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EXACT SOLUTION OF DISPERSION IN A PIPE UNDER THE INFLUENCE OF A MAGNETIC FIELD

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ABSTRACT

The dispersion of a solute in a Newtonian fluid flowing through a tube under the influence of magnetic field is studied. The generalized dispersion model is used to solve unsteady convective diffusion equation. As a result, the total process of dispersion can be described in terms of a simple diffusion process with effective diffusion coefficient as a function of time. The effect of magnetic field on the dispersion coefficient and mean concentration is discussed. It is observed that the magnetic field reduces the dispersion coefficient. The time taken for the mean concentration to attain the peak value is found to increase with magnetic field.

Key Words: *Generalized dispersion model, Newtonian fluid, dispersion coefficient.*

1. INTRODUCTION:

The study of dispersion of a solute in flowing fluids has several applications in Industries, Chemical engineering, Biomedical engineering, environmental sciences, physiological fluid dynamics and various other branches of science. The study of dispersion facilitates to understand the transport of nutrients in blood and various artificial devices [5, 11, 12, 14]. Hence, the dispersion theory is of great value to know the rate of dispersion of drugs.

The concept of longitudinal dispersion was introduced by Taylor [17]. Using the method of moments, Aris [3] extended Taylor theory by considering axial diffusion. Ananthakrishnan *et al*. [1] obtained a numerical solution for the complete convective diffusion equation considering both radial and molecular diffusion. Gill [7] generalized the work of Taylor by giving a series expansion about the mean concentration to describe the local concentration. This theory was extended by Gill and Ananthakrishnan [8] by including the effect of finite slug inputs on the dispersion process. In their subsequent paper [9] showed that the method of series solution mentioned above provides an exact solution to the unsteady convective diffusion problem for laminar flow in a circular tube provided that the coefficients in dispersion model are obtained as a suitable function of time ‘t’. This model was widely known as generalized dispersion model. The applications of magnetohydrodynamic principles in biology and medicine are abundant. It is known that the Lorentz’s force opposes the motion of a conducting fluid.

The dispersion of a solute in a laminar flow of an electrically conducting fluid in a two dimensional channel in the presence of a transverse magnetic field has been studied by [10] using both Taylor’s theory and Aris analysis. Annapurna and Gupta [2] studied the dispersion of a solute in an electrically conducting fluid flow between two parallel plates in the presence of a uniform transverse magnetic field. The importance of dispersion in hydromagnetic flows has been discussed by [4]. Deshikachar [6] studied the axial molecular diffusion of a solute in the laminar flow of an electrically conducting fluid oscillating with zero mean velocity, between two parallel plates in the presence a transverse magnetic field using perturbation analysis.

In this paper the dispersion of a solute in a Newtonian fluid flowing through a pipe is studied under the influence of a transverse magnetic field with a motivation to understand the influence of magnetic field on the rate of dispersion. The mathematical formulation of the problem in pipe flow and the corresponding solutions are presented in section 2. The effect of magnetic field (Hartmann number) on the dispersion coefficient and the overall dispersion process is discussed in section 3. The conclusions are presented in section 4.

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2. MATHEMATICAL FORMULATION:

Consider the dispersion of a solute that is initially of z_s units in length distributed in a straight circular tube of radius '*a*'. The unsteady convective diffusion equation which describes the local concentration C of the solute as a function of longitudinal (axial) coordinate z , transverse (radial) coordinate r and time t can be written in non- dimensional form as

$$\frac{\partial C}{\partial t} + w\frac{\partial C}{\partial z} = \left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{Pe^2}\frac{\partial^2}{\partial z^2}\right]C \tag{1}$$

with non dimensional variables

$$C = \frac{\overline{C}}{C_0}, \quad w = \frac{\overline{w}}{w_0}, \quad r = \frac{\overline{r}}{a}, \quad z = \frac{D_m \overline{z}}{a^2 w_0}, \quad t = \frac{D_m \overline{t}}{a^2}$$

where w is the axial velocity of the fluid in pipe and D_m is coefficient of molecular diffusion (molecular diffusivity) which is assumed to be constant.C_0 is the reference concentration and $w_0 = -\frac{a^2}{4\mu}\frac{d\overline{p}}{d\overline{z}}$ is the characteristic velocity (centerline velocity in a Poiseuille flow), μ is the Newtonian viscosity of the fluid and $\frac{d\overline{p}}{d\overline{z}}$ is the applied pressure gradient along the axis of the pipe, $Pe = \frac{aw_0}{D_m}$, Peclet number. The variables with bars represent the corresponding dimensional quantities.

The initial and boundary conditions in dimensionless form, are given by

$$C(0, r, z) = 1 \qquad if \ \left|z\right| \le z_s/2 \tag{2a}$$

$$C(0, r, z) = 0 \qquad if \ \left|z\right| > z_s/2 \tag{2b}$$

$$C(t, r, \infty) = 0 \tag{2c}$$

$$\frac{\partial C}{\partial r}(t, 0, z) = 0 = \frac{\partial C}{\partial r}(t, 1, z) \tag{2d, e}$$

Consider the flow of Newtonian fluid in a circular pipe. Assume that the flow is axi-symmetric, fully developed, steady and laminar. A uniform magnetic field B_o is applied in the transverse direction. Following [15] and [16], the equation of motion is given by

$$0 = -\frac{\partial \overline{p}}{\partial \overline{z}} + \mu\overline{\nabla}^2\,\overline{w} - \sigma B_o^2\,\overline{w} \tag{3}$$

Solving the equation (3) along with the no slip condition and the velocity distribution of a fluid in a pipe, in non-dimensional form can be obtained as

$$w = \frac{p}{M^2}\left[1 - \frac{I_o(M\,r)}{I_o(M\,)}\right] \tag{4}$$

Where $M = B_o a\sqrt{\frac{\sigma}{\mu}}$ is the Hartmann number, B_o is strength of the magnetic field, σ is the electrical conductivity of the medium, μ is the co-efficient of viscosity of blood. I_o is the modified Bessel function of order zero of the first kind. The mean velocity of the fluid in dimensionless form is given by

$$w_m = \frac{P}{M^2} - \frac{2P}{M^3}\frac{I_1(M)}{I_o(M)} \tag{5}$$

3. METHOD OF SOLUTION:

Consider the convection across a plane moving with an average velocity w_m of the fluid. For this, define a new coordinate system moving with new axial coordinate z_1, given by

$$z_1 = z - w_m\,t \tag{6}$$

The solution of equation (1) along with the conditions (2) is formulated as a series expansion in $\frac{\partial^j C_m}{\partial z^j}$, following [9] is given by

$$C = C_m + \sum_{j=1}^{\infty} f_j(t,r)\frac{\partial^j C_m}{\partial z^j} \tag{7}$$

where $C_m = 2\int_0^1 C\, r\, dr$ (8)

is the mean concentration over a cross section.

On transforming the unsteady convective diffusion equation (1) into the moving co-ordinate system (r, z_1, t) where z_1 is given in equation (6) and substituting equation (7) into the transformed unsteady convective equation, we get

$$\frac{\partial C_m}{\partial t} + (w - w_m)\frac{\partial C_m}{\partial z_1} - \frac{1}{Pe^2}\frac{\partial^2 C_m}{\partial z_1^2} + \sum_{j=1}^{\infty}\left[\left(\frac{\partial f_j}{\partial t} - \frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial f_j}{\partial r})\right)\frac{\partial^j C_m}{\partial z_1^j} + (w - w_m) f_j \frac{\partial^{j+1} C_m}{\partial z_1^{j+1}}\right.$$

$$\left. - \frac{1}{Pe^2} f_j \frac{\partial^{j+2} C_m}{\partial z_1^{j+2}} + f_j \frac{\partial^{j+1} C_m}{\partial t\, \partial z_1^j}\right] = 0 \tag{9}$$

It is assumed that the process of distributing C_m is diffusive in nature from the time 'zero', then following [9], the generalized dispersion model for C_m can be written as

$$\frac{\partial C_m}{\partial t} = \sum_{i=1}^{\infty} K_i(t)\frac{\partial^i C_m}{\partial z_1^i} \tag{10}$$

with dispersion coefficient K_i as suitable functions of time t. The first two terms in the right hand side of equation (10) describe the transport of C_m in axial direction z_1 through convection and diffusion respectively, and therefore, the coefficients K_1 and K_2 are termed as the longitudinal convection and diffusion coefficients for C_m.

Substituting equation (10) in equation (9) and rearranging the terms, we get an infinite set of differential equations given by

$$K_1(t) + \frac{\partial f_1}{\partial t} - \frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial f_1}{\partial r}) + (w - w_m) = 0 \tag{11}$$

$$K_2(t) - \frac{1}{Pe^2} + \frac{\partial f_2}{\partial t} - \frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial f_2}{\partial r}) + [(w - w_m) + K_1(t)] f_1 = 0 \tag{12}$$

$$K_{j+2}(t) + \frac{\partial f_{j+2}}{\partial t} - \frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial f_{j+2}}{\partial r}) + (w - w_m) f_{j+1} - \frac{1}{Pe^2} f_j + \sum_{i=1}^{j+1} K_i(t) f_{j+2-i} = 0 \tag{13}$$

for j =1, 2… with $f_0 = 1$

we get the initial and boundary conditions on f_j's from equation (2) and (7), as

$$f_j(0, r) = 0 \qquad j = 1, 2\ldots. \tag{14a}$$

$$\frac{\partial f_j}{\partial r}(t,0) = 0 = \frac{\partial f_j}{\partial r}(t,1) \qquad j = 1, 2 \ldots \tag{14 b, c}$$

and from equation (7) and (8), the solvability condition is obtained as

$$\int_0^1 f_j\, r\, dr = 0 \qquad j = 1, 2,\ldots. \tag{15}$$

Multiplying equations (11) (12) and (13) by r and integrating from 0 to 1, and using the condition (14), we have

$$K_1(t) = -2\int_0^1 (w - w_m)\, r\, dr = 0 \tag{16}$$

$$K_2(t) = \frac{1}{pe^2} - 2\int_0^1 f_1(t,r)\, w(r)\, r\, dr \tag{17}$$

and $$K_{j+2}(t) = -2\int_0^1 f_{j+1}(t,r)\, w(r)\, r\, dr \quad , j = 1, 2, \ldots\ldots \tag{18}$$

Solution for f_1:

In the series expansion of equation (7), the function f_1 is the most important coefficient as it gives the measure of deviation of the local concentration C from the mean concentration C_m. The solution to the non-homogeneous parabolic partial differential equation (11) and the conditions (14) can be written in the form

$$f_1(t, r) = f_{1s}(r) + f_{1t}(t, r) \tag{19}$$

where $f_{1s}(r)$ is the large time solution which corresponds to Taylor-Aris's dispersion theory and f_{1t} is the transient part which describes the time-dependent nature of the dispersion phenomena corresponding to a Newtonian model. From equations (11) and (14) and using (16), we have

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f_{1s}}{\partial r}\right) = -(w - w_m) \tag{20}$$

$$\frac{\partial f_{1t}}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f_{1t}}{\partial r}\right) \tag{21}$$

with the boundary and initial conditions

$$\frac{df_{1s}}{dr}(r=0) = 0 = \frac{df_{1s}}{dr}(r=1) \tag{22 a, b}$$

$$\frac{\partial f_{1t}}{\partial r}(t,0) = 0 = \frac{df_{1t}}{dr}(t,1) \tag{23a, b}$$

$$f_{1t}(0, r) = -f_{1s}(r) \tag{24c}$$

From the solvability condition (2.19), we have

$$\int_0^1 f_{1t}\ r\,dr = = -\int_0^1 f_{1s}\ r\,dr = 0 \tag{25}$$

The solution for f_{1s} is, obtained from equation (20) subject to the conditions (22) and (25), is given by

$$f_{1s}(r) = \frac{P_o}{M^2}\left[\frac{1}{M}\frac{I_1(M)}{I_o(M)}\frac{r^2}{2} - \frac{1}{M^2}\frac{I_o(M\ r)}{I_o(M)}\right] + B \tag{26}$$

where $B = -\dfrac{2\ p_o}{M^3}\dfrac{I_1(M)}{I_o(M)}\left[\dfrac{1}{8} - \dfrac{1}{M^2}\right]$ (27)

From equation (21) subject to the conditions (23) and (25), the solution for f_{1t} is obtained as

$$f_{1t} = \sum_{m=1}^{\infty} A_m\, J_0(\lambda_m r)\, e^{-\lambda_n^2 t} \tag{28}$$

where $$A_m = -\frac{\int_0^1 f_{1s}\, J_0(\lambda_m\ r)\, r\, dr}{\int_0^1 r\left(J_0(\lambda_m\ r)\right)^2 dr} = \frac{-2}{J_o(\lambda_m)}\frac{P}{M^3}\frac{I_1(M)}{I_o(M)}\left[\frac{1}{\lambda_m{}^2} - \frac{1}{\lambda_m{}^2 + M^2}\right] \tag{29}$$

J_0 and J_1 are the Bessel functions of first kind of order zero and one respectively and λ_m's are the solutions of the equation $J_1(x) = 0$

Solution for K_2:

The coefficient $K_2(t)$ has a very significant role in the generalized dispersion model given by equation (10). It is known from equation (17), that K_2 depends on the function f_1. Substituting the expression of f_{1s} and f_{1t} and simplifying the equation (17), we can obtain K_2 . Once $K_2(t)$ is known, then $f_2(t, r)$ can be obtained from equation (12) in a similar manner to that $f_1(t, r)$. Following similar procedure we can find $K_3(t)$, $f_3(t, r)$, $K_4(t)$, $f_4(t, r)$ … etc. Since the expression for f_1 (t, r) and K_2 (t) are complicated in nature, it is very difficult to evaluate f_2 (t, r), K_3 (t) ,…… and so on. It was shown that in the absence of magnetic field [9], K_3 ($t \to \infty$) = -1/ 23040 and the magnitude of higher order coefficients decrease further. We have not evaluated these coefficients which are likely to decrease further in magnitude due to the presence of magnetic field.

Solution for mean concentration:

Neglecting $K_3(t)$ and higher order coefficients, the generalized dispersion model leads to

$$\frac{\partial C_m}{\partial t} = K_2(t)\frac{\partial^2 C_m}{\partial z_1^2} \tag{30}$$

The initial and boundary conditions for C_m are given by

$$C_m(0,\ z_1) = 1 \quad if \quad |\ z_1| \le \frac{z_s}{2} \tag{31a}$$

$$C_m(0,\ z_1) = 0 \quad if \quad |\ z_1| > \frac{z_s}{2} \tag{31b}$$

$$C_m(t,\infty) = 0 \tag{31c}$$

From equation (30) along with the help of the initial and boundary conditions (31) the solution for mean concentration can be obtained as

$$C_m = \frac{1}{2}\left[erf\left(\frac{\frac{1}{2}z_s - z_1}{2\sqrt{\xi}}\right) + erf\left(\frac{\frac{1}{2}z_s + z_1}{2\sqrt{\xi}}\right)\right] \tag{32}$$

where $$\xi = \int_0^t K_2(t)\ dt \tag{33}$$

4 RESULTS AND DISCUSSION:

The objective of the present study is to understand the effect of magnetic field on the dispersion of a solute in a Newtonian fluid flowing in a pipe. This study facilitates to know the dispersion of drugs and nutrients in circulatory system. This analysis can also be utilized to artificial blood handling devices such as blood oxygenators and hemodialysers.

The dispersion coefficient K_2 is found to be influenced significantly by the magnetic field. The values of (K_2 -1/Pe^2) for different values of M are given in table 1. It is observed that the values of dispersion coefficient are oscillatory for very small values of Hartmann number M and this oscillatory behaviour gradually increases with increases in M. This oscillatory behavior disappears from M = 0.001 onwards and dispersion coefficient decreases with increases in M. In the absence of magnetic field such fluctuations in dispersion coefficient are not present [9]. This might be due to the possibility that the distribution of solute in the flow under pressure gradient, and by convection which might be effected by Lorentz force and viscous force.

Table- 1

M	$K_2 - 1/Pe^2$	M	$K_2 - 1/Pe^2$
0.00001	-1.2696×10^{15}	0.1	3.2492×10^{-4}
0.00003	-3.9084×10^{12}	0.5	2.9643×10^{-4}
0.00005	1.3052×10^{11}	1	2.2862×10^{-4}
0.0001	2.7435×10^{9}	2	9.7878×10^{-5}
0.0003	3.0459×10^{5}	3	3.7021×10^{-5}
0.0005	$- 4.5049 \times 10^{4}$	4	1.4546×10^{-5}
0.001	1.8642×10^{2}	5	6.0960×10^{-6}
0.005	4.9595×10^{-2}	10	2.3203×10^{-7}
0.01	8.1341×10^{-4}	20	5.4142×10^{-9}

The time dependent nature of the dispersion coefficient K_2 versus time for different values of magnetic field (Hartmann number) for dispersion in pipe flow is described in Fig.1. The dispersion coefficient (K_2 -1/Pe^2) becomes essentially a constant for large values of time. Taylor's theory is applicable to the dispersion of the passive tracer in flow after the time at which (K_2 -1/Pe^2) attains the asymptotic value of the dispersion coefficient, while for small values of time the approximation corresponding to Lighthill [13] holds good. It is also observed that the time beyond which the Taylor's theory is applicable is unaffected by the presence of magnetic field. The time taken to reach the steady state is observed to be dependent on the magnetic field. In the absence of magnetic field, the time to reach the steady state is 0.5 [9]. In the presence of magnetic field this steady state is reached faster and this critical value reduces as Hartmann number increases. When M = 3 the time to reach this critical value is almost half of the time corresponding to the case when M = 1. The presence of magnetic field in a pipe reduces the dispersion coefficient. Increase in the Hartmann number still decreases the dispersion coefficient. When Hartmann number is 2 the dispersion is reduced by 2 times of the corresponding value for M = 1. When M = 3 this reduction factor is observed to be 6. From Fig 2, it is noticed that the dispersion coefficient in pipe flow analysis decreases with increase in Hartmann number and as M approaches 5, K_2 approaches the value zero. In this case flow becomes more plug like and the dispersion disappears.

The time evaluation of the function f_1 for dispersion is described in Fig 3. f_1 provides a measure of deviation in the local concentration C from the mean concentration C_m. At time t = 0 f_1 is uniformly zero over the entire cross-section of the pipe. f_1 is noted to attain its steady state value f_{1s} as t increases which is also shown in Fig 4.

The effect of magnetic field on f_1 is shown in fig 5. The presence of magnetic field is seen to reduce the magnitude of the peak values of f_1. when M = 3 there is a 3 fold reduction in the magnitude of f_1 at r = 0 corresponding to the case when M = 1. It is noticed from Fig 4 and Fig 5 that the functions f_1 and f_{1s} pass through a common point for all times and for all values of the magnetic field. At this point f_1 and f_{1s} are zero and the local concentration C of the solute becomes equal to the mean concentration C_m. Therefore, this point shall be considered as the centre of mass of the solute over a cross section of the pipe. This centre of mass of the solute is independent of time and Hartmann number.

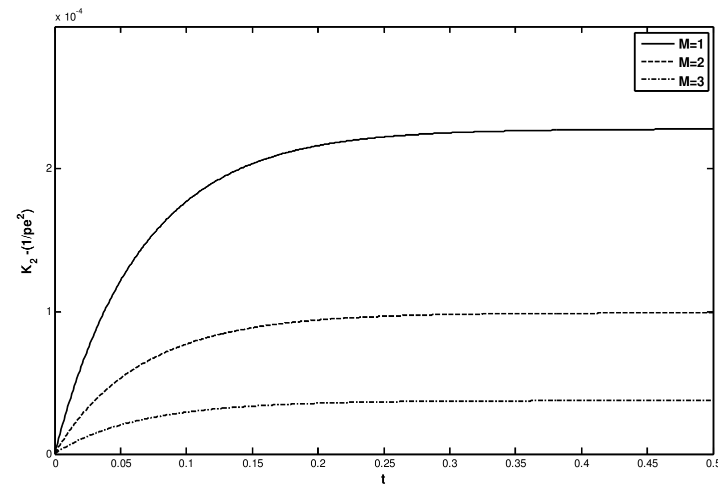


Fig- 1

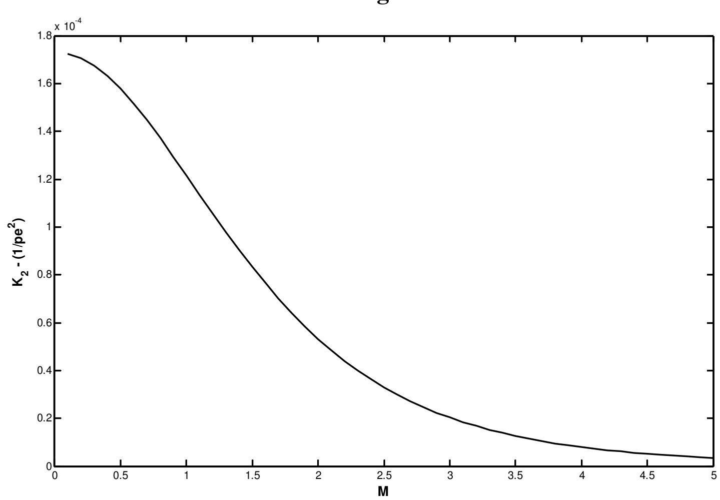


Fig- 2

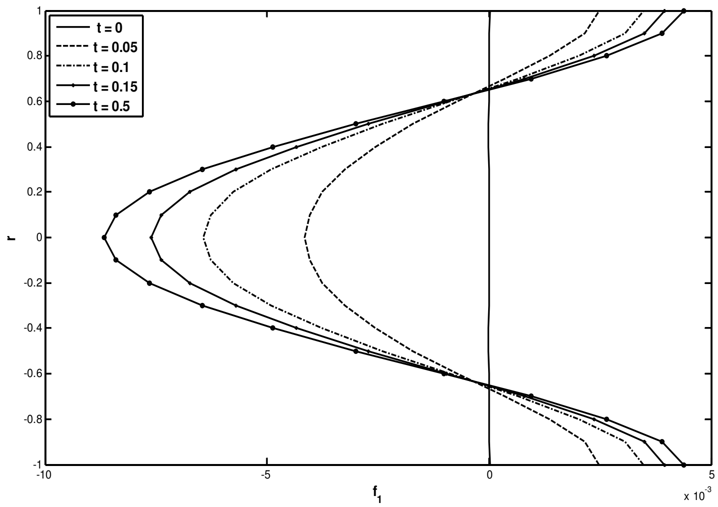


Fig- 3

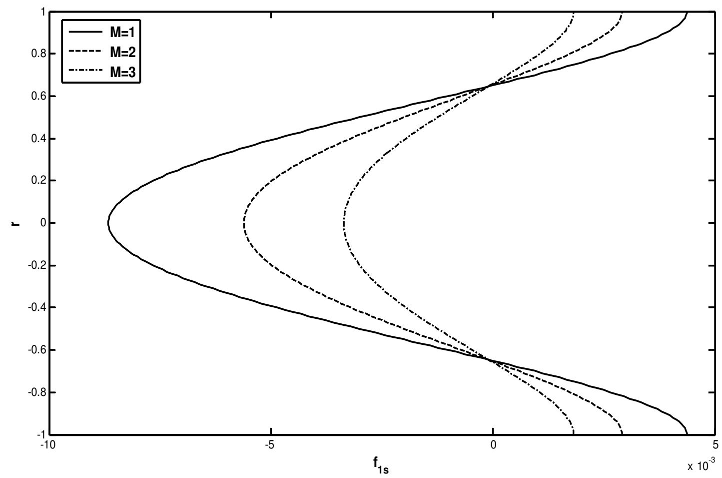


Fig- 4

Fig 1 Variation of dispersion coefficient K_2-($1/Pe^2$) verses time t for different values of M
Fig 2 Variation of dispersion coefficient K_2-($1/Pe^2$) verses M for t = 0.05
Fig 3 Distribution of dispersion function f_1 for different values of time t when M=1
Fig 4 Distribution of steady state dispersion function f_{1s} for different values of M when t=0.5

Fig 6 describes the variation of mean concentration with time when Hartmann number M = 1, pressure gradient P = 1 and axial distance z = 0.5. It is observed that the peak values of the mean concentration C_m occurs at t = 4.64 for different lengths of slug inputs of solute. The peak value of C_m increases with increase in slug input length. There is a fivefold enhancement in C_m when z_s is increased from 0.004 to 0.019 and a two fold increase is noticed when $z_s = 0.008$.

Fig 7 depicts the variation of mean concentration for different values of Hartmann number. It is observed that as M increases the value of C_m is also increased and the time taken to attain this peak value of C_m also increases. The peak value in the absence of the magnetic field occurs at t = 1 [9]. The presence of magnetic field takes more time to attain the peak value. When M = 1 the peak value of C_m occurs at t = 4.65 while it is at t = 9.75 when M = 3.

The variation of mean concentration C_m with axial distance z for different slug input lengths of solute is presented in Fig 8. The peak value of concentration occurs at z = 0.003. As the length of slug input of the solute increases the peak value also increase. In the pipe flow analysis when M = 1 the peak value increases from 0.761 to 1 when z_s changes its value from 0.004 to 0.019. The plot of the variation of C_m versus z for different values of M is presented in Fig 9. It is noticed that the peak value of C_m increases with increase in M. However, the peak value is drifted to right of the origin as M takes higher values.

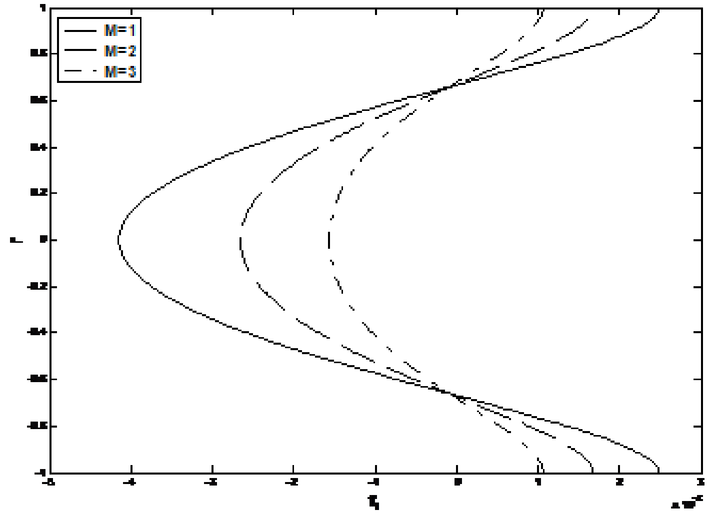


Fig- 5

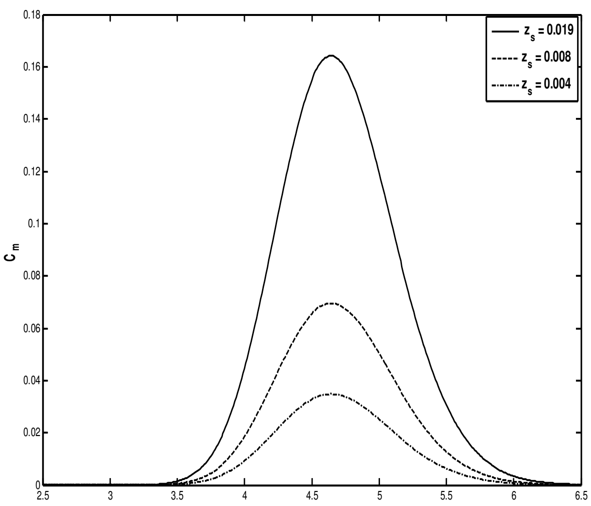


Fig- 6

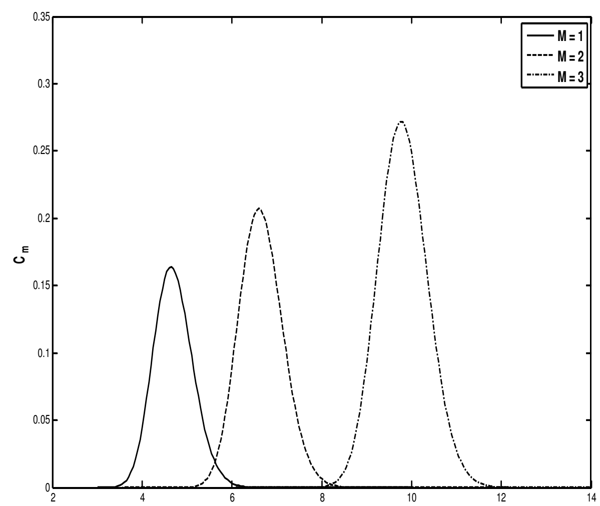
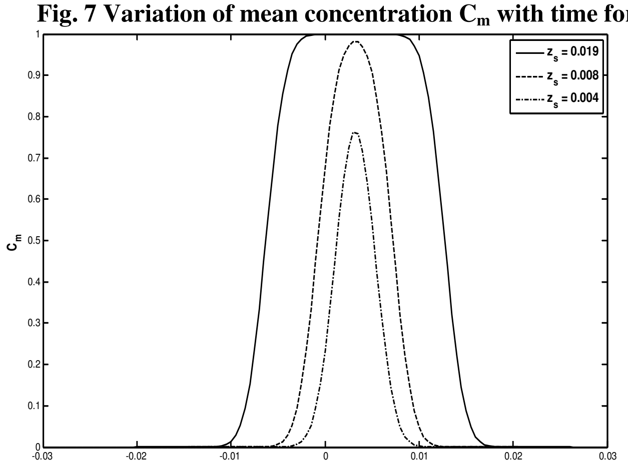


Fig- 7

Fig. 5 Distribution of dispersion function f_1 for different values of M when t = 0.05

Fig. 6 Distribution of dispersion function f_1 for different values of M when t = 0.05

Fig. 7 Variation of mean concentration C_m with time for different values of M when $z_s = 0.019$ and z = 0.5



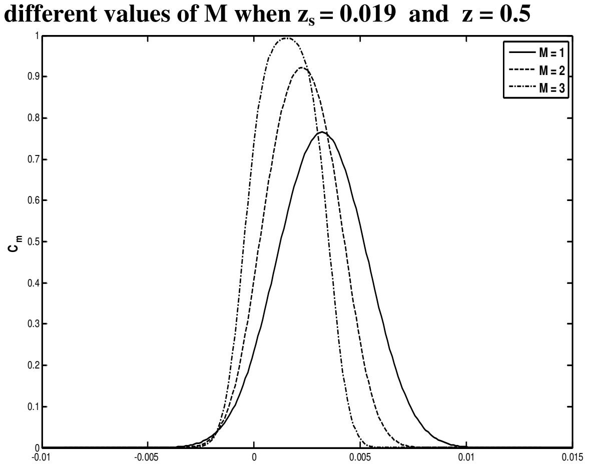


Fig- 8

Fig- 9

Fig 8 Variation of mean concentration C_m with axial distance z for different values of Z_s when t = 0.03 and M =1

Fig 9 Variation of mean concentration C_m with axial distance z for different values of M when t = 0.03 and z_s= 0.04

5. CONCLUSIONS:

The objective of the present investigation is to study the effect of a magnetic field on the process of dispersion. The convective dispersion process is analysed applying the generalized dispersion model. It is observed that the diffusion coefficient, which describes the dispersion process, is influenced by a magnetic field. It is observed that the results on (K_2 -1/Pe^2) agree with that of Taylor's theory for large times and for small values of time the results agree with Lighthill. It is also observed that the time beyond which the Taylor theory is applicable is unaffected by the presence of magnetic field. The effect of magnetic field is to reduce the rate of dispersion of the solute in the fluid flow. It is observed that the presence of magnetic field, time taken for the dispersion coefficient to reach a steady state is more and the time further increases with increase in the magnetic field. Enhancement in magnetic field reduces the dispersion. The time taken for the mean concentration to attain the peak value is found to increase in magnetic field. The values of C_m are drifted to the right of origin along the axial direction as M increases.

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