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HYPERSURFACE-HOMOGENEOUS INFLATIONARY COSMOLOGICAL MODELS IN GENERAL RELATIVITY

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ABSTRACT

We study the inflationary hypersurface-homogeneous cosmological models with massless scalar field with a flat potential. The characteristic feature of inflationary era is found by considering a flat region of a constant potential V. We have shown that exact solutions of Einstein's field equations are solvable for any arbitrary cosmic scale function. The physical and kinematical behavior of the models are also discussed and studied.

Keywords: *Inflationary Universe, Cosmology, Scalar fields, Space-time, flat potential.*

1. INTRODUCTION:

In cosmology, inflation is based on the assumption that there was a period in the very early universe during which space underwent accelerated, exponential expansion. Inflation provides a plausible explanation for several puzzles of standard big-bang cosmology. In trying to understand the universe, the standard big-bang model faces two well-known problems – the horizon problem and the flatness problem. To solve these, the big-bang theory is modified by the inflation theory, which state that the universe is expanded rapidly after it was created. The horizon problem is that the cosmic microwave background radiation temperatures throughout the universe are almost exactly the same temperature in every direction. In flatness problem, the universe appears to have a flat geometry. Guth[1] proposed the inflation theory to solve these major problems. Several versions of the inflationary models are studied by Linde[2], La and Steinhardt[3], Abbott and Wise[4]. In these models, flatness problem is well understood and solved, but this is not so clear about isotropy and homogeneity to solve such problem.

In inflationary universe scenarios, it is assumed that general relativity is the correct theory of gravitation, and the matter is generally taken to be a homogeneous scalar field φ, with a potential $V(\varphi)$ acting as the vacuum energy to drive the accelerated expansion. In general relativity, for example, if the potential is simply a constant, $V(\varphi)=V_0$, then the space time is deSitter and the expansion is exponential. If the potential is exponential, $V(\varphi)=V_0 e^{-\lambda\varphi}$. Then there is a power-law inflationary solution. In general relativity, scalar fields help in explaining the creation of matter in cosmological theories, and can also describe the uncharged field. Wald[5], Burd and Barrow[6], Barrow[7], Stein-Schabes[8], Ellis and Madsen[9], Heusler[10], Bhattacharjee and Baruah[11], Bali and Jain[12] and Rahaman et al.[13] have studied different aspect of scalar fields in inflationary cosmology. Recently, Reddy et al.[14] Reddy and Naidu[15], Reddy et al.[16] Katore and Rane[17] have discussed the inflationary universe models in different space-time in general relativity. Therefore we propose to study of such inflation theory in general relativity.

In this paper, we study the inflationary-homogeneous cosmological models in the presence of massless scalar field with a flat potential in general relativity. To get an inflationary solution, a flat region is assumed in which the potential V is considered to be constant. We have shown that Einstein field equations are solvable for any arbitrary cosmic scale function. The physical and kinematical properties of the investigated models are studied.

2. THE METRIC AND FIELD EQUATIONS:

The general form of a hypersurface-homogeneous space-time can be described by the metric

$$ds^2 = dt^2 - A^2dx^2 - B^2[dy^2 + f_K^2(y)dz^2], \tag{2.1}$$

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where A and B are functions of time t. The function f_K depends on the geometry of 3-D hypersurfaces. The function $f_K^2(y)$ is associated with the group acting on hypersurface-homogeneous space-time and $f_K^2(y)$ = $(\sin y, y, \sinh y)$ for $K = (1,0,-1)$ with *K* is the curvature.

In the case of gravity minimally coupled to a scalar field $V(\varphi)$, the Langrangian *L* is

$$L = \int \sqrt{-g}[R - \frac{1}{2} g^{ij} \varphi_{,i} \varphi_{,j} - V(\varphi)] d^4 x ,$$

where g is the determinant of the space-time metric, and $V(\varphi)$ is the effective potential that describes the self-interaction of the scalar field. The variation of *L* with respect to dynamical fields lead to Einstein field equations

$$R_{ij} - \frac{1}{2} R g_{ij} = -T_{ij} \tag{2.2}$$

with stress-energy tensor associated with *L* has the form

$$T_{ij} = \varphi_{,i} \varphi_{,j} - [\frac{1}{2} \varphi_{,k} \varphi^{,k} + V(\varphi)] g_{ij} \tag{2.3}$$

and
$$\frac{1}{\sqrt{-g}} \partial_{,i} (\sqrt{-g} \partial^{,i} \varphi) = -\frac{dV(\varphi)}{d\varphi} , \tag{2.4}$$

where comma (,) indicates the ordinary differentiation. The function φ depends on t only due to homogeneity. Units are taken such that $8\pi G = c = 1$.

The Einstein field equations (2.2) with the help of (2.3) for the metric (2.1) lead to the following set of equations

$$2\frac{\ddot{B}}{B} + \frac{K + \dot{B}^2}{B^2} = -[\frac{1}{2}\dot{\varphi}^2 + V(\varphi)], \tag{2.5}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -[\frac{1}{2}\dot{\varphi}^2 + V(\varphi)], \tag{2.6}$$

$$2\frac{\dot{A}\dot{B}}{AB} + \frac{K + \dot{B}^2}{B^2} = \frac{1}{2}\dot{\varphi}^2 - V(\varphi) \tag{2.7}$$

and the equation (2.4) for scalar field is

$$\ddot{\varphi} + \dot{\varphi}(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}) = -\frac{dV(\varphi)}{d\varphi} , \tag{2.8}$$

where the dot (.) denotes the differentiation with respect to t.

The average scale factor for the metric (2.1) is defined by

$$a(t) = (AB^2)^{1/3} . \tag{2.9}$$

A volume scale factor is given by

$$V = a^3(t) = AB^2 . \tag{2.10}$$

The expansion scalar θ and shear scalar σ are defined as

$$\theta = (\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}) \tag{2.11}$$

and
$$\sigma = \frac{1}{\sqrt{3}}(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}) . \tag{2.12}$$

The deceleration parameter q is defined as

$$q = -\frac{\ddot{V}V}{\dot{V}^2} \quad . \tag{2.13}$$

3. INFLATIONARY MODELS:

Katore and Rane[17] have obtained the inflationary Kantowski-Sachs cosmological model in the presence of a massless scalar field with a flat potential. They have shown that the field equations are solvable for any arbitrary cosmic scale function. We follow the same approach to find exact solutions of the field equations (2.5) to (2.8).

The flat part of the potential is associated with a vacuum energy with an effective cosmological constant. Therefore we assume the flat region when the potential is constant, i.e.

$$V(\varphi) = \text{constant} = V_0 \text{ (say).} \tag{3.1}$$

The equation (2.8) on integration gives

$$\dot{\varphi} = \frac{d}{AB^2} \ , \tag{3.2}$$

where d is a constant of integration.

The field equations (2.5) and (2.6) in view of (3.1) lead to

$$\frac{\ddot{B}}{B} - \frac{\ddot{A}}{A} + \frac{\dot{B}^2}{B^2} - \frac{\dot{A}\dot{B}}{AB} + \frac{K}{B^2} = 0$$

i.e. $$d(-B^2\dot{A} + AB\dot{B}) = -KA$$

which on integration gives

$$-B^2\dot{A} + AB\dot{B} = -K\int A dt + d_1 \ , \tag{3.3}$$

where d_1 is the constant of integration.

Considering equation (3.3) as a linear differential equation for $B(t)$ where $A(t)$ is an arbitrary function. Therefore equation (3.3) becomes

$$\frac{2}{A}[-B^2\dot{A} + AB\dot{B}] = \frac{2}{A}[-K\int A dt + d_1]$$

i.e. $$\frac{d}{dt}(B^2) - 2\frac{\dot{A}}{A}B^2 = F(t) \ , \tag{3.4}$$

where $$F(t) = \frac{2}{A}\left[-K\int A dt + d_1\right] .$$

The linear differential equation (3.4) has the general solution given by

$$B^2 = A^2\left[\int \frac{F(t)}{A^2} dt + d_2\right] , \tag{3.5}$$

where d_2 is the constant of integration.

We now obtain the solution for a simple choice of the function $A(t)$. We choose

$$A(t) = t^n \ , \tag{3.6}$$

where n is real number $(n \neq -1)$.

Equation (3.5) yields

$$B^2 = \frac{K}{n^2-1}t^2 + \frac{2d_1}{1-3n}t^{1-n} + d_2 t^{2n}. \tag{3.7}$$

Without loss of generality, we take $d_1 = d_2 = 0$, the solution (3.7) becomes

$$B^2 = \frac{K}{n^2-1}t^2. \tag{3.8}$$

Hence the geometry of the inflationary hypersurface-homogeneous metric corresponding to the solutions (3.6) and (3.8) takes the form

$$ds^2 = dt^2 - t^{2n}dx^2 - \frac{K}{n^2-1}t^2[dy^2 + f_K^2(y)dz^2]. \tag{3.9}$$

This model is well-defined when $n^2 - 1 > 0$.

The scalar field φ equation (3.2) with the solutions (3.6) and (3.8) on integration gives

$$\varphi = d(1-n)t^{-(1+n)} + \varphi_0 , \tag{3.10}$$

where φ_0 is the constant of integration.

Model I : For $K = 1$, the metric (3.9) takes the form

$$ds^2 = dt^2 - t^{2n}dx^2 - \frac{t^2}{n^2-1}[dy^2 + \sin^2 ydz^2] . \tag{3.11}$$

This model is well-defined when $n^2 - 1 > 0$.

The physical and kinematical parameters of the model (3.11) are given by the following expressions:

Spatial Volume $V^3 = \frac{t^{n+2}}{n^2-1}$. (3.12)

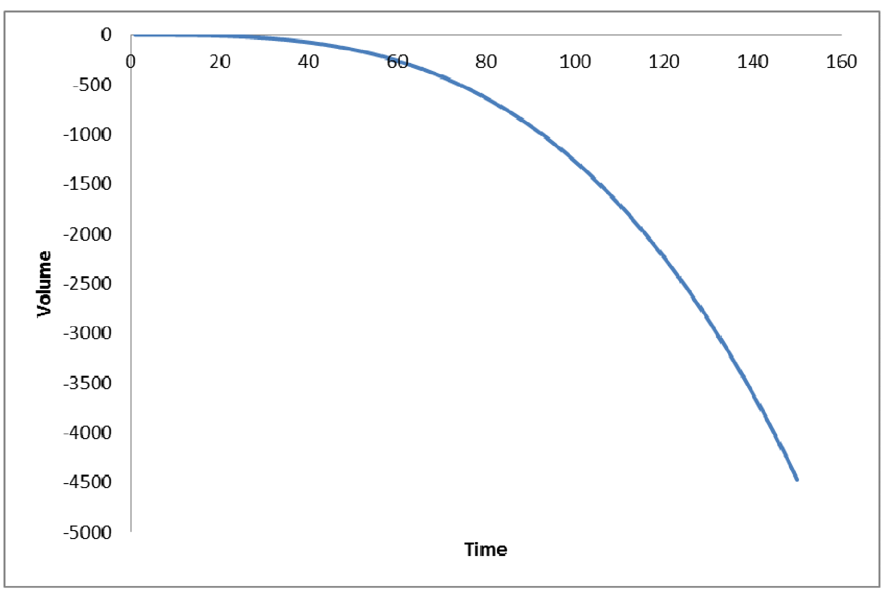

Figure: 1 Volume Vs Time.

The average scale factor $a(t) = \left(\dfrac{t^{n+2}}{n^2 - 1} \right)^{1/3}$. (3.13)

Expansion scalar $\theta = \dfrac{n+2}{t}$. (3.14)

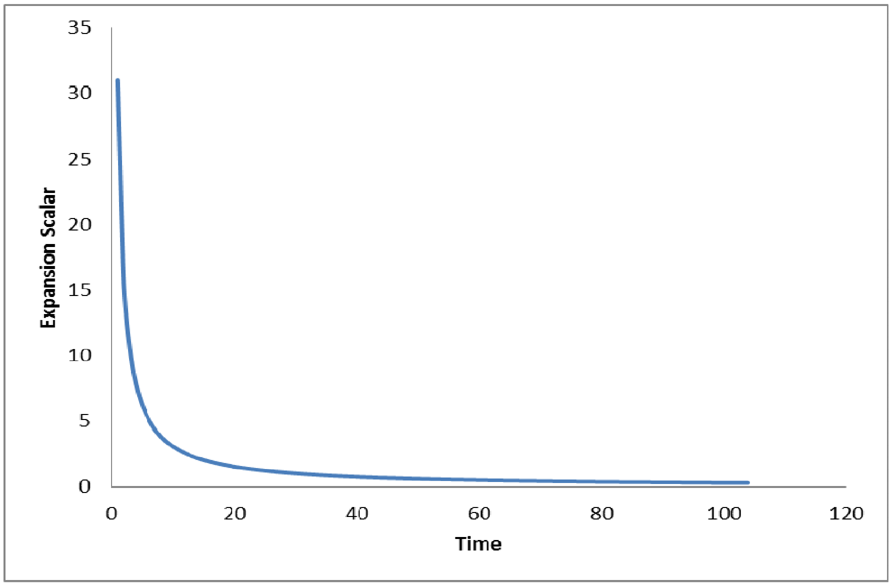

Figure: 2. Expansion Scalar Vs Time

Shear Scalar $\sigma = \dfrac{1}{\sqrt{3}} \left(\dfrac{n-1}{t} \right)$. (3.15)

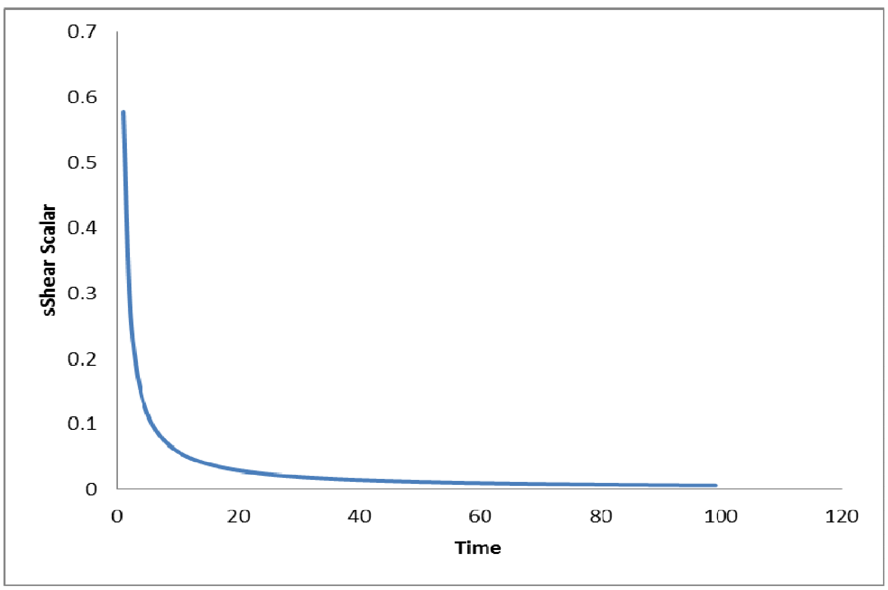

Figure: 3 Shear Scalar Vs Time.

The deceleration parameter q is given by

$$q = \frac{1-n}{2+n}\ , \quad \text{if } n \neq -2. \tag{3.16}$$

For model (3.11), we observed that the spatial volume increases with increase of time t when $(n+2) > 0$ and it becomes infinite for large value of t. The expansion scalar and shear scalar become infinite at $t = 0$ but they vanish for large *t*. Scalar field φ diverges for $t=0$ and it becomes zero for large *t*. The deceleration parameter *q* is positive for $-2<n<-1$. In this case the model (3.11) represents decelerating universe. When $n>1$, the deceleration parameter

q is negative and thus the model (3.11) corresponds to inflationary accelerating model of the universe. Universe must undergo a phase of acceleration i.e. $\ddot{a}>0$. Hence the scalar factor of the universe is accelerating. The model (3.11) starts with big-bang at $t=0$.

Model II: For $K=-1$, the metric (3.9) takes the form

$$ds^2 = dt^2 - t^{2n}dx^2 - \frac{t^2}{1-n^2}[dy^2 + \sinh^2 ydz^2] \; . \tag{3.17}$$

This model is well-defined when $1-n^2>0$.

For model (3.17), the expansion scalar θ, shear scalar σ and the deceleration parameter *q* have the same expressions given by an equations (3.14), (3.15) and (3.16) respectively. The spatial volume and the average scale factor are given by

$$\text{Spatial Volume } V^3 = \frac{t^{n+2}}{1-n^2} \; . \tag{3.18}$$

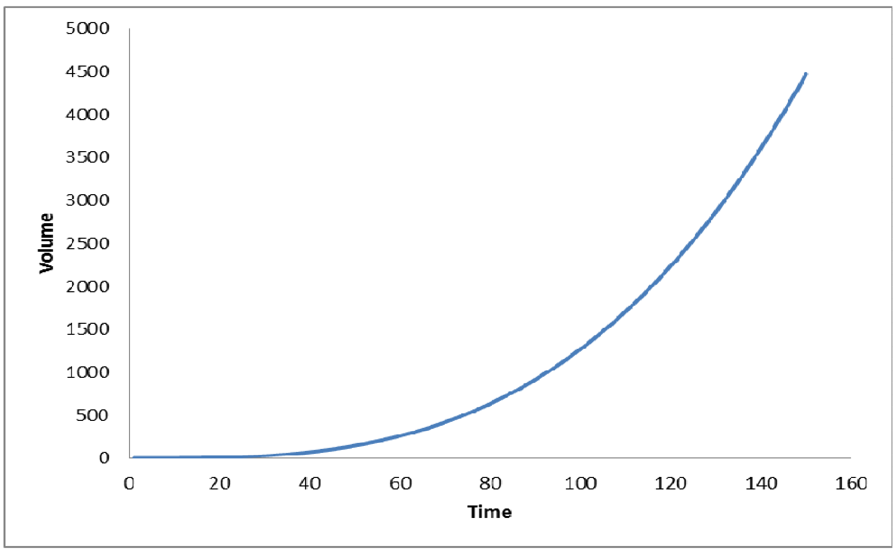

Figure: 4 Volume Vs Time.

$$\text{The average scale factor } a(t) = \left(\frac{t^{n+2}}{1-n^2}\right)^{1/3} \; . \tag{3.19}$$

Where $(n+2)>0$. The physical and kinematical behaviors of the model (3.17) are same as that of the model (3.11).

For $K=0$, the Einstein field equations are not solvable for any arbitrary cosmic scale function. Therefore we can not derive exact solutions of the field equations (2.5) to (2.8) by the method used in this paper.

4. CONCLUSION:

The models (3.11) and (3.17) represent an inflationary hypersurface-homogeneous cosmological models in general relativity when the scalar field is minimally coupled to the gravitational field in which the flat region of potential is constant which is generally associated with vacuum energy. These models have no singularity at $t=0$. The characteristic feature of an inflationary era is that space-time expands exponentially. It is also observed that the negative value of the deceleration parameter is associated with the inflationary accelerating model of the universe. The study of inflationary hypersurface-homogeneous universal model will be astrophysical significance to new researchers in view of the scalar fields in general relativity.

REFERENCES:

[1] Guth, A. H., *Phys. Rev* D23 (1981), 347.

[2] Linde, A. D., *Phys. Lett*. B 108 (1982), 389.

[3] La, D. and Ssteihardt, P. J. , *Phys. Rev. Lett*., 62 (1989), 2376.

[4] Abbott, L. F. and Wise, M. B., *Nucl. Phys*. B 244 (1984), 541.

[5] Wald, R., *Phys. Rev*. D28 (1983) 2118.

[6] Burd, A. B. and Barrow, J. D., *Nucl. Phys*. B 308, (1988), 923.

[7] Barrow, J. D*., Phys. Lett*. B 187, (1987) 12.

[8] Stein-Schabes, J. A., *Phys. Rev*. D35 (1987), 2345.

[9] Ellis, G. F. R. and Madsen, M. S., *Class. Quant. Gravity*, 8 (1991) 667.

[10] Heusler, M., *Phys. Lett*. B 253, (1991) 33.

[11] Bhattacharjee, R. and Baruah, K. K., *Indian J. Pure Appl. Math*., 32 (2001), 47.

[12] Bali, R. and Jain, V. C., *Pramana J. Phys*. 59 (2002) 1.

[13] Rahaman, F., Bag, G., Bhui, B. C. and Das, S., *Fizika B* 12 (2003) 193.

[14] Reddy, D. R. K., Naidu, R. L. and Rao, A. S., *Int. J. Theo. Phys*., 47 (2008), 1016.

[15] Reddy, D. R. K.and Naidu, R. L., Int. *J. Theo. Phys*., 47 (2008) 2339.

[16] Reddy, D. R. K., Adhav, K. S., Katore, S. D. and Wankhade, K. S. *Int. J. Theo. Phys*., 48 (2009),2884.

[17] Katore, S. D. and Rane, R. S., *Astrophys. Space Sci*., 323 (2009) 293.

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