International Journal of Mathematical Archive-3(1), 2012, Page: 113-117
Available online through www.ijma.info ISSN 2229 – 5046

BLOOD FLOW IN ARTERIOLES: A MATHEMATICAL STUDY

**Varun Mohan*, *Dr. V. Prasad, #Janamejay Singh, #Dr. N. K. Varshney*

**Department of mathematics D. S. Degree College, Aligarh*
#Department of mathematics S.V. College, Aligarh
E-mail: Varun0503ind39@gmail.com

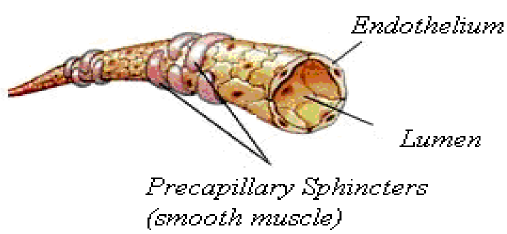
(Received on: 27-12-11; Accepted on: 14-01-12)

ABSTRACT

Blood flow is a study of measuring of the blood pressure and finding the flow through the blood vessel. In human body the arterioles have the greatest collective influence on both local blood flow and on the over all blood pressure. They are primary "adjustable nozzles" in the blood circulation system in human body, across which the greatest pressure drop occurs. In this paper a mathematical modeling of blood flow through arterioles is presented which is derived from the Navier-Stock equations and some assumptions. A system of non linear differential equations for blood flow and the cross sectional area of the arterioles was obtained. MATLAB programming techniques were adopted to solve the equations. The results obtained are very sensitive to the values of the initial conditions and this helps to explain the condition of hypertension.

Key words: *Arterioles, Hypertension, Blood Flow, Lumen*

INTRODUCTION:



Arterioles are the smallest vessels of the arterial system, with a diameter of about 1/3 millimeter or smaller. They serve as the major determinant of blood pressure and blood flow to the individual organs. Arterioles have a much smaller diameter then arteries and thus provide significant resistance to the flow of blood. This resistance creates pressure in circulatory system. Pressure is required to provide adequate flow of blood to all parts of the body. Blood flow to individual organs can be regulated by controlling the diameter of the arterioles. Vasodilatation (the term vasodilation refers to the dilation or relaxation of the arterioles to allow more blood to an area) of an arteriole lowers the resistance and results in an increase in flow through that particular arterioles. In this problem the arteriole is assumed as a tapered and blood is assumed to be a Newtonian fluid. In order to model this problem, Navier-Stock equations will be used to derive the governing equations that represent this problem.

FORMULATION OF THE GOVERNING EQUATIONS:

.
We have adopted Yang, Zhang and Asada's[3] local arterial flow model. This includes the assumptions that blood is considered as the incompressible Newtonian fluid and the flow is axially symmetric. The model approach is to use the two dimensional Navier-Stock equations and continuity equation for a Newtonian and incompressible fluid in cylindrical coordinate (r, z, t):

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial z}+\frac{\mu}{\rho}\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{\partial^2 w}{\partial z^2}\right) \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial r}+\frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{\partial^2 u}{\partial z^2}-\frac{u}{r^2}\right) \tag{2}$$

****Corresponding author:*** **Varun Mohan *,* ****E-mail:*** *Varun0503ind39@gmail.com*

$$\frac{1}{r}\frac{\partial}{\partial r}(ru)+\frac{\partial w}{\partial z}=0 \tag{3}$$

Where u and w = Radial velocity component and axial velocity component respectively
p = Pressure of the blood
ρ = The constant density
r = Radial co-ordinate
z = Axial co-ordinate
μ =Viscosity of blood.

Here, the magnitude of radial velocity (u) is very less in comparison to the magnitude of axial velocity(w) i.e. u<<w, variation of velocity gradient in z direction is less in comparison to velocity gradient in r direction. i.e.

$\frac{\partial w}{\partial z} << \frac{\partial w}{\partial r}$ and $\frac{\partial u}{\partial r}, \frac{\partial^2 u}{\partial r^2}, \frac{\partial^2 u}{\partial z^2}$ also neglected, therefore eq.1 and 2 are reduced as

$$\frac{\partial p}{\partial z}=\frac{\mu}{r}(r\frac{\partial^2 w}{\partial r^2}+\frac{\partial w}{\partial r}) \tag{4}$$

$$-\frac{1}{\rho}\frac{\partial p}{\partial r}=0 \tag{5}$$

It is seen that from eq. 5 the pressure gradient function z is a function of z(p=p(z)) only which caused the motion of flow, therefore: $\frac{\partial p}{\partial z}=\frac{dp}{dz}$

The boundary conditions are as follows

$$\frac{\partial w}{\partial r}=0 \quad r=0 \tag{6a}$$

$$w=0 \quad r=R(z) \tag{6b}$$

By applying the boundary conditions, maximum velocity at the center line 6(a) and no slip velocity at the wall (for finite velocity 6(b)), on eq. 4 the velocity profile of the blood through arteriole is shown as:

$$w=\left(\frac{dp/dz}{4\mu}\right)\left(R^2-r^2\right) \tag{7}$$

Sanjeev Kumar and Sanjeet Kumar[4] discussed the physical model of tapered artery. Here the geometry of of the arteriole (fig-1) is modeled mathematically as follows:

$$R(z)=R_1-m(z+L);\ 0\le z\le d_0 \tag{8}$$

Where
$R(z)$ = Effective radius of tapered arteriole
R_1 = The radius of untapered arteriole
m= tan ϕ = The slope of tapered arteriole
ϕ = Tapering angle

Flow rate: Flow rate for Newtonian fluid (blood) ids defined as:

$$Q=\int_0^R 2\pi w r dr \tag{9}$$

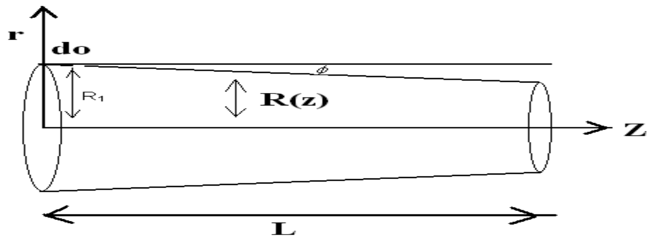


Fig-1 shows the physical model and co-ordinate.

Using eq. 7 into eq. 9, flow rate is shown as:

$$Q = \frac{\pi G R^4}{8\mu} \tag{10}$$

Where G= $\frac{dp}{dz}$. Thus, the expression of total volumetric flow flux for tapered arteriole is shown as:

$$Q_L = \frac{\pi G}{8\mu}\int_0^L R^4 dz \tag{11}$$

By using the geometry of fig-1 as shown in eq. 8 into eq. 11 the expression of non dimensional flow rate is shown as:

$$\bar{Q}_L = \frac{40 m\mu Q_L}{\pi G} A \tag{12}$$

Where $A = \left[(R_1 - mL)^5 - (R_1 - m(d_0 + L))^5\right]$

Wall shear stress: The constitutive relationship for the Newtonian fluid is given as

$$\lambda = \mu\left(-\frac{\partial w}{\partial r}\right) \tag{13}$$

From eq 4 and 10, the wall shear stress,

$$\tau_L = \frac{8\mu Q}{\pi}\int_0^L R^{-3} dz \tag{14}$$

Using eq.8 in 15 the expression of non dimensional wall shear stress

$$\overline{\tau_L} = \frac{m\pi\tau_L}{2Q\mu} A_1 \tag{15}$$

Where $A_1 = \left[\frac{1}{(R_1 - m(d_0 + L))^2} - \frac{1}{(R_1 - mL)^2}\right]$

Resistance parameter: The resistance to flow λ (resistance parameter) is defined as follows:

$$\lambda = \frac{p_1 - p_0}{Q} \tag{16}$$

From eq 10 and using the conditions that the inlet pressure $p=p_1$ at z=0 and outlet pressure $p=p_0$ at z=L, the resistance parameter is shown as:

$$\lambda = \frac{8\mu}{\pi}\int_0^L \frac{1}{R^4} dz \tag{17}$$

Thus the non dimensional resistance parameter $\bar{\lambda}$ for the tapered arteriole is given as

$$\bar{\lambda} = \frac{R_1^4}{3mL} A_2$$

Where $A_2 = \dfrac{1}{\left[R_1 - m(d_0 + L)\right]^3} - \dfrac{1}{(R_1 - mL)^3}$

RESULTS:

Figure 1 and figure 2 show that flux and axial velocity increases with the radius of the arterioles for different tapering angles.

Table-1

Flux			
Radius(mm)	$\phi = 0^o$	$\phi = 1^o$	$\phi = 1^o 30'$
0.1	0.0000125	0.000015	0.000016
0.125	3.05176E-05	6.00E-05	8.60E-05
0.15	6.32813E-05	1.18E-04	1.89E-04
0.175	0.000117236	0.000225	0.000326
0.2	0.0002	0.000314	0.000436
0.225	0.000320361	0.00044	0.000574
0.25	0.000488281	0.00057	0.00077

Figure: 1 Plot of flux against radius of tapered arterioles for Newtonian flow

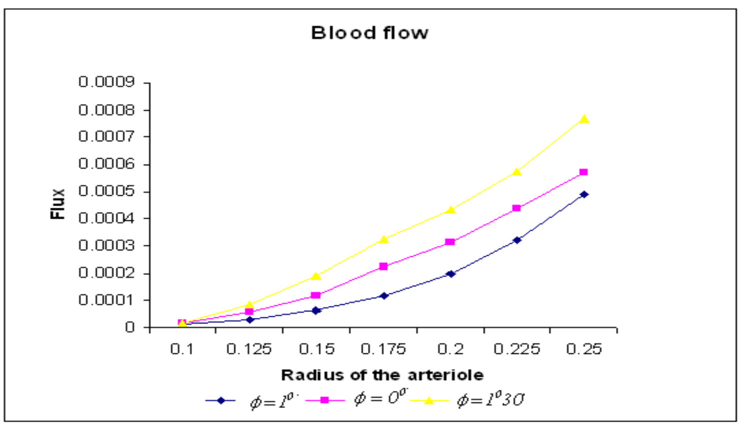
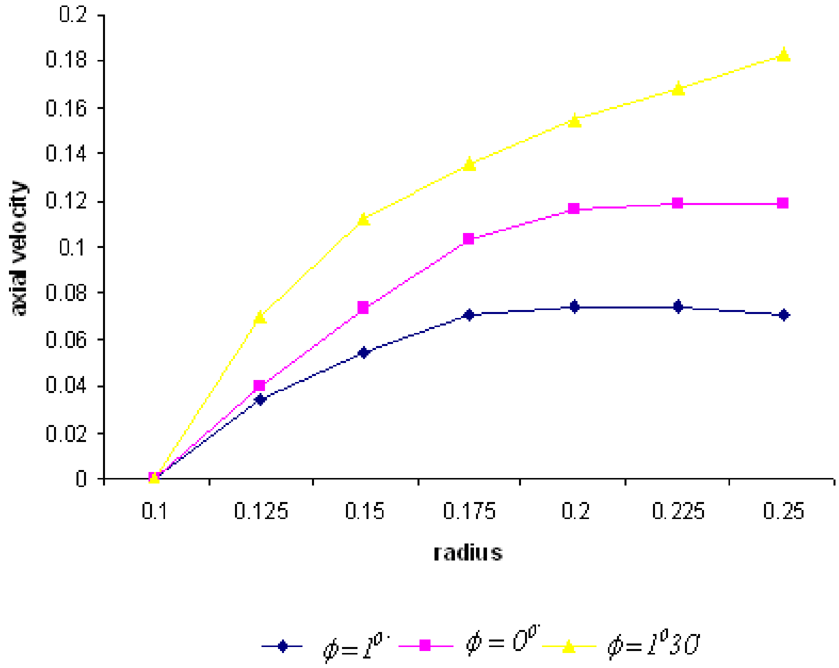


Table-2

Axial Velocity(for different tapering angles)				
Sl. No.	radius	$\phi = 0^0$	$\phi = 1^0$	$\phi = 1^o 30'$
1	0.1	0	0	0
2	0.125	0.034	0.04	0.07
3	0.15	0.055	0.0735	0.112
4	0.175	0.0712	0.103	0.136
5	0.2	0.0746	0.116	0.155
6	0.225	0.0742	0.1185	0.168
7	0.5	0.0709	0.1192	0.1827

Fig 2 Variation of Axial velocity with Arteriole radius for different tapering angles



CONCLUSION:

The flux and variation of axial velocity is demonstrated in figure 1 and figure 2 respectively are obtained theoretically. These patterns have a close resemblance with the patterns obtained by simulation techniques[6]. [6]. In the future, this study of blood flow in arterioles will lead to the prediction of individual homodynamic flows in any patient, the development of diagnostic tools to quantify disease, and the design of devices that mimic or alter blood flow.

REFERENCES:

[1] Walawender,W.P., Tien, C and Creny, L.C.(1972): " Experimental studies on the blood flow throughout tapered tubes", international Journal Engg. Science, vol.10, pp 1123-1135.

[2] Walawender, W.P. and Chen, T.Y.(1975): " Blood flow in tapered tubes." Microvasc. Res. Vol.9 pp 190-205.

[3] Yang, B.H., Asada, H.H., Zhang, Yi. 1999, cuffless Continuous Monitoring of blood pressure using hemodynamic model, the home automation and health care consortium progress report no. 2-3.

[4] Sanjeev Kumar and Sanjeet Kumar (2009) "A mathematical model of newtonian and non newtonian flow through tapered tube." Indian Journal Biomechanics., special issue (NCBM-78 march 2009).

[5] Nidhi Verma and R.S. Parihar (2010) "A mathematical modeling of blood flow through a tapered artery with mild stenosis and hematocrit" ,Journal of modern mathematics and statistics. 4(1) pp 38-43.

[6] Blood Flow in Microchannels Lennart Bitsch(page 71. chapter 5, Master Thesis, c960370).

****