# PERISTALTIC TRANSPORT OF A MAGNETIC FLUID IN A UNIFORM AND NON-UNIFORM ANNULUS 

Asha. S. K*1 ${ }^{*}$ and V. P. Rathod ${ }^{2}$<br>Department of studies and Research in Mathematics, Gulbarga University, Gulbarga-585106, Karnataka, India<br>E-mail: as.kotnur2008@gmail.com ${ }^{1}$, rathod_vijaykumar@rediffmail.com ${ }^{2}$<br>(Received on: 23-11-11; Accepted on: 13-12-11)


#### Abstract

The aim of the present investigation is to study the peristaltic transport through the gap between coaxial tubes, where the outer tube is non uniform and the inner tube is rigid. The necessary theoretical results such as viscosity, pressure gradient and friction force on the inner and outer tubes have been obtained in terms of couple stress parameter. Out of these theoretical results the numerical solution of pressure gradient, outer friction, inert friction and flow rate are shown graphically for the better understanding of the problem.


Keywords: Peristaltic transport, magnetic parameter, and flow rate.

## 1. INTRODUCATION:

Peristalsis is now well known to physiologists to be one of the major mechanisms for fluid transport in many biological systems. In particular, a mechanism may be involved in swallowing food through the esophagus, in urine transport form the kidney to the bladder through the urethra, in movement of chyme in the gastro -intestinal tract, in the transport of spermatozoa in the ductus efferent of the male reproductive tracts and in the cervical canal, in movement of ovum in the female fallopian tubes, in the transport of lymph in the lymphatic vessels, and in the vasomotion of small blood vessel such as arterioles, venules and capillaries. In addition, peristaltic pumping occurs in many practical applications involving biomechanical system. Also, finger and roller pumps are frequently used for pumping corrosive or very pure materials so as to prevent direct contact of the fluid with the pump's internal surfaces.

A number of analytical $[3,6,7,8,9,12,13,21]$, numerical and experimental $[1,10,18,19,20]$ studies of peristaltic flows of different fluids have been reported. A summary of most of the investigation reported up to the year 1983, has been presented by Srivastava and Srivastava [14], and some imported contribution of recent year, are reference in Srivastava and Saxsen [16]. Physiological organs are generally observed have the form of a non-uniform duct [11, 21]. In particular, the vas deferens in rhesus monkey is in the form of a diverging tube with a ration of exit to inlet dimensions of approximately four [4]. Hence, peristaltic analysis of a Newtonian fluid in a uniform geometry cannot be applied when explaining the mechanism of transport of fluid in most bio-systems. Recently, Srivastava et al [16] and Srivastava and Srivastava [15] studied peristaltic transport of Newtonian and non-Newtonian fluids in non-uniform geometries.

With the above discussion in mind, we propose to study the peristaltic transport of a viscous incompressible fluid (creeping flow) through the gap between coaxial tubes, where the outer tube is non-uniform and has a sinusoidal wave traveling down its wall and the inner one is a rigid, uniform tube and moving with a constant velocity. This investigation may have application in many clinical applications such as the endoscopes problem.

With the above discussion in mind, we propose to study the peristaltic transport of a viscous incompressible fluid (creeping flow) through the gap between coaxial tubes, where the outer tube is non-uniform and has a sinusoidal wave traveling down its wall and the inner one is a rigid, uniform tube and moving with a constant velocity. This investigation may have application in many clinical applications such as the endoscopes problem.

## 2. FORMULATION OF THE PROBLEM:

Consider the flow of an incompressible Newtonian fluid through coaxial tubes such that the outer tubes is non-uniform and has a sinusoidal wave traveling down and inner one rigid, and moving with a constant velocity. The geometry of the wall surface is
*Corresponding author: Asha. S. $K^{* 1}$, *E-mail: as.kotnur2008@gmail.com

$$
\begin{align*}
& r_{1}^{\prime}=a_{1}  \tag{2.1}\\
& r_{2}^{\prime}=a_{2}+b \operatorname{Sin}\left(\frac{2 \pi}{\lambda}\left(x^{\prime}-c t\right)\right) \tag{2.2}
\end{align*}
$$

With
$a_{2}\left(z^{\prime}\right)=a_{20} k z^{\prime}$

With $\mathrm{a}_{1}$ is the radius of the inner tube $a_{2}\left(z^{\prime}\right)$ is the radius of the outer tube at axial distance $z^{\prime}$ form inlet, $\mathrm{a}_{20}$ is the radius of the outer tube at the inlet, $\mathrm{k}(\ll 1)$ is the constant whose magnitude depends on the length of the outer tube, b is the amplitude, $\lambda$ is the wave length, c is the propagation velocity and t is the time. Choose a cylindrical coordinate system $\left(r^{\prime}, z^{\prime}\right)$ where the z-axis lies along the centreline of the inner and the outer tubes and $r^{\prime}$ is the distance measured racially.

The equation of motion of the flow in the gap between the inner and the outer tubes are

$$
\begin{align*}
& \frac{1}{r^{\prime}} \frac{\partial\left(r^{\prime}, u^{\prime}\right)}{\partial r^{\prime}}+\frac{\partial\left(w^{\prime}\right)}{\partial z^{\prime}}=0  \tag{2.3}\\
& \rho\left\{\frac{\partial u^{\prime}}{\partial t^{\prime}}+u^{\prime} \frac{\partial u^{\prime}}{\partial r^{\prime}}+w^{\prime} \frac{\partial u^{\prime}}{\partial z^{\prime}}\right\}=-\frac{\partial p^{\prime}}{\partial r^{\prime}}+\mu\left\{\frac{\partial}{\partial r^{\prime}}\left(\frac{1}{r^{\prime}} \frac{\partial\left(r^{\prime}, u^{\prime}\right)}{\partial r^{\prime}}\right)+\frac{\partial^{2} u^{\prime}}{\partial z^{\prime 2}}\right\}-\sigma B_{0}^{2}\left(u^{\prime}\right)  \tag{2.4}\\
& \rho\left\{\frac{\partial w^{\prime}}{\partial t^{\prime}}+u^{\prime} \frac{\partial w^{\prime}}{\partial r^{\prime}}+w^{\prime} \frac{\partial w^{\prime}}{\partial z^{\prime}}\right\}=-\frac{\partial p^{\prime}}{\partial z^{\prime}}+\mu\left\{\frac{\partial}{\partial r^{\prime}}\left(\frac{1}{r^{\prime}} \frac{\partial\left(r^{\prime}, w^{\prime}\right)}{\partial r^{\prime}}\right)+\frac{\partial^{2} w^{\prime}}{\partial z^{\prime 2}}\right\}-\sigma B_{0}^{2}\left(w^{\prime}\right) \tag{2.5}
\end{align*}
$$

Where $u^{\prime}$ and $w^{\prime}$ are the velocity components in the $r^{\prime}$ and $w^{\prime}$ direction respectively, $\rho$ is the density, $p^{\prime}$ is the pressure and $\mu$ is the viscosity, $\sigma$ is Electric conductivity and $\mathrm{B}_{0}$ is applied magnetic field.
The boundary conditions are
$\begin{array}{llll}u^{\prime}=0, & w^{\prime}=V_{0}^{\prime}, & \text { at } & r^{\prime}=r_{1}^{\prime} \\ u^{\prime}=\frac{\partial r_{2}^{\prime}}{\partial t^{\prime}} & w^{\prime}=0 & \text { at } & r^{\prime}=r_{2}^{\prime}\end{array}$
It is convenient to non dimensionalize the variable appearing in equation (1-6) and introducing Reynolds number Re, wave number ratio $\boldsymbol{\delta}$, and velocity parameter $V_{0}$ and as follows:

$$
\begin{align*}
& z=\frac{z^{\prime}}{\lambda^{\prime}}, \quad r=\frac{r^{\prime}}{c}, \quad u=\frac{\lambda u^{\prime}}{a_{20} c}, \quad p=\frac{a_{20}^{2}}{\lambda \mu c} p^{\prime}\left(z^{\prime}\right), \quad t=\frac{t^{\prime} c}{\lambda}, \quad \operatorname{Re}=\frac{\rho c a_{20}}{\mu}, \\
& \delta=\frac{a_{20}}{\lambda}, \quad V_{0}=\frac{V_{o}^{\prime}}{c} \quad r_{1}=\frac{r_{1}^{\prime}}{a_{20}}=\varepsilon, \quad r_{2}=\frac{r_{2}^{\prime}}{a_{20}}=1+\frac{\lambda k z}{a_{20}}+\phi\left(\frac{2 \pi}{\lambda}(z-t)\right. \tag{2.7}
\end{align*}
$$

$$
\text { where } \phi \quad \text { amplitude }=\frac{b}{a_{20}} \leq 1
$$

The equation of motion and boundary conditions in the dimensionless form becomes
$\frac{1}{r} \frac{\partial(r u)}{\partial r}+\frac{\partial w}{\partial z}=0$
$\operatorname{Re} \delta^{3}\left\{\frac{\partial u}{\partial t}+u \frac{\partial u}{\partial r}+w \frac{\partial u}{\partial z}\right\}=-\frac{\partial p}{\partial r}+\delta^{2} \frac{\partial}{\partial r}\left(\frac{1}{r} \frac{\partial(r u)}{\partial r}\right)+\delta^{4} \frac{\partial^{2} u}{\partial z^{2}}-\delta^{2} M^{2} u$

Asha. S. $K^{* 1}$ and V. P. Rathod ${ }^{2} /$ Peristaltic Transport of a Magnetic fluid In a Uniform and Non-Uniform Annulus / IJMA- 2(12),
$\operatorname{Re} \delta\left\{\frac{\partial w}{\partial t}+u \frac{\partial w}{\partial r}+w \frac{\partial w}{\partial z}\right\}=-\frac{\partial p}{\partial z}+\frac{\partial}{\partial r}\left(\frac{1}{r} \frac{\partial(r w)}{\partial r}\right)+\delta^{2} \frac{\partial^{2} w}{\partial z^{2}}-M^{2} w$
where $M=\sqrt{\frac{\sigma}{\mu}} B_{0} a_{20}$ is the Hartmann number
The boundary conditions are

$$
\begin{array}{ll}
\mathrm{u}=0 & \mathrm{w}=\mathrm{V}_{0} \quad \text { at } \mathrm{r}=\mathrm{r}_{1}=\mathcal{E}, \\
\mathrm{u}=\frac{\partial r_{2}}{\partial y} & w=0 \quad \text { at } r=r_{2}=1+\frac{\lambda k z}{a_{20}}+\phi \sin [2 \pi(z-t)] \tag{2.11b}
\end{array}
$$

Using the long wavelength approximation and dropping terms of order $\boldsymbol{\delta}$ it follows from equation (5.2.8-5.2.11) that the appropriate equation describing the flow in the laboratory frame of reference are

$$
\begin{align*}
& \frac{\partial p}{\partial r}=0  \tag{2.12}\\
& \frac{\partial p}{\partial z}=\frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial w}{\partial r}\right)-M^{2} w \tag{2.13}
\end{align*}
$$

with dimensionalessl boundary condition

$$
\begin{array}{ll}
\mathrm{w}=\mathrm{V}_{0} & \text { at } \mathrm{r}=r_{1}=\mathcal{E} \\
w=0 & \text { at } r=r_{2}=1+\frac{\lambda k z}{a_{20}}+\phi \sin [2 \pi(z-t)] \tag{2.14}
\end{array}
$$

Integrating equation and using the boundary condition one finds the expression for the velocity profile as

$$
\begin{equation*}
w(z, t)=-\frac{1}{4}\left(\frac{\partial p}{\partial z}\right)\left[\left(r_{2}^{2}-r_{1}^{2}\right)\left(\frac{\operatorname{In}\left(r / r_{1}\right)}{\operatorname{In}\left(r_{2} / r_{1}\right)}\right)-r^{2}+r_{1}^{2}\right]-\frac{V_{0}}{\operatorname{In}\left(r_{2} / r_{1}\right)} \operatorname{In}\left(r / r_{2}\right)\left[1-\frac{r_{1}}{4} M^{2}\right] \tag{2.15}
\end{equation*}
$$

The instantaneous volume flow rate $\mathrm{Q}(\mathrm{z}, \mathrm{t})$ is given by

$$
\begin{gather*}
Q(z, t)=\int_{r_{1}}^{r_{2}} 2 \pi r w d r=-\frac{\pi}{8} \frac{\partial p}{\partial z}\left\{\left(r_{2}^{2}-r_{1}^{2}\right)\left[r_{2}^{2}+r_{1}^{2}-\left(\frac{r_{2}^{2}-r_{1}^{2}}{\operatorname{In}\left(r_{2} / r_{1}\right)}\right)\right]\right\} \\
-\pi V_{o}\left\{\frac{r_{1}^{2}-r_{2}^{2}}{\operatorname{In}\left(r_{2} / r_{1}\right)}+r_{1}^{2}\right\}\left[1-\frac{r_{1}}{4} M^{2}\right]  \tag{2.16}\\
\frac{\partial p}{\partial z}=-8\left(\frac{\left.Q / \pi \operatorname{In}\left(r_{2} / r_{1}\right)\right)+\frac{V_{0}}{2}\left(r_{1}^{2}-r_{2}^{2}\right)+V_{0} r_{1}^{2} \operatorname{In}\left(r_{2} / r_{1}\right)\left[1-\frac{r_{1}}{4} M^{2}\right]}{\left(r_{2}^{4}-r_{1}^{4}\right) \operatorname{In}\left(r_{2} / r_{1}\right)-\left(r_{2}^{2}-r_{1}^{2}\right)^{2}}\right) \tag{2.17}
\end{gather*}
$$

The pressure rise $\Delta p_{L}(t)$ and friction force (at the wall) on the outer and the inner tubes $F_{L}^{(o)}(t)$ and $F_{L}^{(i)}(t)$ respectively, in a tube of length $L$, in their non-dimensional forms, are given by

$$
\begin{equation*}
\Delta p_{L}(t)=\int_{0}^{A} \frac{\partial p}{\partial z} d z \tag{2.18}
\end{equation*}
$$

Asha. S. $K^{*^{1}}$ and V. P. Rathod ${ }^{2} /$ Peristaltic Transport of a Magnetic fluid In a Uniform and Non-Uniform Annulus / IJMA- 2(12),

$$
\begin{align*}
& \Delta F_{L}^{(o)}(t)=\int_{0}^{A} r_{2}^{2}\left(-\frac{\partial p}{\partial z}\right) d z  \tag{2.19}\\
& \Delta F_{L}^{(i)}(t)=\int_{0}^{A} r_{1}^{2}\left(-\frac{\partial p}{\partial z}\right) d z \tag{2.20}
\end{align*}
$$

Where $\mathrm{A}=\mathrm{L} / \lambda$,
Substituting from equation (2.17) in equation (2.18-2.20) and with $r_{1}=\mathcal{E}$ and
$r_{2}(z, t)=1+\frac{\lambda k z}{a_{20}}+\phi \sin [2 \pi(z-t)]$, we get

$$
\begin{align*}
& \Delta p_{L}(t)=\int_{0}^{A}-8\left\{\frac{Q(z, t)}{\pi} \operatorname{In}\left[\frac{1+\frac{\lambda k z}{a_{20}}+\phi \sin 2 \pi(z-t)}{\varepsilon}\right]\right. \\
&+\frac{V_{0}}{2}\left[\varepsilon^{2}-\left(1+\frac{\lambda k z}{a_{20}}+\phi \sin 2 \pi(z-t)\right)^{2}\right] \\
&\left.+V_{0} \varepsilon^{2} \operatorname{In}\left[\frac{1+\frac{\lambda k z}{a_{20}}+\phi \sin 2 \pi(z-t)}{\varepsilon}\right]\right\}\left[1-\frac{r_{1}}{4} M^{2}\right] \\
&\left\{1 /\left[\left(\left(\left[1+\frac{\lambda k z}{a_{20}}+\phi \sin 2 \pi(z-t]\right)^{4}-\varepsilon^{4}\right) \operatorname{In}\left[\frac{1+\frac{\lambda k z}{a_{20}}+\phi \sin 2 \pi(z-t)}{\varepsilon}\right]\right.\right.\right. \\
&\left.\left(\left(1+\frac{\lambda k z}{a_{20}}+\phi \sin 2 \pi(z-t)^{2}-\varepsilon^{2}\right)^{2}\right)\right\} d z \tag{2.21}
\end{align*}
$$

$$
\Delta F_{L}^{(o)}(t)=\int_{0}^{A} 8\left\{\left(1+\frac{\lambda k z}{a_{20}}+\phi \sin 2 \pi(z-t)\right)^{2}\right\}+
$$

$$
\frac{V_{0}}{2}\left[\varepsilon^{2}-\left(1+\frac{\lambda k z}{a_{20}}+\phi \sin 2 \pi(z-t)\right)^{2}\right]_{+}
$$

$$
1+\frac{\lambda k z}{a_{20}}+\phi \sin 2 \pi(z-t)
$$

$$
\left.V_{0} \varepsilon^{2} \operatorname{In}\left[\frac{a_{20}}{\varepsilon}\right]\right\}\left[1-\frac{r_{1}}{4} M^{2}\right]
$$

$$
\left\{1 /\left[\left(\left(1+\frac{\lambda k z}{a_{20}}+\phi \sin 2 \pi(z-t)\right)^{4}-\varepsilon^{4}\right)\right.\right.
$$

$$
\operatorname{In}\left[\frac{1+\frac{\lambda k z}{a_{20}}+\phi \sin 2 \pi(z-t)}{\varepsilon}\right]-
$$

$$
\begin{equation*}
\left.\left(\left(1+\frac{\lambda k z}{a_{20}}+\phi \sin 2 \pi(z-t)^{2}-\varepsilon^{2}\right)^{2}\right)\right\} \mathrm{d} z \tag{2.22}
\end{equation*}
$$

$$
\begin{gather*}
\Delta F_{L}^{(i)}(t)=\int_{0}^{A} 8 \varepsilon^{2}\left\{\frac{Q(z, t)}{\pi} \operatorname{In}\left[\frac{1+\frac{\lambda k z}{a_{20}}+\phi \sin 2 \pi(z-t)}{\varepsilon}\right]\right. \\
\frac{V_{0}}{2}\left[\varepsilon^{2}-\left(1+\frac{\lambda k z}{a_{20}}+\phi \sin 2 \pi(z-t)\right)^{2}\right]+ \\
\left.V_{0} \varepsilon^{2} \operatorname{In}\left[\frac{1+\frac{\lambda k z}{a_{20}}+\phi \sin 2 \pi(z-t)}{\varepsilon}\right]\left[1-\frac{r_{1}}{4} M^{2}\right]\right\} \\
\left\{1 /\left[\left(\left(1+\frac{\lambda k z}{a_{20}}+\phi \sin 2 \pi(z-t)\right)^{4}-\varepsilon^{4}\right)\right.\right. \\
\operatorname{In}\left[\frac{1+\frac{\lambda k z}{a_{2} 0}+\phi \sin 2 \pi(z-t)}{\varepsilon}\right] \\
\left.\left.-\left(\left(1+\frac{\lambda k z}{a_{20}}+\phi \sin 2 \pi(z-t)\right)^{2}-\varepsilon^{2}\right)^{2}\right]\right\} d z \tag{2.23}
\end{gather*}
$$

The limiting of equation (2.15-2.17) as $r_{1}$ tends to zero gives the forms of the axial velocity and the pressure gradient for peristaltic flow in non uniform tube(without endoscope, $\mathcal{E}=0$ ), these are

$$
\begin{align*}
& w(r, z, t)=-\frac{1}{4}\left(\frac{\partial p}{\partial z}\right)\left(r_{2}^{2}-r^{2}\right)  \tag{2.24}\\
& \frac{\partial p}{\partial z}=-\frac{8 Q}{\pi r_{2}^{4}} \tag{2.25}
\end{align*}
$$

Hence the pressure rise and the outer friction force, in this case respectively, take the form

$$
\begin{align*}
& \Delta p_{L}(t)=-8 \int_{0}^{A} \frac{Q(z, t) / \pi}{\left(1+\frac{\lambda k z}{a_{20}}+\phi \sin 2 \pi(z-t)\right)^{4}} d z  \tag{2.26}\\
& \Delta F_{L}^{(o)}(t)=8 \int_{0}^{A} \frac{Q(z, t) / \pi}{\left(1+\frac{\lambda k z}{a_{20}}+\phi \sin 2 \pi(z-t)\right)^{2}} d z \tag{2.27}
\end{align*}
$$

Equation (2.26) and equation (2.27) are the same result as those obtained by Gupta and Seshadri [16], Srivastava and Srivastava for non Newtonian fluid [15] when the power law index and also those obtained by Srivastava et al.[14] for a constant viscosity if $\delta=0$. Further, if $\mathrm{k}=0$ in equations (2.26) and (2.27), we get expression for the pressure rise and friction force in a uniform tube. The analytical interpretation of our analysis with other theories are difficult to make at this stage, as the integrals in equation (2.21-2.23) and equation (2.26) and (2.27) are not integrable in closed form, neither for non-uniform nor uniform geometry ( $\mathrm{k}=0$ ). Thus further studies of our analysis are only possible after numerical evaluation of these integrals.

### 5.3. NUMERICAL RESULT AND DISCUSSION:

To discuss the results obtained above quantitatively we shall assume the form of the instantaneous volume rate of the flow $\mathrm{Q}(\mathrm{z}, \mathrm{t})$, periodic in $(\mathrm{z}-\mathrm{t})$ as $[19,21]$

$$
\frac{Q(z, t)}{\pi}=\frac{\bar{Q}}{\pi}-\frac{\phi^{2}}{2}+2 \phi \sin (2 \pi(z-t))+\frac{2 \lambda k z}{a_{20}} \phi \sin (2 \pi(z-t))+\phi^{2} \sin ^{2}(2 \pi(z-t))
$$

where $\bar{Q}$ is the time average of the flow over one period of the wave. This form $\mathrm{Q}(\mathrm{z}, \mathrm{t})$ has been assumed in view of the fact that the constant value of $\mathrm{Q}(\mathrm{z}, \mathrm{t})$ gives $\Delta P_{L}(t)$ always negative, and hence will be no pumping action. Using this form of $\mathrm{Q}(\mathrm{z}, \mathrm{t})$, we shall now compute the dimensionless pressure rise $\Delta P_{L}(t)$, the inner friction force $F_{L}^{(i)}(t)$ (on the
inner surface) and the outer friction force $F_{L}^{(o)}(t)$ (on the outer tube) over the tube length for various value of the dimensionless time t , dimensionless flow average $\bar{Q}$, amplitude ratio $\phi$, radius ratio $\mathcal{E}$, magnetic field parameter M and the velocity of the inner tube $\mathrm{V}_{0}$. The average rise in pressure $\Delta \bar{P}_{L}$, outer friction force $F_{(L)}^{-(o)}$ and the inner friction force $F_{L}^{-(i)}(t)$ are then evaluated by averaging $\Delta P_{L}(t), F_{L}^{(o)}(t)$ and $F_{L}^{(i)}(t)$ over one period of the wave. As integrals in equation (2.21-2.23) are not integrable in closed form, they are evaluated numerically using digital computer. Following Srivastava [15],], using the value of the various parameters in equation (2.21-2.23) as:

$$
\mathrm{a}_{20}=1.25 \mathrm{~cm}, \quad \mathrm{~L}=\lambda=8.01 \mathrm{~cm} \quad k=\frac{3 a_{20}}{\lambda}
$$

Furthermore, since most routine upper gastrointestinal endoscopes are between $8-11 \mathrm{~mm}$ in diameter as reported Cotton and Williams [17] and the radius of the small intestine is 1.25 cm as reported in Srivastava [15] then the radius ratio $\mathcal{E}$, take the values $0.32,0.38$, and 0.44 .

In figures (1) and (3) we plot the variation of Hartmann number and radius ratio on the pressure rise over the length of a non uniform annulus when the magnitude of the velocity is zero. We note that increasing the Hartmann number the pressure is also increases. In Fig (2) when both the Hartmann number and radius ratio kept constant here it is observed that increasing the velocity decreases the pressure rise when radius ratio is at $\mathcal{E}=0.38$ and Hartmann number at $\mathrm{M}=5$

In Figures (4) (5) and (6) we consider the effects of a variable Hartmann number, velocity and radius ratio on the inner friction force over the length of a non-uniform annulus. The effects of varying velocity and Hartmann number on the inner friction force are same as indicate in outer friction force. From fig (4) it is observed that the as the radius ratio increase the inner friction force is also increases when the magnitude of velocity is zero and Hartmann number is at $\mathrm{M}=5$. In fig. (7) we plotted the variation of pressure rise over the length of a uniform annulus for different value of velocity it is clear that as velocity increase the pressure is also increases for different value of flow rate $\bar{Q}=0.0,0.22$, 0.66 , when magnetic field is placed at $\mathrm{M}=5$ and radius ratio $\mathcal{E}$ at 0.38 In figures (8), (9) (10) and (11) we plot the effects of variation in Hartmann number velocity and radius ratio on the pressure rise, inner and outer friction. From fig (8) and (9) it is observed that as the velocity increase the pressure is also increases and as the magnetic field increases the velocity is decreases. From fig (10) it is clear that as the velocity increase the pressure is also increases for different value of radius ratio $\mathcal{E}=0.32,0.38,0.44$, it has been observed that as the radius ratio increase there is a decreasing in the pressure when magnetic field is placed at $\mathrm{M}=5$. In fig (11) it is clear that as the velocity increase the inner friction force is decreases for different value of magnetic field $\mathrm{M}=10,50,100$ when radius ratio is constant and magnitude of the velocity is varying and is not affected when velocity is zero.


Fig (1): Variation of pressure rise over the length of a non-uniform annulus at $\phi=0.4, \mathrm{~V} 0=0$, and different values of M


Fig (2): Variation of pressure rise over the length of a non-uniform annulus at $\phi=0.4, \varepsilon=0.38$, and $\mathrm{M}=1$ different values of $\mathrm{V}_{0}$


Fig (3): Variation of pressure rise over the length of a non-uniform annulus at different values of $\varepsilon \phi=0.4, \mathrm{~V}_{0}=0$ and $\mathrm{M}=1$ '

Asha. S. $K^{* 1}$ and V. P. Rathod ${ }^{2} /$ Peristaltic Transport of a Magnetic fluid In a Uniform and Non-Uniform Annulus / IJMA- 2(12),
Dec.-2011, Page: 2802-2812


Fig (4): Variation of inner friction force over the length of a non-uniform annulus at $\phi=0.4$, and different values of $\varepsilon$ at $\mathrm{V}_{0}=0, \mathrm{M}=3$


Fig (5): Variation of inner friction force over the length of a non-uniform annulus at $\phi=0.4$, and different values of $V_{0}$, $\varepsilon=0.38 \mathrm{M}=3$


Fig (6): Variation of inner friction force over the length of a non-uniform annulus at $\phi=0.4$, and different values of M at $\mathrm{V}_{0}=0, \varepsilon=0.38$

Asha. S. $K^{*^{1}}$ and V. P. Rathod ${ }^{2} /$ Peristaltic Transport of a Magnetic fluid In a Uniform and Non-Uniform Annulus / IJMA- 2(12), Dec.-2011, Page: 2802-2812


Fig (7): Variation of pressure rise over the length of a uniform annulus at $\phi=0.4, \mathrm{M}=5, \varepsilon=0.38$ and different values of

 valnes ofic atic = 0.3, and for difinem wabees of Yat $\mathrm{E}=0.38$





 vilues of Mat = 03 and for ot berl witus of Yal $E=0.38$

## ACKNOWLEDGEMENT:

The author Miss. Asha is grateful to the Rajive Gandhi National Fellowship For financial support in publication.

## REFERENCES:

[1] Brown T.D and Hung T.K, "Computational and Experimental Investigations of Two Dimensional Non-Linear Peristaltic Flows", J. Fluid Mech., 83 (1977), p. 249.
[2] Cotton P.B and Williams C.B, Practical Gastrointestinal Endoscopy, London: Oxford University Press Third Edition, (1990).Paper Received 18 March 2003,Revised 12 October 2003; Accepted 13 January 2004.
[3] Elshehawey L.T and Kh.S. Mekheimer, "Couple Stress in Peristaltic Transport of Fluids" J. Phys, D: Appl. Phys,. 27 (1994), p. 1163.
[4] Guha S.K. Kaur and Ahmed A , Mechanics of Spermatic Fluid
[5] Gupta B. B. and Sheshadri V, Peristaltic Pumping in Non-Uniform Tubes", J.Biomech. 9 (1976), pp. 105-109.
[6] Kh.S. Mekheimer Elsayed, L.El. Shehawey, and A.M.Elaw, "Peristaltic Motion of a Particle fluid Suspension in a Planar Channel", Int.J. of Theor .Phys. (IJTP), 37 (11) (1998), p. 2895.
[7] Kh.S. Mekhemier, "Non Linear Peristaltic Transport a Porous Medium in an Inclined Planar Channel", J. Porous. Media, 6(3), (2003), pp. 189-201.
[8] Kh.S.Mekhemier, "Non-Linear Peristaltic Transport of Magneto-Hydrodynamic Flow in an Inclined Planar Channel" AISE , 28 (2A) (2003) pp 183-201.
[9] Kh.S. Mekhemier, "Peristaltic Tranport of a Couple Stress fluid in a Uniform and Non-Uniform Channels", Biorheol., 39 (2002), pp 755-765.
[10] Latham T.W, "Fluid Motion in a Peristaltic Pump", M.Sc. Thesis, MIT, Cambridge MA (1966).
[11] Lee J.S. and.Hung Y.C "Flow in Non - Uniform Small Blood Vessel", Microcir. Res., 3 (1971), pp. 272-279.
[12] Ramachandra R.A and Usha S "Peristaltic Transport of Two Immiscible Viscous Fluids in a Circular Tube", J. Fluid Mech., 298 (1995), p. 271
[13] Shapiro H.A, Jaffrin M.Y, and Weinberg S.L., "Peristaltic pumping Long Wave length at Low Reynolds Number" J. Fluid Mech., 37(1969), pp. 799-825.
[14] Srivastava L.M and Srivastava V.P, "Peristaltic Tranport of Blood Casson-II", J. Biomech. 17(1984), p. 821-829.
[15] Srivastava L. M.and Srivastava V.P, "Peristaltic Transport of a Power-Law Fluid: Application to the Ductus Efferentes of the Reproducative Tract" Rheo. Acta, 27 (1988), pp 428-433.
[16] Srivastava L.M. and Srivastava V.P., and Sinha S.K, "Peristaltic Transport of a Physiological Fluid: Part I Flow in Non- Uniform Geometry" Biorheol., 20(1983), pp. 428-433.
[17] Srivastava V.P and M.Saxena "A Two Fluid Model of Non-Newtonian Blood Flow Induced by Peristaltic Waves". Rheol.Acta, 33(1994), p.111.
[18] Takabatake S and Ayukawa K "Peristaltic Pumping in Circular Cylindrical Tubes: a Numerical Study of Fluid Transport and its Efficiency", J. Fluid Mech., 193 (1988).
[19] Takabatake S and Ayukawa K, "Numerical Study of two Dimensional Peristaltic Flows", J. Fluid Mech.,122 (1982), p 439.
[20] Tang D. and Shen M, Non Stationary peristaltic and transport of a heat conducting fluid, J .of Math. Anal. And Appl., 174 (1) (1993), p 265.
[21] Wiedeman M.P, Dimension of Blood Vessel from Distributing Artery to Collecting Vein Circ. Res., 12(1963), pp 375-381.
[22] Zien T.T and Ostrach S.A, "A Long Wave Approximation to Peristaltic Motion", J. Biomech, 3 (1970), p. 63.

