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On Quasi (1, 2)*-πgα-open and Quasi (1, 2)*-πgα-closed Functions

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ABSTRACT

The purpose of this paper is to give a new type of open functions called quasi (1, 2)-πgα-open function. Also, we obtain characterizations and its basic properties.*

Mathematics Subject Classification: *54E55.*

Keywords: *(1, 2)*-πgα-open set, quasi (1, 2)*-πgα-open function, quasi (1, 2)*-πgα-closed function.*

1. INTRODUCTION:

Functions and of course open functions stand among the most important notions in the whole of mathematical science. Many different forms of open functions have been introduced over the years. Various interesting problems arise when one considers openness. Its importance is significant in various areas of mathematics and related sciences. O. Ravi et al [8, 9] discussed quasi sg-open and quasi sg-closed functions, quasi $\tilde{g}$ -open and quasi $\tilde{g}$ p-closed functions in topological spaces. Recently, as a generalization of closed sets, the notion of (1, 2)*-πgα-closed sets in bitopological spaces introduced and studied by I. Arockiarani and K. Mohana [1, 2, 3, 6, 7]. In this paper, we will continue the study of related functions by involving (1, 2)*-πgα-open sets. We introduce and characterize the concept of quasi (1, 2)*-πgα-open functions in Bitopological space.

2. PRELIMINARIES:

Throughout this paper (X, τ_1, τ_2), (Y, σ_1, σ_2) and (Z, η_1, η_2) (or X, Y and Z) represent bitopological spaces on which no separation axioms are assumed unless otherwise mentioned.

We recall the following definitions which are useful in the sequel.

Definition: 2. 1 [4] A subset S of a bitopological space X is said to be $\tau_{1,2}$ -open if S=A $\cup$ B where $A \in \tau_1$ and $B \in \tau_2$ .A subset S of X is said to be

1. $\tau_{1,2}$ -closed if the complement of S is $\tau_{1,2}$ -open.
2. $\tau_{1,2}$ -clopen if S is both $\tau_{1,2}$ -open and $\tau_{1,2}$ -closed.

Definition: 2. 2 [4] Let S be a subset of the bitopological space X. Then

1. The $\tau_{1,2}$ -interior of S, denoted by $\tau_{1,2}$ -int(S) is defined by $\cup$ {G: G $\subseteq$ S and G is $\tau_{1,2}$ -open}
2. The $\tau_{1,2}$ -closure of S, denoted by $\tau_{1,2}$ -cl(S) is defined by $\cap$ {F: S $\subseteq$ F and F is $\tau_{1,2}$ -closed}

Remark: 2. 3 [4] $\tau_{1,2}$ -open sets need not form a topology.

Definition: 2. 4 [5] A subset A of a bitopological space X is called

1. $(1,2)^*$ -regular open if $A = \tau_{1,2} - \mathrm{int}(\tau_{1,2} - cl(A))$.

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2. $(1,2)^*$-α -open if $A \subseteq \tau_{1,2} - \text{int}(\tau_{1,2} - cl(\tau_{1,2} - \text{int}(A)))$.

The complement of the sets mentioned from (i) and (ii) are called their respective closed sets.

Definition: 2. 5 [1] The finite union of (1, 2)*-regular open set is called $\tau_{1,2}$ -π-open.

Definition: 2. 6- [1] A subset A of a bitopological space X is called $(1,2)^*$- πgα –closed if $(1,2)^* - \alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\tau_{1,2}$ -π-open in X.

The complement of the $(1,2)^*$- πgα –closed is called $(1,2)^*$- πgα –open set.

Definition: 2. 7 [2] Let S be a subset of the bitopological space X. Then
1. The (1, 2)*-πgα-interior of S denoted by (1, 2)*-πgα-int(S) is defined by $\cup$ {G: G $\subseteq$ S and G is (1,2)*-πgα-open}

2. The (1, 2)*-πgα-closure of S denoted by (1, 2)*-πgα-cl(S) is defined by $\cap$ {F: S $\subseteq$ F and F is (1,2)*-πgα-closed}

Definition: 2. 8 [2] A function f: X → Y is called
1. $(1,2)^* - \pi g\alpha -$continuous function if $f^{-1}(V)$ is $(1,2)^* - \pi g\alpha -$closed in X for every $\sigma_{1,2}$-closed in Y.
2. $(1,2)^* - \pi g\alpha$ **-**irresolute if $f^{-1}(V)$ is $(1,2)^* - \pi g\alpha$ closed in X, for every $(1,2)^* - \pi g\alpha$ closed set V of Y.

Definition: 2. 9 A map f: X → Y is called
1. $(1,2)^* - \pi g\alpha$ **-**closed [3] if f (U) is $(1,2)^* - \pi g\alpha$ closed set in Y for every $\tau_{1,2} -$ closed set U in X.
2. strongly $(1,2)^* - \pi g\alpha -$closed [7] if the image f(A) is $(1,2)^* - \pi g\alpha -$ closed in Y for every $(1,2)^* - \pi g\alpha -$ closed set A in X.

3. QUASI (1, 2)*-πgα-OPEN FUNCTIONS:

Definition: 3. 1 A function f: X $\rightarrow$ Y is said to be quasi (1, 2)*-πgα-open if the image of (1, 2)*-πgα-open set in X is $\sigma_{1,2}$-open in Y.

Remark: 3. 2 It is evident that, the concepts quasi (1, 2)*- πgα-openness and (1, 2)*- πgα-continuity coincide if the function is a bijection.

Theorem: 3. 3 A function f: X $\rightarrow$ Y is quasi (1, 2)*-πgα-open if and only if for every subset U of X, f ((1, 2)*- πgα-int(U)) ⊂ $\sigma_{1,2}$-int-(f(U)).

Proof: Let f be a quasi (1, 2)*-πgα-open function. Now, we have $\tau_{1,2}$-int (U) ⊂U and (1, 2)*- πgα-int(U) is a (1, 2)*-πgα-open set.

Hence, we obtain that f ((1, 2)*- πgα-int (U)) ⊂ f (U). As f ((1, 2)*- πgα-int(U)) is $\sigma_{1,2}$-open, f((1, 2)*- πgα-int(U)) ⊂ $\sigma_{1,2}$-int-(f(U)).

Conversely, assume that U is a (1, 2)*-πgα-open set in X. Then, f (U) = f ((1, 2)*- πgα-int(U)) ⊂ $\sigma_{1,2}$-int-(f(U)), but $\sigma_{1,2}$-int-(f(U)) ⊂ f(U). Consequently, f (U) = $\sigma_{1,2}$-int-(f (U)) and hence f is quasi (1, 2)*-πgα-open.

Lemma: 3. 4 If a function f: X $\rightarrow$ Y is quasi (1, 2)*-πgα-open, then (1, 2)*- πgα-int(f^{-1}(G)) ⊂ f^{-1}($\sigma_{1,2}$-int-(G)) for every subset G of Y.

Proof: Let G be any arbitrary subset of Y. Then, (1, 2)*- πgα-int(f^{-1}(G)) is a (1, 2)*-πgα-open set in X and f is quasi (1, 2)*-πgα-open, then f((1, 2)*- πgα-int(f^{-1}(G))) ⊂ $\sigma_{1,2}$-int-(f(f^{-1}(G))) ⊂ $\sigma_{1,2}$-int-(G).

Thus, (1, 2)*- πgα-int(f^{-1}(G)) ⊂ f^{-1}($\sigma_{1,2}$-int-(G)).

Recall that a subset S is called a (1, 2)*- πgα- neighbourhood [6] of a point x of X if there exists a (1, 2)*- πgα-open set U such that $x \in U \subset S$.

Theorem: 3. 5 For a function f: X $\rightarrow$ Y, the following are equivalent:

1. f is quasi (1, 2)*- πgα-open.

2. For each subset U of X, f ((1, 2)*- πgα-int(U)) ⊂ $\sigma_{1,2}$-int-(f(U)).

3. For each $x \in X$ and each (1, 2)*- πgα-neighbourhood U of x in X, there exists a (1, 2)*-neighbourhood f(U) of f(x) in Y suchthat f(V) ⊂f(U).

Proof:

1 => 2: If follows from Theorem 3.3.

2=>3: Let $x \in X$ and U be an arbitrary (1, 2)*- πgα-neighbourhood of x in X. Then there exists a (1, 2)*- πgα-open set V in X such that $x \in V \subset U$. Then by(ii), we have f(V)= f((1, 2)*- πgα-int(V)) ⊂ $\sigma_{1,2}$-int-(f(V)) and hence f(V)= $\sigma_{1,2}$-int-(f(V)). Therefore, it follows that f(V) is $\sigma_{1,2}$-open in Y such that $f(x) \in f(V) \subset f(U)$.

3=>1: Let U be an arbitrary (1, 2)*- πgα-open set in X. Then for each $y \in f(U)$, by (3) there exists a (1, 2)*-neighbourhood V_y of y in Y such that V_y⊂f(U). As V_y is a (1, 2)*-neighbourhood of y, thereexists an $\sigma_{1,2}$-open W_y in Y such that $y \in W_y \subset V_y$. Thus f(U)= U {W_y : $y \in f(U)$ }. Which is a $\sigma_{1,2}$-open set in Y. This implies that f is Quasi (1, 2)*- πgα-open function.

Theorem: 3. 6 A function f : X $\rightarrow$ Y is quasi (1, 2)*- πgα-open if and only if for any subset B of Y and for any (1, 2)*-πgα-closed set F of X containing f^{-1}(B), there exists a $\sigma_{1,2}$-closed set G of Y containing B such that f^{-1}(G) ⊂F.

Proof: Suppose f is quasi (1, 2)*- πgα-open. Let B⊂Y and F be a (1, 2)*- πgα-closed set of X containing f^{-1}(B). Now, put G=Y-f(X-F). It is clear that f^{-1}(B) ⊂F implies B ⊂G. Since f is quasi (1, 2)*- πgα-open, we obtain G as a $\sigma_{1,2}$-closed set of Y. Moreover, we have f^{-1}(G) ⊂F.

Conversely, let U be a (1, 2)*- πgα-open set of X and put B=Y \ f(U). Then X\U is a (1, 2)*- πgα-closed set in X containing f^{-1}(B). By hypothesis, there exists a $\sigma_{1,2}$-closed set F of Y such that B⊂F and f^{-1}(F) ⊂X\U.

Hence, we obtain f (U) ⊂ Y \ F. On the other hand, it follows that B⊂F, Y\F ⊂ Y \ B= f (U).

Thus, we obtain f (U) =Y\F which is $\sigma_{1,2}$-open and hence f is a quasi (1, 2)*- πgα-open function.

Theorem: 3. 7 A function f : X $\rightarrow$ Y is quasi (1, 2)*- πgα-open if and only if f^{-1}($\sigma_{1,2}$-cl-(B))⊂(1, 2)*- πgα-cl(f^{-1}(B)) for every subset B of Y.

Proof: Suppose that f is quasi (1, 2)*- πgα-open. For any subset B of Y, f^{-1}(B) ⊂ (1, 2)*- πgα-cl(f^{-1}(B)). Therefore by Theorem 3.5, there exists a $\sigma_{1,2}$-closed set F in Y such that B⊂F and f^{-1}(F) ⊂ (1, 2)*- πgα-cl(f^{-1}(B)). Therefore, we obtain f^{-1}($\sigma_{1,2}$-cl-(B)) ⊂f^{-1}(F) ⊂ (1, 2)*- πgα-cl(f^{-1}(B)). Conversely, let B⊂Y and F be a (1, 2)*- πgα-closed set of X containing f^{-1}(B).

Put W= $\sigma_{1,2}$-cl-(B), then we have B⊂W and W is $\sigma_{1,2}$-closed and f^{-1}(W)⊂(1, 2)*- πgα-cl(f^{-1}(B)) ⊂F. Then by Theorem 3. 6, f is quasi (1, 2)*- πgα-open.

Lemma: 3. 8 Let f: X $\rightarrow$ Y and g: Y $\rightarrow$ Z be two functions. Given that gof: X $\rightarrow$ Z is quasi (1, 2)*- πgα-open. If g is (1, 2)*-continuous injective, then f is quasi (1, 2)*- πgα-open.

Proof: Let U be a (1, 2)*- πgα-open set in X. Then (gof)(U) is $\eta_{1,2}$-open set in Z, since gof is quasi (1, 2)*- πgα-open. Again, g is an injective (1, 2)*-continuous function, f(U)=g^{-1}(gof(U)) is $\sigma_{1,2}$-open in Y. This shows that f is quasi (1, 2)*- πgα-open.

Theorem: 3. 9 Let f: X $\rightarrow$ Y and g: Y $\rightarrow$ Z be two functions. Given that f: X $\rightarrow$ Y is quasi (1, 2)*- πgα-open. If g is (1, 2)*-closed functions, then gof is quasi (1, 2)*- πgα-open.

Proof: The proof is obvious.

4. QUASI (1, 2)*- πgα-CLOSED FUNCTIONS:

Definition: 4.1 A function f: X $\to$ Y is said to be quasi (1, 2)*-πgα-closed if the image of (1, 2)*-πgα-closed set in X is $\sigma_{1,2}$-closed in Y.

Clearly, every quasi (1, 2)*- πgα-closed function is (1, 2)*-closed as well as (1, 2)*- πgα-closed.

Remark: 4. 2 Every (1, 2)*-πgα-closed ((1, 2)*-closed) function need not be quasi (1, 2)*-πgα-closed as shown by the following example.

Example: 4. 3 Let X=Y={a, b, c}, τ_1 = {ϕ, X, {a, b}}, τ_2 = {ϕ, X}, σ_1={ϕ, Y, {a}}, σ_2={ϕ, Y,{b, c}}. Define a function $f : X \to Y$ by f(a)=b, f(b)=c, f(c) = a. Then clearly, f is (1, 2)*-πgα-closed map as well as (1, 2)*-closed but not quasi (1, 2)*-πgα-closed.

Lemma: 4. 4 If a function f: X $\to$ Y is quasi (1, 2)*-πgα-closed, then $f^{-1}(\sigma_{1,2}\text{-int-}(G)) \subset$ (1, 2)*- πgα-int(f^{-1}(G)) for every subset G of Y.

Proof: This proof is similar to the proof of Lemma 3.4.

Theorem: 4. 5 A function f : X $\to$ Y is quasi (1, 2)*- πgα-closed if and only if for any subset B of Y and for any (1, 2)*- πgα-open set G of X containing f^{-1}(B), there exists a $\sigma_{1,2}$-open set U of Y containing B such that $f^{-1}(U) \subset G$.

Proof: The proof is similar to the proof of Theorem 3. 6.

Theorem: 4. 6 If f: X $\to$ Y and g: Y $\to$ Z are two quasi (1, 2)*- πgα-closed functions, then the gof: X $\to$ Z is a quasi (1, 2)*- πgα-closed function.

Proof: Obvious.

Furthermore, we have the following theorem:

Theorem: 4. 7 Let f: X $\to$ Y and g: Y $\to$ Z be any functions.

1. If f is (1, 2)*- πgα-closed map and g is quasi (1, 2)*- πgα-closed function, then gof: X $\to$ Z is (1, 2)*-closed function.

2. If f is quasi (1, 2)*- πgα-closed and g is (1, 2)*- πgα-closed map, then gof: X $\to$ Z is strongly (1, 2)*-πgα-closed map.

3. If f is strongly (1, 2)*-πgα-closed map and g is quasi (1, 2)*- πgα-closed function, then gof: X $\to$ Z is quasi (1, 2)*- πgα-closed function.

Proof: The proof is obvious.

Theorem: 4. 8

1. If f: X $\to$ Y is (1, 2)*- πgα-irresolute surjective and gof: X $\to$ Z is quasi (1, 2)*- πgα closed function, then g: Y $\to$ Z is (1, 2)*- closed function.

2. If g: Y $\to$ Z is (1, 2)*- πgα-continuous injective and gof: X $\to$ Z is quasi (1, 2)*- πgα closed function, then f: X $\to$ Y is strongly (1, 2)*- πgα-closed.

Proof: 1. Suppose that F is an arbitrary $\sigma_{1,2}$-closed set in Y. As f is (1, 2)*- πgα-irresolute, f^{-1}(F) is (1, 2)*- πgα-closed in X. Since g o f is quasi (1, 2)*- πgα-closed and f is surjective, (g o f (f^{-1}(F))) = g(F), which is $\eta_{1,2}$-closed in Z. This implies that g is a (1, 2)*-closed function.

2. Suppose F is any (1, 2)*- πgα-closed set in X. Since g o f is quasi (1, 2)*- πgα-closed, (g o f)(F) is $\eta_{1,2}$-closed in Z. Again g is a (1, 2)*- πgα-continuous injective function, g^{-1}(g o f (F)) = f (F), which is (1, 2)*- πgα-closed in Y. This shows that f is strongly (1, 2)*- πgα-closed map.

Theorem: 4. 9 Let X and Y be bitopological spaces. Then the function g : X $\rightarrow$ Y is a quasi (1, 2)*- πgα-closed if and only if g(X) is $\sigma_{1,2}$- closed in Y and g(V) \ g(X \ V) is $\sigma_{1,2}$- open in g(X) whenever V is (1, 2)*- πgα-open in X.

Proof: Necessity: Suppose g: X $\rightarrow$ Y is a quasi (1, 2)*- πgα-closed function. Since X is (1, 2)*- πgα-closed, g(X) is $\sigma_{1,2}$- closed in Y and g(V) \ g(X \ V) = g(V) ∩g(X) \ g(X \ V) is $\sigma_{1,2}$- open in g(X) when V is (1, 2)*- πgα-open in X.

Sufficiency: Suppose g(X) is $\sigma_{1,2}$- closed in Y, g(V) \ g(X \ V) is $\sigma_{1,2}$- open in g(X) when V is (1, 2)*- πgα-open in X, and let C be $\tau_{1,2}$–closed set in X. Then g(C) = g(X) \ (g(X \ C) \ g(C)) is $\sigma_{1,2}$- closed set in g(X) and hence, $\sigma_{1,2}$- closed set in Y.

Corollary: 4.10 Let X and Y be bitopological spaces. Then a surjective function g : X $\rightarrow$ Y is quasi (1, 2)*- πgα-closed if and only if g(V) \ g(X \V) is $\sigma_{1,2}$- open set in Y whenever V is (1, 2)*- πgα-open set in X.

Proof: Obvious.

Definition: 4.11 A bitopological space X is said to be (1, 2)*- πgα*-normal if for any pair of disjoint (1, 2)*- πgα-closed subsets F_1 and F_2 of X, there exist disjoint $\tau_{1,2}$–open sets U and V such that $F_1 \subset U$ and $F_2 \subset V$.

Theorem: 4.12 Let X and Y be bitopological spaces with X is (1, 2)*- πgα*- normal. If g : X $\rightarrow$ Y is a (1, 2)*- πgα - continuous quasi (1, 2)*- πgα-closed surjective function, then Y is (1, 2)*-normal.

Proof: Let K and M be disjoint $\sigma_{1,2}$- closed subsets of Y. Then $g^{-1}(K)$, $g^{-1}(M)$ are disjoint (1, 2)*- πgα -closed subsets of X. Since X is (1, 2)*- πgα*-normal, there exist disjoint $\sigma_{1,2}$- open sets V and W such that $g^{-1}(K) \subset V$ and $g^{-1}(M) \subset W$. Then K ⊂ g(V) \ g (X \ V) and M ⊂ g(W) \ g(X \ W). Further by Corollary 4.12, g(V) \ g(X \ V) and g(W) \ g(X \ W) are $\sigma_{1,2}$-open sets in Y and clearly (g(V) \ g(X \ V)) ∩ (g (W) \ g(X \ W)) = Φ. This shows that Y is (1, 2)*-normal.

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