International Journal of Mathematical Archive-2(12), 2011, Page: 2757-2761
Available online through www.ijma.info ISSN 2229 – 5046

CALCULATING THE TIME FOR A SILENT EVENT THROUGH THE THRESHOLD LEVEL OF HIV SEROCONVERSION PERIOD

Subramanian, C., Rajivgandhi, R* and R. Vinoth

Department of Statistics, Annamalai University, Annamalai nagar-608 002, Tamil Nadu, India
*E-mail:*statrajiv@gmail.com*

(Received on: 17-11-11; Accepted on: 04-12-11)

ABSTRACT

As the world enters into the 30th year of the AIDS epidemic the common people have more response in preventing this dreadful disease. Mathematical and Statistical model is developed to obtain the expected time to reach the threshold level, in the context of HIV/AIDS with the assumptions that the times between decisions epochs are independent and identically distributed (i.i.d) random variable. The numbers of exits at each period time are i.i.d random variables and that the threshold level is a random variable following Generalized Rayleigh distributions. From the obtained result one can estimate the time period of an infected patient.

Keywords: *Expected Time, Inter arrival Time, Seroconversion, Threshold.*

INTRODUCTION:

The worldwide HIV/AIDS epidemic of the past several decades has stimulated a very impressive amount of methodological innovation in biostatistics, epidemiology, mathematical biology, and related disciplines. In HIV models, interest is sometimes focused on the distribution of time until an event that cannot be directly observed and whose occurrence is assessed by performing a diagnostic test periodically during follow-up.

Early surveys of mathematical and statistical methods developed for HIV/AIDS transmission are numerous; see Anderson et al., (1986), Busenberg et al., (1995). The expected time to reach the seroconversion threshold level of a random variable is found out. Generalized Rayleigh distribution follows the random variable. Graphical illustrations are provided for the support of the model. One can see for more detail in Esary et al., (1973), Rajivgandhi, et al., (2010), Subramanian and Rajivgandhi (2011) discussed the expected time to cross threshold level of seroconversion period. In this paper taking $\alpha = 2$, i.e., the extension of the work from Subramanian and Rajivgandhi (2011) is studied and the expected time of the event to cross the threshold is discussed.

ASSUMPTION:

These assumptions are somewhat artificial, but are made because of the lack of detailed real-world information on one hand and in order to illustrate the proceedings on the other hand.

- Sexual contacts are the only source of HIV infection.
- The threshold threshold of any individual is a random variable.
- If the total damage crosses a threshold level Y which itself is a random variable, the seroconversion occurs and a person is recognized as an infected.
- The inter-arrival times between successive contacts, the sequence of damage and the threshold are mutually independent.

NOTATIONS:

X_i : a continuous random variable denoting the amount of contribution to the antigenic diversity due to the HIV transmitted in the i[th] contact, in other words the damage caused to the immune system in the i[th] contact, with p. d. f g (.) and c. d. f G (.).
Y : a continuous random variable denoting the threshold which follows Generalized Rayleigh distribution.
U_i : a random variable denoting the inter-arrival times between contact with c. d. f. $F_i(.)$,
$i = 1,2,3 \ldots k$.

***Corresponding author: Rajivgandhi, R*, *E-mail:** *statrajiv@gmail.com*

$g(.)$: The probability density functions of X_i
$g^*(.)$: Laplace transform of g (.)
$g_k(.)$: The k- fold convolution of g (.) i.e., p.d.f. of $\sum_{j=1}^{k} X_i$
$f(.)$: p.d.f. of random variable denoting between successive contact with the corresponding c.d.f. F (.)
$F_k(.)$: k-fold convolution of F (.)
$V_k(t)$: Probability of exactly k successive contact.
$S(.)$: Survival function, i.e., $P\,[T > t]$
$L(t)$: 1 - S (t)

RESULT:

Let Y be the random variable which has the cdf defined as

$$F(x) = \left(1 - e^{-(\lambda x)^2}\right)^2$$

The corresponding survival function is

$$\bar{H}(x) = 1 - \left(1 - e^{-(\lambda x)^2}\right)^2$$

$$= 2e^{-(\lambda x)^2} - e^{-2(\lambda x)^2}$$

The shock survival probability are given by

$$P(X_i < Y) = \int_0^\infty g_k(x)\bar{H}(x)dx$$

$$= \int_0^\infty g_k(x)\left[2e^{-(\lambda x)^2} - e^{-2(\lambda x)^2}\right]dx$$

$$= 2[g^*(\lambda)^2]^k - [g^*2(\lambda)^2]^k \quad (1)$$

The survival function which gives the probability that the cumulative threshold will fail only after time t.
$S(t) = P(T > t)$ = Probability that the total damage survives beyond t

$$= \sum_{k=0}^{\infty} P\ \{\text{there are exactly k contacts in } (0,t] * P\ (\text{the total cumulative threshold } (0,t]\}$$

It is also known from renewal process that

$P(\text{exactly k policy decesions in } (0,t]) = F_k(t) - F_{K+1}(t)$ with $F_0(t) = 1$

$$P(T > t) = \sum_{k=0}^{\infty} V_k(t)\, P(X_i < Y)$$

$$= 2\sum_{k=0}^{\infty}[F_k(t) - F_{k+1}(t)][g^*(\lambda)^2]^k - \sum_{k=0}^{\infty}[F_k(t) - F_{k+1}(t)][g^*2(\lambda)^2]^k \quad (2)$$

Now, the life time is given by

$$L(T) = 1 - S(t)$$

$$= 1 - \left\{2\sum_{k=0}^{\infty}[F_k(t) - F_{k+1}(t)][g^*(\lambda)^2]^k - \sum_{k=0}^{\infty}[F_k(t) - F_{k+1}(t)][g^*2(\lambda)^2]^k\right\}$$

$$= 2[1 - g^*(\lambda)^2]\sum_{k=1}^{\infty} F_k(t)\,[g^*(\lambda)^2]^{k-1} - [1 - g^*2(\lambda)^2]\sum_{k=1}^{\infty} F_k(t)\,[g^*2(\lambda)^2]^{k-1} \quad (3)$$

Taking Laplace transform of $L(t)$, we get

$$l^*(s) = \frac{2[1-g^*(\lambda)^2]f^*(s)}{[1-g^*(\lambda)^2 f^*(s)]} - \frac{[1-g^*2(\lambda)^2]f^*(s)}{[1-g^*2(\lambda)^2 f^*(s)]} \quad (4)$$

$$E(T) = -\frac{d}{ds} l^*(s) \; given \; s = 0$$

$$E(T^2) = \frac{d^2}{ds^2} l^*(s) \; given \; s = 0$$

From which $V(T)$ can be obtained.

Let the random variable U denoting inter arrival time which follows exponential with parameter c. Now $f^*(s) = \left(\frac{c}{c+s}\right)$, substituting in the above equation (4) we get,

$$l^*(s) = \frac{2[1-g^*(\lambda)^2]\frac{c}{c+s}}{\left[1-g^*(\lambda)^2\frac{c}{c+s}\right]} - \frac{[1-g^*2(\lambda)^2]\frac{c}{c+s}}{\left[1-g^*2(\lambda)^2\frac{c}{c+s}\right]}$$

$$= \frac{2c[1-g^*(\lambda)^2]}{[c+s-g^*(\lambda)^2 c]} - \frac{c[1-g^*2(\lambda)^2]}{[c+s-g^*2(\lambda)^2 c]}$$

$$E(T) = -\left[-\frac{2c[1-g^*(\lambda)^2]}{[c+s-g^*(\lambda)^2 c]^2} - \frac{c[1-g^*2(\lambda)^2]}{[c+s-g^*2(\lambda)^2 c]^2}\right]$$

$$= \frac{2}{c[1-g^*(\lambda)^2]} - \frac{1}{c[1-g^*2(\lambda)^2]} \qquad on \; simplification \quad (5)$$

$$E(T^2) = \frac{d}{ds}\left[\frac{4c[1-g^*(\lambda)^2]}{[c+s-g^*(\lambda)^2 c]^3} - \frac{2c[1-g^*2(\lambda)^2]}{[c+s-g^*2(\lambda)^2 c]^3}\right]$$

$$= \frac{4}{c^2[1-g^*(\lambda)^2]^2} - \frac{2}{c^2[1-g^*2(\lambda)^2]^2} \qquad on \; simplification \quad (6)$$

$$V(T) = E(T^2) - [E(T)]^2$$

$$V(T) = \frac{4}{c^2[1-g^*(\lambda)^2]^2} - \frac{2}{c^2[1-g^*(\lambda)^2]^2} - \left[\frac{2}{c[1-g^*(\lambda)^2]} - \frac{1}{c[1-g^*2(\lambda)^2]}\right]^2$$

$$g^*(.) \sim exp(\mu)\,, g^*(\lambda)^2 = \frac{\mu}{\mu+2\lambda}\,, g^*2(\lambda)^2 = \frac{\mu}{\mu+4\lambda}$$

$$E(T) = \frac{2}{c\left[1-\frac{\mu}{\mu+\lambda^2}\right]} - \frac{1}{c\left[1-\frac{\mu}{\mu+2\lambda^2}\right]}$$

$$= \left[\frac{2(\mu+\lambda^2)}{\lambda^2 c} - \frac{(\mu+2\lambda^2)}{2\lambda^2 c}\right] \qquad on \; simpilification \quad (7)$$

$$V(T) = \left[\frac{4(\mu+\lambda^2)^2}{c^2\lambda^4} - \frac{2(\mu+2\lambda^2)^2}{c^2 4\lambda^4}\right] - \left[\frac{2(\mu+\lambda^2)}{\lambda^2 c} - \frac{(\mu+2\lambda^2)}{2\lambda^2 c}\right]^2$$

$$= \frac{2}{c}\left[\frac{2(\mu+\lambda^2)}{\lambda^2} - \frac{(\mu+2\lambda^2)}{2\lambda^2}\right] - \frac{3(\mu+2\lambda^2)^2}{c^2 4\lambda^4} \qquad on \; simpilification \quad (8)$$

NUMERICAL ILLUSTRATION:

On the basis of the expressions derived for the expected time and variance the behavior of the same due to the change in different parameters are given in Figures 1 to 4 that follow.

Figure-1

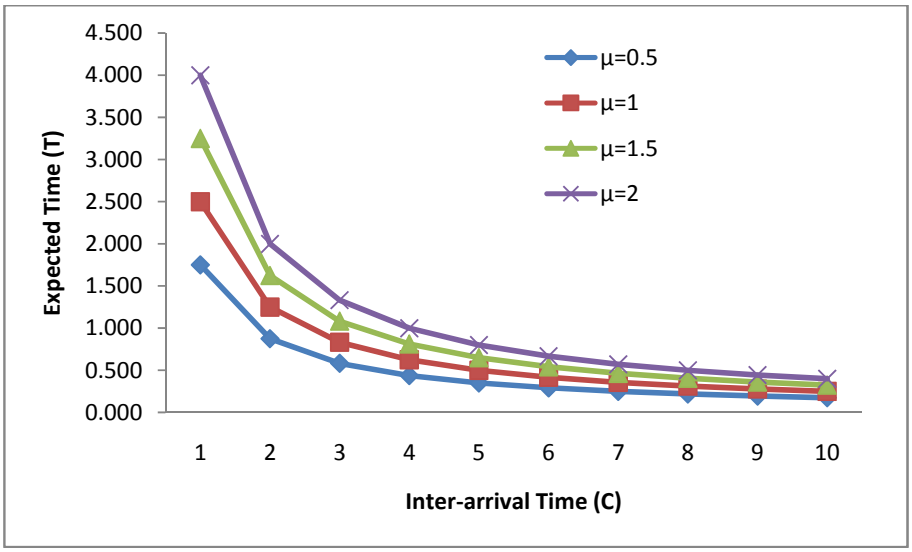


Figure-2

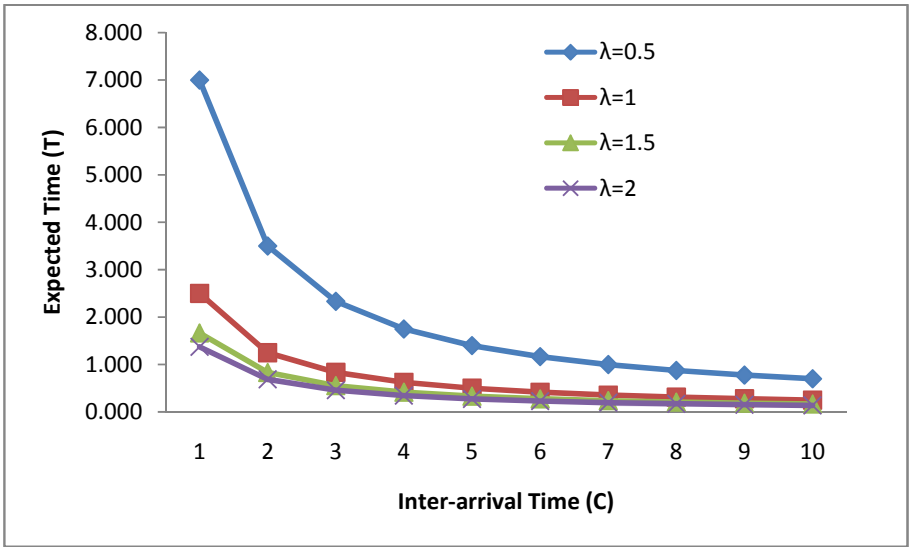


Figure-3

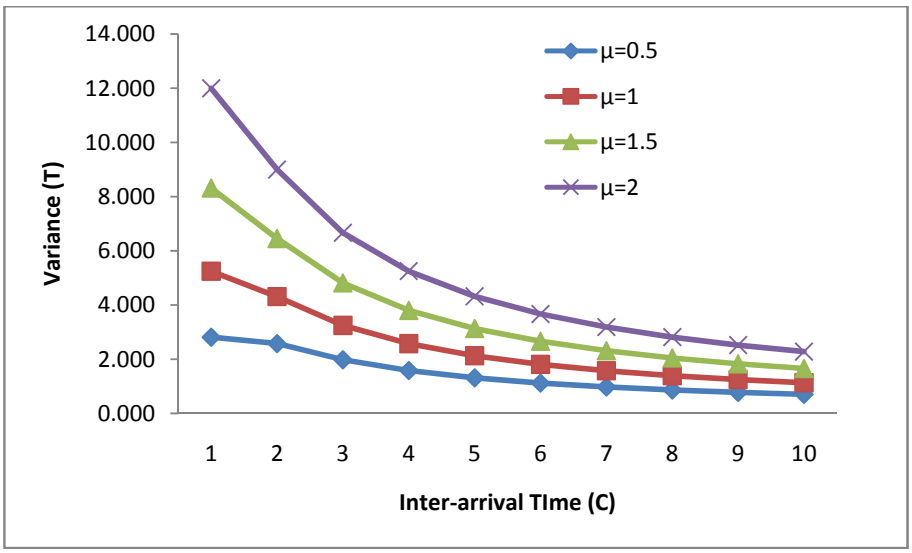
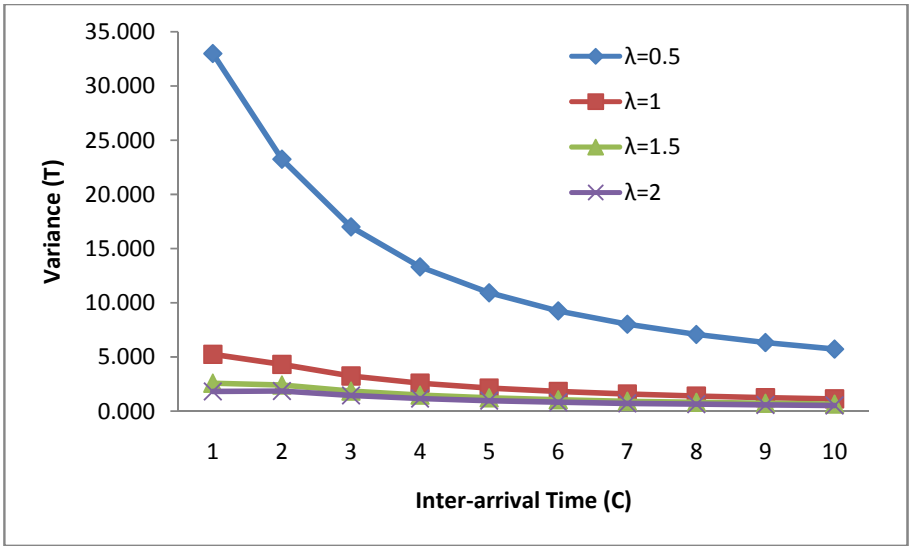


Figure-4



CONCLUSION:

The mathematical models have been discussed by various authors taking into consideration, many hypothetical assumptions. Such models provide the possible clues relating to the consequences of infections, the time taken for seroconversion etc., but these models serve as useful suggestion for the medical personnel to suitably develop the drugs and medicine, and also the methods of treatment.

When μ is kept fixed, the inter-arrival time $'c'$, which follows exponential distribution, is an increasing parameter. Therefore, the value of the expected time $E(T)$ to cross the threshold of seroconversion is decreasing, for all cases of the parameter value $\mu = 0.5,1,1.5,2$. When the value of the parameter μ increases, the expected time is also found decreasing, this is observed in Figure 1. The same case is found in Variance V (T) which is observed in Figure 3.

When λ is kept fixed and the inter-arrival time $'c'$ increases, the value of the expected time E (T) to cross the threshold of seroconversion is found to be decreasing, in all the cases of the parameter value $\lambda = 0.5,1,1.5,2$. When the value of the parameter λ increases, the expected time is found increasing. This is indicated in Figure 2. The same case is observed in the threshold of seroconversion of Variance V (T) which is observed in Figure 4.

REFERENCE:

[1] Anderson, R. M., Medly, G. F., May, R. M., Johnson, A.M., (1986). A preliminary study of the transmission dynamics of the human immunodeficiency virus (HIV), the Causative agent of AIDS. IMAJ. Math. Appl. Med. Biol.3, 229–263.

[2] Busenberg, S., Cooke, K., Ying-Hen, H., (1995). A model for HIV in Asia. Math.Biosci.128, 185–210.

[3] Esary, J.D., A. W. Marshall and F. Proschan. (1973). Shock models and wear processes. Ann. Probability, 1(4), 627-649.

[4] Rajivgandhi, R, Ganesan, A, Vinoth, R and K. Kannadasan. (2010). Left and Right Censoring for Expected Time to Seroconversion – A Shock Model Approach. Journal of Recent Research in Science and Technology, 2(3), 80-85.

[5] Subramanian, C and R. Rajivgandhi, (2011). Extremely Prolonged HIV Seroconversion to Estimate the Expected Time through Generalized Rayleigh Distribution. Asian Journal of Science and Technology, 1(4), pp. 041-043.

****

