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STRONGLY GENERALIZED b-CLOSED MAPS IN TOPOLOGICAL SPACES

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ABSTRACT

*In this paper, we introduce and investigate a new class of sets and maps between topological spaces called strongly generalized b closed sets and g*b continuous maps, respectively. Furthermore, we introduce the concepts of strongly generalized b closed maps and investigate several properties of them.*

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***Key Words and Phrases**: strongly generalized b closed set, strongly generalized b continuity, strongly generalized b-open map, strongly generalized b b-closed maps*

1. INTRODUCTION:

Andrijevic introduced b-open [3] sets in 1996. This is a subclass of semi preopen [2] sets and superset of the class of semi-open sets. Andrejevic studied several fundamental and interesting properties and showed that a rare b-open set is preopen. Variants of continuity related to this concept, studied in [4, 6, 7, 8, 10, 14, 15]. The purpose of the present paper is to introduce and investigate some of the fundamental properties of strongly generalized *b* continuous functions. We obtain characterizations of strongly generalized *b*-continuous functions and investigate relationships between other variants of continuity.

2. PRELIMINARIES:

Throughout this paper (X, τ) and (Y, σ) (or simply X and Y) represent nonempty topological spaces on which no separation axioms are assumed, unless otherwise mentioned. For a subset S of (X, τ), $cl(S)$ and $int(S)$ represent the closure of S with respect to τ and the interior of S with respect to τ , respectively.

A subset S of a space (X, τ) is called semi-open [7] (resp.preopen [2], α-open [12], semi-preopen [2] or β-open [1], b-open [3] or $sp-$open [3])if $S \subset cl(int(S))$;(resp. $S \subset int(cl(S))$, $S \subset int(cl(int(S)))$, $S \subset cl(int(cl(S)))$, $S \subset cl(int(S)) \cup int(cl(S))$) The family of all semi-open (resp.preopen, α- open, β-open, b-open) sets of X denoted by $SO(X)$ (resp. $PO(X)$, $\alpha O(X)$,$\beta O(X)$, $BO(X)$) The complement of a semi-open (resp.preopen, α-open, β-open, b-open) set is said to be semi-closed (resp. preclosed, α-closed, β-closed,b-closed). If S is a subset of a space X, then the b-closure of S, denoted by $bcl(S)$, is the smallest b-closed set containing S. The b-interior of S, denoted by $bint(S)$ is the largest b-open set contained in S. The semiclosure, preclosure,α-closure, β-closure of S are similarly defined and are denoted by $scl(S)$,$pcl(S)$, $\alpha Cl(S)$, $\beta Cl(S)$. The family of all b-closed sets of X is denoted by $BC(X)$ and the family of all b-open (resp. α-open) sets of X containing apoint $x \in X$ is denoted by $BO(X, x)$ (resp. $\alpha O(X, x)$).

Our next definition contains some types of functions used throughout this paper.

Definition: 2.1 A subset A of a space (X, τ) is called
(1) a generalized closed set (briey g-closed) [11] if $Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open;

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(2) a generalized b- closed set (briey g*b -closed) [11] if bCl(A) $\subseteq$U whenever A $\subseteq$U and U is open.

(3) a b-generalized set (briefly bg-closed) if bcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is b-open.

(4) A subset of a topological space (X, τ) is said to be g*b-closed set in (X, τ) if bcl(A) $\subseteq$ G whenever A $\subseteq$ G where G is g-open. The collection of all g*b-closed sets of (X, τ) is denoted by G*bC(X, τ).

3. STRONGLY GENERALIZED b-CONTINUOUS MAPS:

Definition: 3.1 A function f: (X, τ) → (Y, σ) is called*:*

(a) *b*-continuous [5] (resp. *b*-irresolute [6]) if f^{-1} *(V)* is *b*-open in *(X, τ)* for every open set (resp. *b*-open set) *V* of *(Y, σ)*;

Definition: 3.2 A map $f:(X,\tau)\to(Y,\sigma)$ is called strongly generalized b-continuous (g*b-continuous) if the inverse image of every closed set V of (Y,σ) is g*b-closed in (X,τ).

Proposition: 3.3 Every continuous map is g*b-continuous but not conversely.

The class of all g*b-continuous maps properly contains the class of all continuous maps.

Proposition: 3.4 If $f:(X,\tau)\to(Y,\sigma)$ is g*b-irresolute, then f is g*b -continuous.

Proof: Let F be any closed set in Y. Since every closed set is g*b-closed, F is g*b-closed in Y. Since f is g*b-irresolute, f^{-1}(F) is g*b-closed in X. Therefore f is g*b-continuous.

The converse is not true as seen from the following example.

Example: 3.5 Let X = {a, b, c, d}, τ = {X,ϕ,{a},{b},{a, b},{a, b, c}}=σ . Let $f:(X,\tau)\to(Y,\sigma)$ be defined as f (a) =b, f (b) = a, f(c) = d, f (d) = a. Then f is g*b continuous but not g*b irresolute, since f^{-1}(d) = {a} is not g*b- closed in X.

Remark: 3.6 (i) Every g-continuous function is g*b-continuous. (ii) Every super-continuous map is g*b-continuous. The converse need not be true as shown from the following examples.

Example: 3.7 Consider X= { a, b, c}= Y, τ ={ X, ϕ ,{a},{a,b}} and σ ={ Y, ϕ ,{a}}. Let $f:(X,\tau)\to(Y,\sigma)$ be defined as f (a) = a, f (b) = c, f(c) = b, then f is g*b continuous but not super continuous. Since f^{-1}{b, c} = {b, c} is not b-closed in X.

Example: 3.8 Let X = { a, b, c, d}=Y , τ= { X, ϕ ,{a},{b}, {a,b}, {a,b,c}}=σ .Let $f:(X,\tau)\to(Y,\sigma)$ be a map defined as f(a) = a , f(b) = b , f(c) = d , f(d) = c. Then the map is g*b-continuous, but not g-closed in X.

Remark: 3.9 The concept of b-continuity and g*b-continuity are independent concepts. From the above example 3.7,

we find that f is g*b-continuous but not b-continuous. For the b-open set in

Definition: 3.10 A map $f:(X,\tau)\to(Y,\sigma)$ is called strongly g*b-continuous, if the inverse image of every g*b-open set is open in X.

Proposition: 3.11 A map $f:X\to Y$ is strongly g*b-continuous, if and only if the inverse image of every g*b-closed set in Y is closed in X.

Proof: Let F be any g*b-closed set in Y. Then Y\F is g*b-open in Y. By assumption f^{-1}(Y\F) is open in X. But f^{-1}(Y\F) = X\ f^{-1}(F) and so f^{-1}(F) is closed in X. Conversely assume the condition holds. Let V be any g*b-open set in Y. Then Y\V is g*b-closed in Y. Hence f^{-1}(Y\V) is closed in X. Therefore f^{-1}(V) is open in X. Thus f is strongly g*b-continuous.

Proposition: 3.12 Let $f:(X,\tau)\to(Y,\sigma)$ be a map.

(i) If f is strongly g*b-continuous, then it is strongly g-continuous.

(ii) If f is strongly g*b-continuous, then it is continuous.

(iii) If f is strongly g*b continuous, then it is strongly continuous.

Example: 3.13 Let X= { a, b, c}= Y , τ = { X, ϕ, {a}, {b}, {a, b}, { a, b, c}, { a, b, d}}and σ ={ Y, ϕ ,{a}}. Let $f:(X,\tau)\to(Y,\sigma)$ be the identity. Then f is strongly continuous but not strongly g*b-continuous. For if we consider the set {b}, then $f^{-1}\{b\}$ = {b} which is closed, but not open in X.

Example: 3.14 Let X= { a, b, c}= Y , τ = { X, ϕ, {a},{a,b}}=σ .Let $f:(X,\tau)\to(Y,\sigma)$ be defined as f(a) = b, f (b) = a , f(c)= c, f(d) = d , then f is strongly g-continuous but not strongly g*b-continuous. Since for the set A = {a, c}, $f^{-1}\{a, c\}$={b, c} is not g*b-closed in X.

Example: 3.15 Let X= { a, b, c}= Y , τ = { X, ϕ, {a}, {b}, {a,b}}and σ ={ Y, ϕ ,{a},{a, b}}. Let $f:(X,\tau)\to(Y,\sigma)$ be the identity. Then f is continuous. However f is not strongly g*b-continuous. Since for the g*b-open set {b}, the inverse image {b} is not open in X.

Proposition: 3.16 A map $f:(X,\tau)\to(Y,\sigma)$ is g*b-continuous if and only if $f:(X,\tau^{g^{*}b})\to(Y,\sigma)$ is continuous.

Proof: Assume that $f:(X,\tau)\to(Y,\sigma)$ is g*b-continuous. Then $f^{-1}(U)\in\tau^{\omega}$ for every U∈ σ. Therefore, $f:(X,\tau^{g^{*}b})\to(Y,\sigma)$ is continuous.

Conversely, assume that $f:(X,\tau^{g^{*}b})\to(Y,\sigma)$ is continuous. Then $f^{-1}(G)\in\sigma$. Therefore, $f:(X,\tau)\to(Y,\sigma)$ is g*b-continuous.

Definition: 3.17: A topological space (X,τ) is called T_{b**} - space if every g*b-closed set is b-closed.

Proposition: 3.18 A map $f:X\to Y$ is g*b-irresolute iff the inverse image of every g*b-open set is g*b-open in X.

Proposition: 3.19 Let $f:(X,\tau)\to(Y,\sigma)$ and $g:(Y,\sigma)\to(Z,\eta)$ are two maps. Then

(i) $g\cdot f$ is g*b-continuous, if g is continuous and g*b-continuous.
(ii) $g\cdot f$ is g*b-continuous, if g is g*b-continuous and f is g*b-irresolute.
(iii) $g\cdot f$ is g*b-continuous, if g is continuous and f is strongly g*b-continuous.
(iv) Let (Y,σ) be almost weakly Housdroff space. Then $g\cdot f$ is if g is g*b-continuous and f is g*b-continuous.
(v) Let (Y,σ) be a T_b space. Then $g\cdot f$ is super continuous if g is g*b-continuous and f is g*b-continuous.

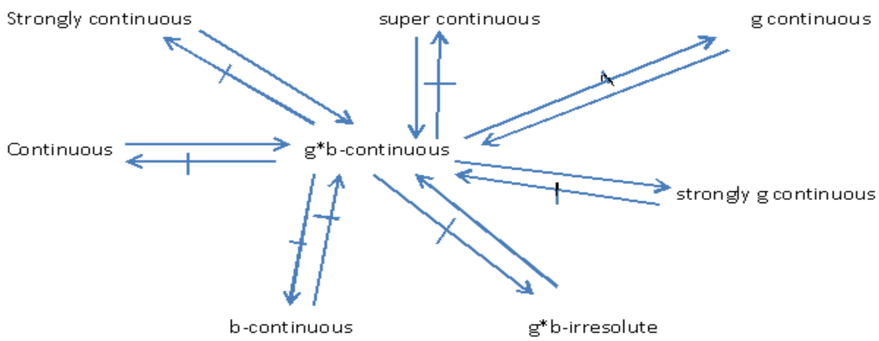
The following example will show that the composition of two g*b-continuous maps need not always be g*b-continuous.

Example: 3.20 Let X={a, b, c, d},Y={a, b, c}=Z, τ = { X,ϕ,{a},{b},{a, b},{a, b, c},{a, b, d}}, σ = { Y,ϕ, {a},{a, b}} and η = { Z, ϕ, {b}, {a, b}}. Define $f:(X,\tau)\to(Y,\sigma)$ by f (a) = b = f(c) , f(b) = b, f(d) = c and $h:(Y,\sigma)\to(Z,\eta)$ by h(a) = a , h(b) = c, h(c) = a. Then f and h are g*b closed maps. However $h\circ f$ is not g*b-closed since for the closed sets {d}, ($h\circ f$) (d) = h (f(d)) = h(c) = {a} is not g*b-closed in Z.

Proposition: 3.21 Let $f:(X,\tau)\to(Y,\sigma)$ be b-continuous and closed. Then for every g*b-closed subset A of (X,τ), f (A) is g*b -closed in (Y,σ).

Proof: Let A ⊂ G, where G is b-open in (Y,σ). Then A ⊂ $f^{-1}(G)$ and $f^{-1}(G)$ is b-open in (X,τ).Since A is g*b-closed, then cl(A) ⊂ $f^{-1}(G)$.That is f(cl(A)) ⊂ G. Since f (cl(A)) is closed, then it is g*b-closed. Hence cl (f(cl(A))) ⊂ G. This implies that cl(f(A)) ⊂ cl (f(cl(A))) = f(cl(A)) ⊂ G. This shows that f (A) is g*b-closed in (Y,σ).

From the Propositions, remarks and the examples above, We have the following implications.



REFERENCES:

[1] Abd El-Monsef, M.E., El-Deeb, S.N. and Mahmoud, R.A., β-open sets and β- Continuous mappings, Bull-Fac. Sci. Assiut. Univ., 12 (1983), 77–90.

[2] Andrijevic,D., Semi-preopen sets, Mat. Vesnik., 38 (1986), pp. 24–32.

[3] Andrijevic,D., On *b*-open sets, Mat. Vesnik., 48 (1996), 59–64.

[4] J. Dontchev and M. Przemski, On the various decompositions of continuous and some weakly continuous functions, Acta Math. Hungary, 71 1–2 (1996), 109–120.

[5] El-Atik AA., A study on some types of mappings on topological spaces, M.Sc. Thesis, Egypt: Tanta University, 1997.

[6] Levine, N., A decomposition of continuity in topological spaces, Am. Math. Mon., 68, (1961), 44-46.

[7] Levine, N., Semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly, 70 (1963), 36–41.

[8] Mashhour,A.S., Abd El-Monsef, M.E.and El-Deeb, S.N. On precontinuous and weak precontinuous functions, Proc Math Phys Soc Egypt 53 (1982), 47–53.

[9] Nasef, A.A., On *b*-locally closed sets and related topics, Chaos Solitons Fractals, 12, No.10, (2001), 1909-1915.

[10] Noiri, T. and Popa, V., On almost β-continuous functions, Acta Math. Hung. 79, No.4, (1998).329-339.

[11] Noiri, Takashi, Almost quasi-continuous functions, Bull. Inst. Math., Acad. Sin. 18, No.4, (1990) 321-332.

1480 *U. S¸eng¨ul* [12] Noiri, T., Weakly α-continuous functions, Int. J. Math. Math. Sci.
10, (1987) 483-490.

[13] Singal, M.K. and Singal, A. R., Almost-continuous mappings, Yokohama Math. J. 16,(1968) 63-73.

[14] S, eng¨ul, U., Weakly *b*-continuous functions, Chaos, Solitons & Fractals (2008), doi: 10.1016/j. chaos. 2008. 04. 036.

[15] Park, J. H., Strongly θ-*b*-continuous functions, Acta Math. Hungar. 110 (4), (2006), 347-359.

[16] Popa, V. and Noiri,T., Weakly β-continuous functions., An. Univ. Timis., Ser. Mat.-Inform., 32, No.2, (1994), 83-92.

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