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TAYLOR SERIES APPROACH FOR SOLVING CHANCE-CONSTRAIED MULTIOBJECTIVE INTEGER LINEAR FRACTIONAL PROGRAMMING PROBLEM

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ABSTRACT

This paper presents the use of a Taylor series for chance-constrained multiobjective integer linear fractional programming problem (CHMOILFP). The Taylor series is a series expansion that a representation of a function. The basic idea in the proposed approach in treating the (CHMOILFP) is to convert the probabilistic nature of this problem into a deterministic version. and thus the problem is reduced to a single objective. Numerical example is provided in order to show the efficiency and the superiority of the proposed approach.

***Keywords:** Multiobjective optimization; Integer programming; Stochastic programming;Fractional programming; Taylor series.*

***MSC No (2001):** 90C29; 90C10; 90C15; 90C32; 90C99.*

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1. INTRODUCTION:

Decision problems of chance- constrained or stochastic optimization arise when certain coefficients of an optimization model are not fixed or known but instead, to some extent, probabilistic quantities. In most of the real life problems in mathematical programming, the parameters are considered as random variables.

The branch of mathematical programming which deals with the theory and methods for the solution of conditional extreme problems under incomplete information about the random parameters is called stochastic programming [12].

In recent years methods of stochastic optimization have become increasingly important in scientifically based decision-making involved in practical problems arising in practical problems arising in transportation, scheduling, agriculture, military purposes and technology.

Fractional programming (FP), which has been used as an important planning tool for the last four decades, is applied to different disciplines such as engineering, business, economics. Fractional programming is generally used for modeling real life problems with fractional objective such as profit / cost, inventory / sales, actual cost / standard cost and output / employee etc [5, 8 and 13].

Integer linear fractional programming problem with multiple objectives (MOILFP) is an important field of research and has not received as much attention as did multiple objective linear fractional programming since some computational difficulties are posed in solving such problems. For comprehensive review of the work in this direction of studies, we refer to [1, 3, 7, 9 and10].

In [4], Farag presented a solution algorithm to multiobjective integer linear fractional programming problem (MOILFP) using the 1^{st} order Taylor polynomial series [14, 15] which obtains polynomial objective functions equivalent to the fractional objective functions. Then, by the use of the nonnegative weighted sum method explained in [2], the (MOILFP) can be reduced to a single-objective problem and thus an optimal integer solution can be found via the branch and bound method [6].

Saad and Emam in a recent work [11] proposed a solution approach to bi-level integer linear fractional programming problem with individual chance constraints (CHBLIFP). It has been assumed that the randomness was in the right-hand side of the constraints only and that the random variables were normally distributed.

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2. PROBLEM FORMULATION AND THE SOLUTION CONCEPT:

The purpose of this paper is to develop a method based on Taylor series for solving the following chance-constrained multiobjective integer linear fractional programming problem (CHMOILFP):

$$(\text{CHMOILFP}): \begin{cases} \max z_1(x) = \dfrac{c^1 x + \alpha^1}{d^1 x + \beta^1} \\ \max z_2(x) = \dfrac{c^2 x + \alpha^2}{d^2 x + \beta^2} \\ \quad \vdots \\ \max z_k = \dfrac{c^k x + \alpha^k}{d^k x + \beta^k} \end{cases}$$

Subject to

$$x \in X,$$

where, in the above problem, $k \geq 2, c^r, d^r$ are $1 \times n$ vectors; α^r, β^r are scalars for each $r \in \{1,2,...,k\}$.In addition,

$$X = \left\{ x \in R^n \,\middle|\, P\{\ g_i(x) = \sum_{j=1}^{n} a_{ij} x_j \leq b_i\ \} \geq \alpha_i, i = 1,2,....,m, \right.$$
$$\left. x_j \geq 0 \text{ and integer, j} = 1,2,..\text{n} \right\},$$

Here *X* is the vector of the integer decision variables and *Z(x)* is a vector of *k*-linear real-valued objective functions to be maximized. Furthermore, *P* means probability and α_i is a specified probability value. This means that the linear constraints may be violated some of the time and at most 100(1-α_i) % of the time. For the sake of simplicity, we assume that the random parameters b_i, *(i =1, 2… m)* are distributed normally with known means $E\{b_i\}$ and variances $Var\ \{b_i\}$ and independently of each other.

Definition: 1 A point $x^* \in X$ is said to be an efficient solution to problem (CHMOILFP) with probability $\prod_{i=1}^{m} \alpha_i$ if there does not exist another $x \in X$ such that $Z(x) \succ Z(x^*)$ and $Z(x) \neq Z(x^*)$.

The basic idea in treating problem (CHMOILFP) is to convert the probabilistic nature of this problem into a deterministic version. Here, the idea of employing deterministic version will be illustrated by using the interesting technique of chance-constrained programming [11, 12]. In this case, the set of constraints *X* can be rewritten in the deterministic form as:

$$X' = \left\{ x \in R^n \,\middle|\, \sum_{j=1}^{n} a_{ij} x_j \leq E\{b_i\} + K_{\alpha_i} \sqrt{Var\{b_i\}},\ i = 1, 2, ..., m \right.$$
$$\left. x_j \geq 0 \text{ and integer, j} = 1,2,..\text{n} \right\},$$

where K_{α_i} is the standard normal value such that $\Phi(K_{\alpha_i}) = 1 - \alpha_i$; and $\Phi(\text{a})$ represents the "cumulative distribution function" of the standard normal distribution evaluated at a.

3. DETERMINISTIC MULTIOBJECTIVE INTEGER LINEAR FRACTIONAL PROGRAMMING PROBLEM:

Now before we go any further, problem (CHMOILFP) can be understood as the following deterministic multiobjective integer linear fractional programming problem (MOILFP):

$$(\text{MOILFP}): \begin{cases} \max \quad z_1(x) = \dfrac{c^1 x + \alpha^1}{d^1 x + \beta^1} \\ \max \quad z_2(x) = \dfrac{c^2 x + \alpha^2}{d^2 x + \beta^2} \\ \quad\vdots \\ \max \quad z_k = \dfrac{c^k x + \alpha^k}{d^k x + \beta^k} \end{cases}$$

Subject to

$$x \in X' = \left\{ x \in R^n \,\Big|\, \sum_{j=1}^{n} a_{ij} x_j \le E\{b_i\} + K_{\alpha_i} \sqrt{Var\{b_i\}},\ i = 1, 2, ..., m, \right.$$
$$\left. x_j \ge 0 \text{ and integer},\ \ \text{j} = 1,2,..\text{n} \right\}$$

4. TAYLOR SERIES APPROACH FOR THE DETERMINISTIC MULTIOBJECTIVE INTEGER LINEAR FRACTIONAL PROGRAMMING PROBLEM (MOILFPP):

In the deterministic multiobjective integer linear fractional programming problem (MOILFP), objective functions are transformed by using Taylor series at first, and then a satisfactory value for the variables of the model is obtained by solving the model, which has a single objective function.

Here, Taylor series obtains polynomial objective functions which are equivalent to fractional objective functions. Then, the (MOILFP) can be reduced into a single objective combined with the use of the branch and bound method [6]. In the compromised objective function, the weight of the first objective is more than the weight of the second objective and so on. The proposed approach to solve multiobjective integer linear fractional programming problem (MOILFP) can be explained as follows:

Step: 1 Determine $x_i^* = (x_{i1}^*, ..., x_{in}^*)$ which is the value that is used to maximized the r[th] objective function $z_r(x), (r = 1,2,...,k)$ where *n* is number of the variables.

Step: 2 Transform the objective functions $z_r(x), (r = 1,2...,k)$ by using the following 1[st] order Taylor series polynomial series in the following form stated in [14, 15] as:

$$z_r(x) \cong \overline{z_r}(x) = z_r(x_i^*) + \sum_{j=1}^{n} (x_j - x_{ij}^*) \frac{\partial z_r(x_i^*)}{\partial x_j}, (j = 1,2,...,n)$$

Step: 3 Find the satisfactory solution by solving the reduced problem to a single objective function. In the compromised objective function, the weight of the first objective is more than the weight of the second objective and so on.

5. AN ILLUSTRATIVE EXAMPLE:

In this section, an illustrative example is given to clarify the proposed solution algorithm. This example is adapted from one appearing in Chergui and Moulaü [3] and the LINGO software package is used in the computational process.

The problem to be solved here is the following chance –constrained multiobjective integer linear fractional programming problem:

(CHMOILFP):

$$\max\ z_1(x_1,x_2) = \left(\frac{2x_1+3x_2}{x_1+4x_2+6}\right),$$
$$\max\ z_2(x_1,x_2) = \left(\frac{3x_1+4x_2}{6x_1+4x_2+3}\right),$$
$$\max\ z_3(x_1,x_2) = (x_1-x_2),$$

Subject to

$$P\{2x_1 - x_2 \le b_1\} \ge 0.95,$$
$$P\{-x_1+3x_2 \le b_2\} \ge 0.90,$$
$$x_1, x_2 \ge 0 \text{ and integers.}$$

where *bi, (i =1, 2)* are independent, normally distributed random parameters with the following means and variances.:

$E\{b_1\} = 1$, $E\{b_2\} = 9$, $Var\{b_1\} = 25$, $Var\{b_2\} = 4$.

and from the standard normal tables, we have: $K_{\alpha 1} = 1.645,\ K_{\alpha 2} = 1.285$.

Problem (CHMOILFP) can be understood as the following deterministic multiobjective integer linear fractional programming problem (MOILFP):

(MOILFP):

$$\max\ z_1(x_1,x_2) = \left(\frac{2x_1+3x_2}{x_1+4x_2+6}\right),$$
$$\max\ z_2(x_1,x_2) = \left(\frac{3x_1+4x_2}{6x_1+4x_2+3}\right),$$
$$\max\ z_3(x_1,x_2) = (x_1-x_2),$$

Subject to

$$2x_1 - x_2 \le 9.225,$$
$$-x_1+3x_2 \le 11.57,$$
$$x_1, x_2 \ge 0 \text{ and integers.}$$

With the help of the branch-and-bound method [6], an equivalent multiobjective linear fractional programming problem (MOLFP) corresponding to the deterministic problem (MOILFP) can be formulated as follows:

(MOLFP):

$$\max\ z_1(x_1,x_2) = \left(\frac{2x_1+3x_2}{x_1+4x_2+6}\right),$$
$$\max\ z_2(x_1,x_2) = \left(\frac{3x_1+4x_2}{6x_1+4x_2+3}\right),$$
$$\max\ z_3(x_1,x_2) = (x_1-x_2),$$

Subject to

$$2x_1 - x_2 \le 9.225,$$
$$-x_1+3x_2 \le 11.57,$$
$$x_1 \le 4,$$
$$x_2 \le 3$$
$$x_1, x_2 \ge 0.$$

If the problem (MOLFP) is solved for each objective function one by one, then $z_1(4,0) = 0.8$, $z_2(0,3) = 0.8$, $z_3(4,0) = 4$. Therefore, the objective functions are transformed by using the 1st order Taylor polynomial series to

$$z_1(x_1,x_2) \cong \bar{z}_1(x_1,x_2) = 0.12x_1 - 0.02x_2 + 0.32,$$

$$z_2(x_1, x_2) \cong \bar{z}_2(x_1, x_2) = -0.12x_1 + 0.053x_2 + 0.64,$$
$$z_3(x_1, x_2) \cong \bar{z}_3(x_1, x_2) = x_1 - x_2.$$

Now, the resulting form of the problem (MOLFP) above is written as:

$$\max\ [0.5\bar{z}_1(x_1, x_2) + 0.3\bar{z}_2(x_1, x_2) + 0.2\bar{z}_3(x_1, x_2)]$$
$$= 0.224x_1 - 0.19x_2 + 0.352$$

Subject to

$$2x_1 - x_2 \le 9.225,$$
$$-x_1 + 3x_2 \le 11.57,$$
$$x_1 \le 4,$$
$$x_2 \le 3$$
$$x_1, x_2 \ge 0.$$

where $w_1 = 0.5, w_2 = 0.3, w_3 = 0.2$ and $w_1 + w_2 + w_3 = 1$

Therefore, the compromise solution can be obtained as: $(x_1^*, x_2^*)_1 = (4,0)$ with the optimum objective value functions:

$$z_1^*(x_1^*, x_2^*) = 0.8,\ z_2^*(x_1^*, x_2^*) = 0.44444,\ z_3^*(x_1^*, x_2^*) = 4$$

6. CONCLUSIONS:

In this paper, a powerful approach based on Taylor series to solve chance-constrained multiobjective integer linear fractional programming problem (CHMOILFP) has been suggested. It has been assumed that there was randomness in the right-hand side of the constraints only and that the random variables were normally distributed.
Certainly, there are many other points for future research in the area of stochastic multiobjective integer linear fractional programming problems and should be studied and explored. One may have to tackle the following open points for future research.

(i) Taylor series approach for solving chance-constrained bi-level multiobjective integer linear fractional programming problem.

(ii) Taylor series approach for solving chance constrained multi-level multiobjective integer linear fractional programming problem.

(iii) Taylor series approach for solving chance constrained bi-level multiobjective mixed-integer and zero-one nonlinear fractional programming problem.

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