_1, \tag{20}$$

$$\frac{\partial^2\theta_1}{\partial y^2}+P_r\frac{\partial\theta_1}{\partial y}-R\theta_1=-P_r\left(\frac{\partial u_0}{\partial y}\right)^2, \tag{21}$$

$$\frac{\partial^2 C_1}{\partial y^2}+S_c\frac{\partial C_1}{\partial y}-S_ck_0C_1=0. \tag{22}$$

The corresponding boundary conditions are as follows:

$$\left.\begin{array}{l} u_0=0,u_1=0,\ \theta_0=1,\theta_1=0,\ C_0=1,C_1=0 \text{ at } y=0\ ; \\ u_0\to 0,u_1\to 0,\ \theta_0\to 0,\theta_1\to 0,\ \theta_0\to 0,\theta_1\to 0 \text{ as } y\to\infty\ . \end{array}\right\} \tag{23}$$

Solving equations (16)-(22) under the boundary conditions (23) and then using (15), we get the solution, which is as under:

$$\begin{aligned} u=&\left(G+EA_2\right)e^{-\lambda y}-G_1e^{-\mu y}-G_2e^{-\delta y}-EG_r\left[\frac{A_1e^{-\delta y}}{\delta^2-\delta-m}-A_3e^{-2\delta y}\right] \\ &+EG_r\left[A_4e^{-2\lambda y}+A_5e^{-2\mu y}-A_6e^{-(\lambda+\delta)y}-A_7e^{-(\mu+\delta)y}-A_8e^{-(\mu+\lambda)y}\right], \end{aligned} \tag{24}$$

$$\theta=\left(1+EA_1\right)e^{-\delta y}-P_rE\left[A_9e^{-2\lambda y}+A_{10}e^{-2\delta y}+A_{11}e^{-2\mu y}+A_{13}e^{-(\mu+\delta)y}\right] \\ +P_rE\left[A_{12}e^{-(\lambda+\delta)y}+A_{14}e^{-(\lambda+\mu)y}\right], \tag{25}$$

$$C=e^{-\mu y}, \tag{26}$$

where $\lambda=\dfrac{1+\sqrt{1+4m}}{2}$, $\mu=\dfrac{S_c+\sqrt{S_c^2+4k_0S_c}}{2}$, $\delta=\dfrac{P_r+\sqrt{P_r{}^2+4R}}{2}$, $G_1=\dfrac{G_m}{\mu^2-\mu-m}$,

$G_2=\dfrac{G_r}{\delta^2-\delta-m}$, $m=M+\dfrac{1}{K}$, $G=G_1+G_2$, $A_1=P_r\left[A_9+A_{10}+A_{11}-A_{12}+A_{13}-A_{14}\right]$,

$A_2=\dfrac{A_1G_r}{\delta^2-\delta-m}+P_rG_r\left(A_4-A_3+A_5-A_6+A_7+A_8\right)$, $A_{14}=\dfrac{2\lambda\mu G_1G}{(\lambda+\mu)^2-(\lambda+\mu)P_r-R}$,

$A_3=\dfrac{\delta^2G_2^2P_r}{\left(4\delta^2-2\delta P_r-R\right)\left(4\delta^2-2\delta-m\right)}$, $A_4=\dfrac{\lambda^2G_1^2P_r}{\left(4\lambda^2-2\lambda P_r-R\right)\left(4\lambda^2-2\lambda-m\right)}$,

$A_5=\dfrac{\mu^2G_1^2P_r}{\left(4\mu^2-2\mu P_r-R\right)\left(4\mu^2-2\mu-m\right)}$, $A_9=\dfrac{\lambda^2G_1^2}{\left(4\lambda^2-2\lambda P_r-R\right)}$, $A_{10}=\dfrac{\delta^2G_2^2}{\left(4\delta^2-2\delta P_r-R\right)}$,

$A_{11}=\dfrac{\mu^2G_1^2}{\left(4\mu^2-2\mu P_r-R\right)}$, $A_6=\dfrac{2\lambda\delta GG_2P_r}{\left[(\lambda+\delta)^2-2(\lambda+\delta)P_r-R\right]\left[(\lambda+\delta)^2-(\lambda+\delta)-m\right]}$,

$A_{13}=\dfrac{2\mu\delta G_1G_2}{(\mu+\delta)^2-2(\mu+\delta)P_r-R}$, $A_7=\dfrac{2\mu\delta G_1G_2P_r}{\left[(\mu+\delta)^2-2(\mu+\delta)P_r-R\right]\left[(\mu+\delta)^2-(\mu+\delta)-m\right]}$,

$A_{12}=\dfrac{2\lambda\delta GG_2}{(\lambda+\delta)^2-2(\lambda+\delta)P_r-R}$, $A_8=\dfrac{2\lambda\mu G_1GP_r}{\left[(\lambda+\mu)^2-(\lambda+\mu)P_r-R\right]\left[(\mu+\lambda)^2-(\mu+\lambda)-m\right]}$.

Skin friction:

The non-dimensional skin friction is given by:

$$\tau=\left(\frac{\partial u}{\partial y}\right)_{y=0}=EG_r\left[\frac{\delta A_1}{\delta^2-\delta-m}+(\lambda+\delta)A_6+(\delta+\mu)A_7+(\lambda+\mu)A_8\right] \\ +\mu G_1+\delta G_2-\left(G+EA_2\right)\lambda-EG_r\left[2\delta A_3+2\lambda A_4+2\mu A_5\right]. \tag{27}$$

Nusselt number:

The non-dimensional Nusselt number is given by:

$$Nu=-\left(\frac{\partial\theta}{\partial y}\right)_{y=0}=EP_r\left[(\lambda+\mu)A_{14}+(\lambda+\delta)A_{12}-(\mu+\delta)A_{13}\right] \\ +(1+EA_1)\delta-2EP_r\left(\lambda A_9+\delta A_{10}+\mu A_{11}\right). \tag{28}$$

RESULTS AND DISCUSSION:

The velocity profiles for different parameters E, M, k_0 and R are shown by figures-1 to 5.

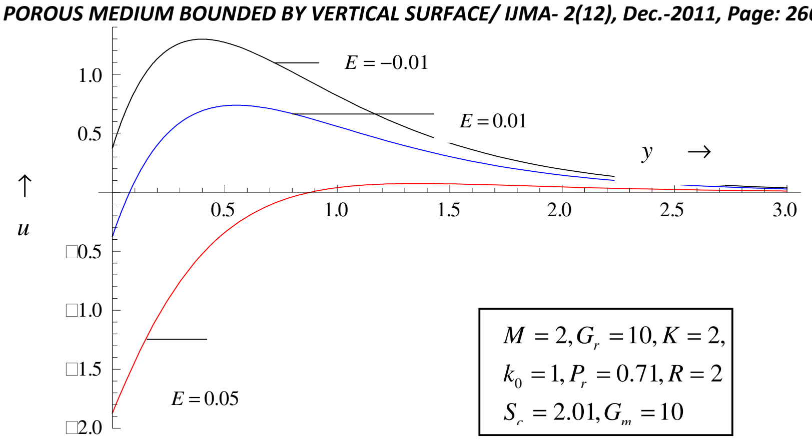

Figure -1: Velocity profile for different Eckert number E

From figure-1, it is clear that the velocity decreases when Eckert number E is increased (keeping other parameters $M = 2,\ G_r = 10, G_m = 10,\ K = 2, k_0 = 1, P_r = 0.71, S_c = 2.01, R = 2$ constant). Velocity profile for different values of Eckert number E is shown in figure-2. It shows that the velocity decreases with increasing Prandtl number P_r (keeping other parameters constant). In figure-3, the velocity increases when chemical parameter k_0 is increased. But in figure-4, velocity increases when magnetic field parameter M is increased (keeping other parameters constant). It is observed in figure-5 that velocity decreases when radiation parameter R is increased.

Temperature profile is shown in figures-6 to 8. From figure-6, it is clear that temperature decreases when Eckert number E is increased (keeping other parameters $M = 2,\ G_r = 10,\ G_m = 10,\ K = 2,\ k_0 = 1,\ P_r = 0.71, S_c = 2.01, R = 2$ constant). In figure-7, it is shown that temperature decreases when magnetic field parameter M is increased. Figure-8 shows that temperature decreases with increasing the radiation parameter R. Figure-9 represents the concentration profile for different chemical parameter k_0.

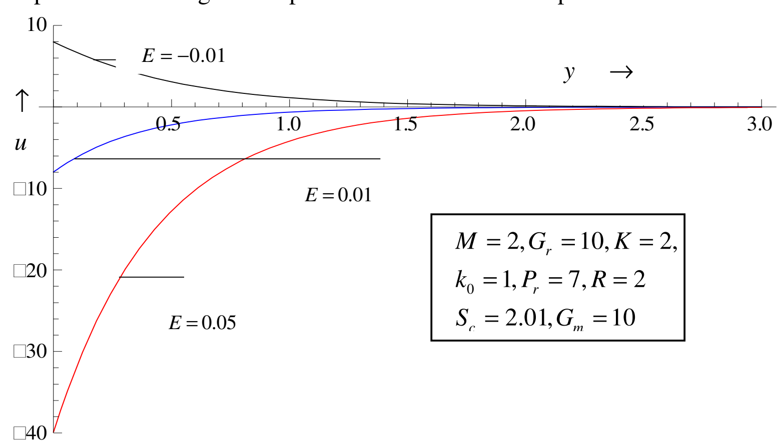

Figure- 2: Velocity profile for different Eckert number E

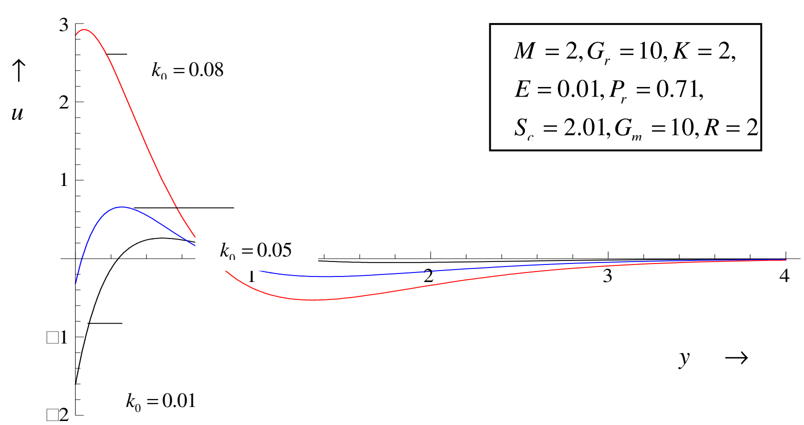

Figure-3: Velocity profile for different chemical parameter k_0

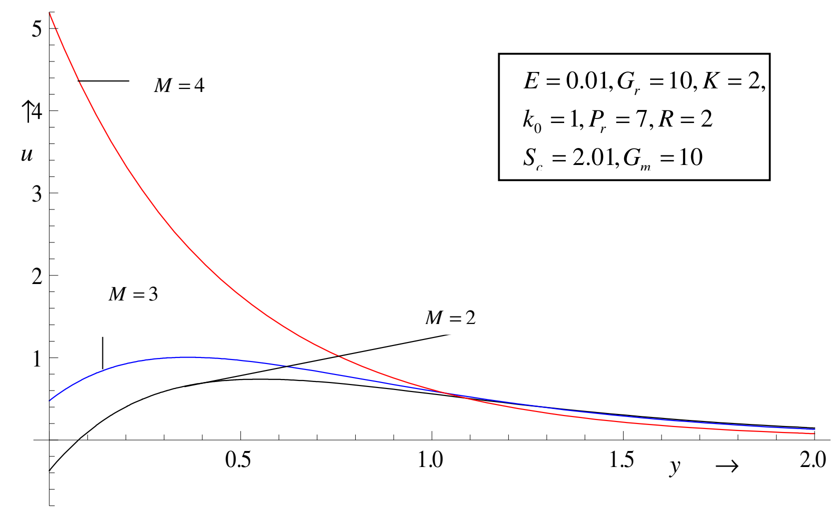

Figure-4: Velocity profile for different magnetic field parameter M

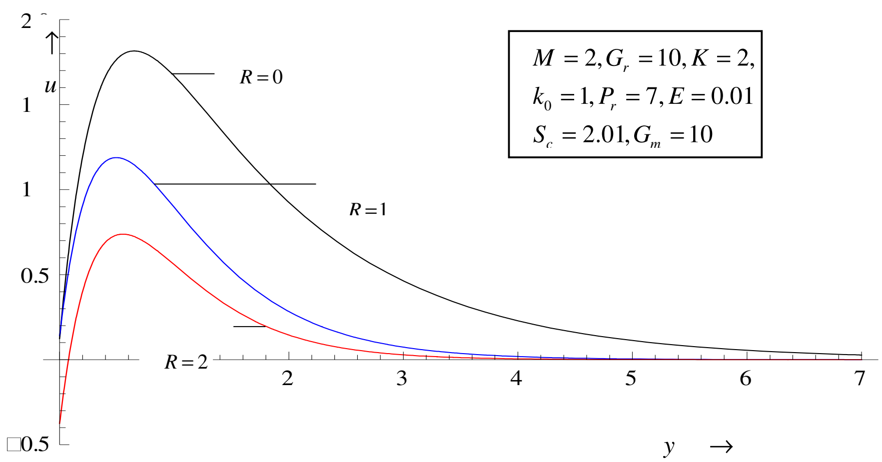

Figure-5: Velocity profile for different radiation parameter R

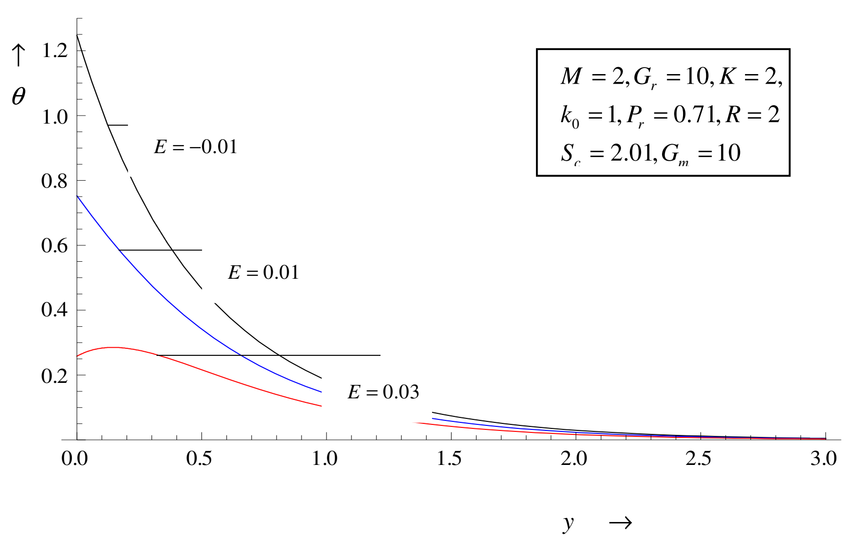

Figure-6: Temperature profile for different Eckert number E

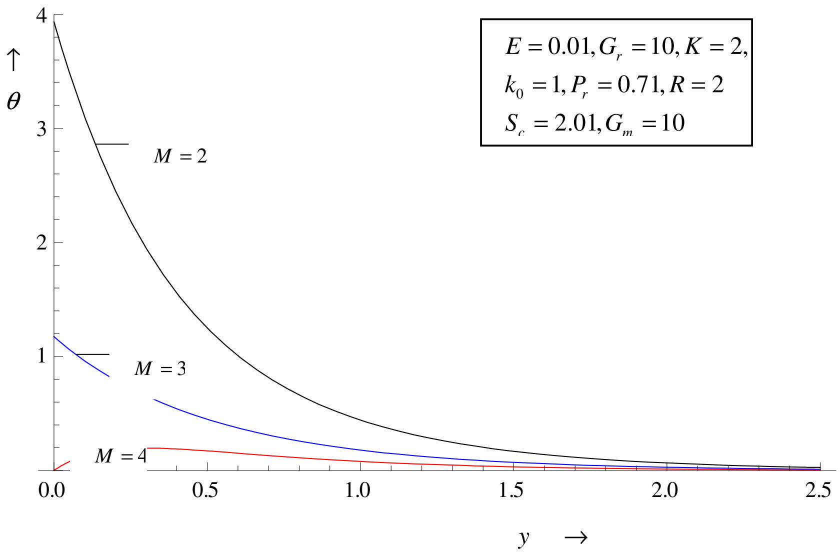

Figure-7: Temperature profile for different magnetic field M

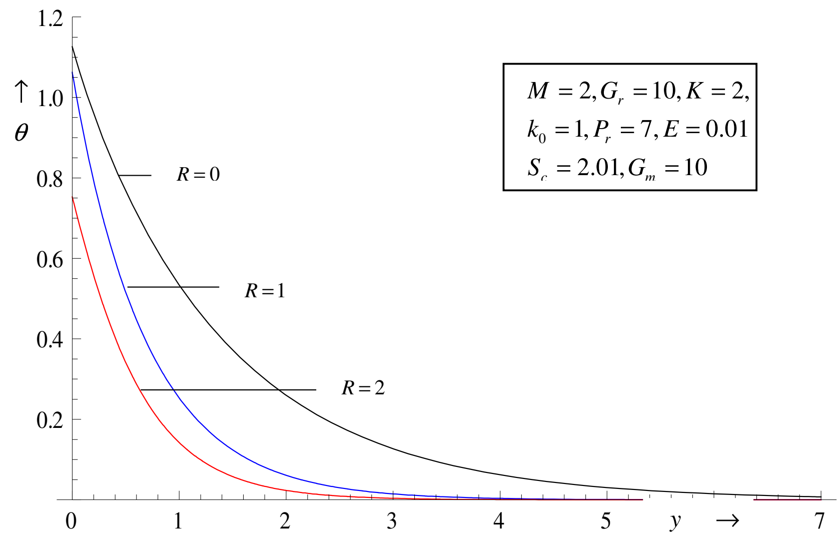

Figure-8: Temperature profile for different radiation parameter R

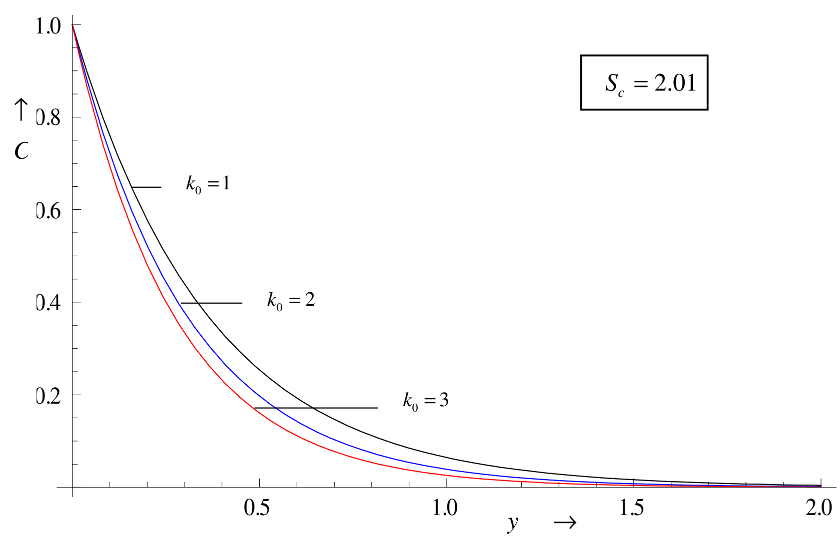

Figure -9: Concentration profile for different chemical parameter k_0

Table-1: τ and Nu for different parameters

$S_c = 2.01, P_r = 0.71, G_r = 10, G_m = 10$						
M	K	R	E	k_0	τ	Nu
2	2	2	0.01	1	-15.18	2.11
3	2	2	0.01	1	15.69	1.57
2	3	2	0.01	1	-16.76	2.42
2	2	1	0.01	1	0.12	1.34
2	2	2	0.05	1	-7.03	3.29
2	2	2	0.01	2	-18.85	2.42

It is shown in figure-9, concentration decreases when increasing the value of chemical parameter k_0 (keeping $S_c = 2.01$ fixed).

The values of skin friction and Nusselt number are tabulated in table-1 for different parameters. When the values of M and E are increased (keeping other parameters constant) the value of τ also gets increased. But if values of K, k_0 and R are increased, the value of τ gets decreased.

When the values of M is increased (keeping other parameters constant) the value of Nu is also get decreased. But if values of K, k_0, E and R are increased, the value of τ gets increased.

CONCLUSIONS:

In this paper a theoretical analysis has been done to study the radiation and chemical reaction effect on free convection MHD flow through a porous medium bounded by vertical surface**.** Solutions for the model have been derived by using two- term perturbation method. Some conclusions of the study are as below:

- Velocity increases with the increase in k_0 and M and decreases with increase in R and E.
- Temperatures of the fluid increase when M, E, R and k_0 are decreased.
- Concentration of the fluid decreases when k_0 is increased.
- Skin friction increases when magnetic field parameter and Eckert number are increased but decreases when chemical parameter, radiation parameter and permeability parameter are increased.
- Nusselt number increases when, chemical parameter, Eckert number, radiation parameter and permeability parameter are increased but decreases when magnetic field parameter is increased.

Notation:

C - Non-dimensional fluid concentration;
C' - Concentration, mol/m^3;
C_∞ - Fluid concentration far away from the wall, mol/m^3;
C_p - Specific heat at a constant pressure, J/(kg.deg);
D - Mass diffusivity, m^2/sec;
E - Eckert number;
G_m - Mass Grashof number;
G_r - Thermal Grashof number;
g - Gravitational acceleration, m/sec^2;
K - Non-dimensional permeability coefficient of a porous medium;
k_0 - Non-dimensional rate of a chemical reaction;
K_c - Rate of chemical reaction, sec^{-1};
K_p - Permeability of a porous medium, m^2;
M - Magnetic field parameter;
Nu - Nusselt number;

P_r - Prandtl number;
R - Non- dimensional radiation parameter;
S_c - Schmidt number;
T_∞ - Fluid temperature far away from the wall, ^{0}C;
T' - Temperature, ^{0}C;
u', v' velocity components, m/sec;
u - Non-dimensional velocity;
v_0 - suction velocity, m/sec;
x', y' - Space coordinates, m;
y - Non-dimensional space coordinates;

Greek Symbols:
α - Thermal conductivity, W/ (m.deg);
β - Coefficient of volume expansion, 1/deg;
β^* - Coefficient of volume expansion with concentration, m^3/mol;
θ - Non-dimensional temperature;
υ - Kinematic viscosity, m^2/sec;
ρ - Fluid density, kg/m^3;
τ - Non-dimensional skin friction.

Subscripts and Superscripts:
w - Wall;
∞ - Far away from the wall;
0 and 1 - zero and first orders.

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