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MODIFIED NEXT TO NEXT MINIMUM PENALTY METHOD FOR FUZZY TRANSPORTATION PROBLEMS

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ABSTRACT

New algorithm named, Modified Next to Next Minimum Penalty Method (MNNMPM) is proposed for solving fuzzy Transportation problem. This algorithm is more efficient than other existing algorithms. The procedure for the solution is illustrated with a numerical example.

Keywords: *Fuzzy transportation Problem, Initial solution, Total opportunity cost, Modified Next to Next Minimum Penalty Method.*

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1. INTRODUCTION:

The transportation problem is one of the oldest applications of linear programming problems. The basic transportation problem was originally developed by Hitchcock [3]. Efficient methods of solution derived from the simplex algorithm were developed in 1947, primarily by Dantzig [4] and then by Charnes and Cooper [1]. The transportation problem can be modeled as a standard linear programming problem, which can be solved by the simplex method. We can get an initial basic feasible solution for the transportation problem by using the North-West corner rule, Row Minima, Column Minima, Matrix minima or the Vogel's Approximation Method. To get an optimal solution for the transportation problem, we use the MODI method (Modified Distribution Method). Charnes and Cooper [1] developed the Stepping Stone Method, which provides an alternative way of determining the optimal solution.

The LINDO (Linear Interactive and Discrete Optimization) package handles the transportation problem in explicit equation from and thus solves the problem as a standard linear programming problem. Consider m origins (or sources) O_i (i=1,…,m) and n destinations D_j (j=1,…,n). At each origin O_i , let a_i be the amount of a homogeneous product that we want to transport to n destinations D_j , in order to satisfy the demand for b_j units of the product there. A penalty c_{ij} is associated with transport in a unit of the product from source I to destination j. The penalty could represent transportation cost, delivery time, quantity of goods delivered, under-used capacity, etc. A variable x_{ij} represent the unknown quantity to be transported from origin O_i to destination D_j. The transportation problem can be represented as a single objective transportation problem or as a multi-objective transportation problem.

Fuzzy transportation problem (FTP)[5] is the problem of minimizing fuzzy valued objective functions with fuzzy source and fuzzy destination parameters. The balanced condition is both a necessary and sufficient condition for the existence of a feasible solution to the transportation problem. Shan Chen [12] introduced the concept of function principle, that is used to calculate the fuzzy transportation cost. The Graded Mean integration Representation Method, Used to defuzzify the fuzzy transportation cost, was also introduced by Shan Chen [11], and [14] was introduced the Next to Next Minimum Method, [15] was introduced the Modified Vogel's Approximation Method (MVAM).

In this paper, we propose an algorithm namely Modified Next to next Minimum Penalty Method is proposed for solving fuzzy transportation problems, which is more efficient than other existing algorithm. The procedure for the solution is illustrated with a numerical example.

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2. PRELIMINARIES:

Consider m origins (or sources) O_i (i=1,…,m) and n destinations D_j (j=1,…,n) . At each origin O_i , let a_i be the amount of a homogeneous product that we want to transport to n destinations D_j , in order to satisfy the demand for b_j units of the product there. A penalty c_{ij} is associated with transport in a unit of the product from source I to destination j. The penalty could represent transportation cost, delivery time, quantity of goods delivered, under-used capacity, etc. A variable x_{ij} represent the unknown quantity to be transported from origin O_i to destination D_j. However, in the real world, all transportation problems are not single objective linear programming problems.

The mathematical form of the above said problem is as follows:

Minimize $Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}\, x_{ij}$

Subject to $\sum_{j=1}^{n} x_{ij} = b_i,\ i = 1,2,...,m$

$$\sum_{i=1}^{m} x_{ij} = a_j,\ j = 1,2,...,n$$

$$x_{ij} \ge 0,\quad i = 1,2,...,m,\ j = 1,2,...,n$$

and $\sum_{i=1}^{m} a_j = \sum_{j=1}^{n} b_i$, (balanced condition). The balanced condition is both a necessary and sufficient condition for the existence of a feasible solution to the transportation problems.

We need the following definition, which can be found in [2, 5].

2.1 Definition:

A fuzzy number ã in a triangular fuzzy number denoted by [a_1, a_2, a_3] where a_1, a_2, a_3 are real numbers and its membership function $\mu_{\tilde{a}}(x)$ is given below

$$\mu_{\tilde{a}}(x) = \begin{cases} (x-a_1)/(a_2-a_1) & for\ \ a_1 \le x \le a_2 \\ (x-a_3)/(a_2-a_3) & for\ \ a_2 \le x \le a_3 \\ 0 & otherwise \end{cases}$$

Consider the following fuzzy transportation problem (FTP),

Minimize $Z = \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}_{ij}\, \tilde{x}_{ij}$

Subject to $\sum_{j=1}^{n} \tilde{x}_{ij} \le \tilde{a}_i,\ i = 1,2,...,m$

$$\sum_{i=1}^{m} \tilde{x}_{ij} \ge \tilde{b}_j,\ j = 1,2,...,n$$

$$\tilde{x}_{ij} \ge 0,\quad i = 1,2,...,m,\ j = 1,2,...,n$$

Where $\tilde{a}_i = (a_1,\ a_2,\ a_3)$, $\tilde{b}_i = (\mathrm{b}_1,\ b_2,\ b_3)$ and $\tilde{c}_{ij} = (\mathrm{c}_{ij},\ c_{ij},\ c_{ij})$ representing the uncertain supply and demand for the transportation problems.

2.2 Definition:

Let ã= [a_1, a_2, a_3] & $\tilde{b} = [b_1, b_2, b_3]$ be two triangular fuzzy numbers then

(i) $\tilde{a} \oplus \tilde{b}$=[$a_1 + b_1, a_2 + b_2, a_3 + b_3$]

(ii) $\tilde{a}\Theta\tilde{b}$ =[$a_1 - b_3, a_2 - b_2, a_3 - b_1$]

(iii) $k\tilde{a}=[ka_1, ka_2, ka_3]$ for $k \geq 0$

(iv) $k\tilde{a}=[ka_3, ka_2, ka_1]$ for $k < 0$

(v) $\tilde{a} \otimes \tilde{b}=[c_1, c_2, c_3]$

Where c_1 = minimum $\{ a_1b_1, a_1b_3, a_3b_1, a_3b_3 \}$

$c_2 = a_2b_2$

c_3 = maximum $\{ a_1b_1, a_1b_3, a_3b_1, a_3b_3 \}$

(vi) $\frac{1}{\tilde{b}} = \left[\frac{1}{b_3}, \frac{1}{b_2}, \frac{1}{b_1} \right]$, where b_1, b_2, b_3 are all non zero real numbers and

(vii) $\frac{\tilde{a}}{\tilde{b}} = \tilde{a} \otimes \frac{1}{\tilde{b}}$, where b_1, b_2, b_3 are all non zero real numbers.

2.3 Definition:

The magnitude of the triangular fuzzy number $\tilde{\mu} = [a, b, c]$ is given by

$$Mag(\tilde{u}) = \frac{a + 10b + c}{12}.$$

3. MODIFIED NEXT TO NEXT MINIMUM PENALTY METHOD:

Step 1: Balance the given transportation problem if either (total supply > total demand) or (total supply < total demand).

Step 2: Obtain the TOC matrix. The TOC matrix is obtained by adding the "row opportunity cost matrix" (for each row, the smallest cost of that row is subtracted from each element of the same row) and the "column opportunity cost matrix" (for each column of the original transportation cost matrix the smallest cost of that column is subtracted from each element of the same column).

Step 3: The smallest entry from the first row is chosen and it is subtracted from the third smallest entry. This value is written against the row on the right. This value is calculated as the penalty for the first row. Similarly, the penalty for each row is computed. Likewise column penalties are calculated and they are written on the bottom of the TOC matrix below their corresponding columns.

Step 4: The highest penalty is selected and the row or column for which this corresponds is verified. Min(a_i, b_j) allocation is made to the cell having the lowest cost from the selected row or column.

Step 5: The satisfied row or column is eliminated fresh penalties for the remaining sub matrix are calculated as in step-3 and allocations made as mentioned in step-4. This is continued until one row or column remains to be satisfied.

Step 6: Compute total transportation cost for the feasible allocations using the original balanced-Transportation cost matrix.

4. NUMERICAL EXAMPLE:

					Supply
	(1, 5, 9)	(4, 9, 14)	(9, 13, 17)	(1, 2, 3)	(20, 50, 80)
	(9, 11, 13)	(9, 18, 27)	(18, 20, 22)	(1, 3, 5)	(25, 50, 75)
	(8, 14, 20)	(9, 13, 17)	(22, 16, 20)	(2, 3, 4)	(30, 50, 70)
Demand	(10, 30, 50)	(20, 40, 60)	(35, 55, 75)	(10, 25, 40)	

Table 1: Computational Results

Problem	MMM	VAM	MVAM	MNNMPM
01	2033	1806	1798	1645

5. CONCLUSION:

From the investigations and the results given in table 1, it is clear that Modified Next to Next Minimum penalty Method is better than any other methods for solving the fuzzy Transportation problem.

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