International Journal of Mathematical Archive-3(1), 2012, Page: 143-152
IJMA Available online through www.ijma.info ISSN 2229 – 5046

SIMILARITY SOLUTION FOR UNSTEADY BOUNDARY LAYER FLOW OF A MICROPOLAR NON- NEWTONIAN POWER LAW FLUID THROUGH A POROUS MEDIUM

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(Received on: 28-12-11; Accepted on: 14-01-11)

ABSTRACT

In the present paper, the two dimensional unsteady boundary layer flow of a micropolar non-Newtonian power law fluid near the rear stagnation point of a plane surface in a porous medium are studied. The similarity solutions are obtained using group theoretic technique. The application of two-parameter group reduce the number of independent variables by two and a set of auxiliary conditions from three to only one independent variable, and consequently the system of governing partial differential equations with boundary conditions reduces to a system of ordinary differential equations with appropriate boundary conditions.

Keywords: *Similarity solution, micro-polar fluid, porous medium, Non-Newtonian.*

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1. INTRODUCTION:

The concept of theory of micro polar fluid including thermo-micro polar fluid was developed by Eringen [9, 10, 11]. This theory includes the effects of micro-inertia and couple stresses and can explain the non-Newtonian behavior of certain polymeric fluid, animal blood and liquid crystals. The concept of boundary-layer theory in micro polar fluid was first introduced by Willson [23].Since then, several authors have studied the steady self similar flow of micro polar fluids for two dimensional bodies with or without pressure gradient. It is well known that most problems encountered in applications are unsteady. In fact, there is no actual flow situation, natural and artificial, which does not involve some unsteadiness. Unsteady flows are frequently encountered in such technological and environmental situation as geophysical and biological flows, and the processing the materials, the spread of pollutants and fires (Telionis). Because of its great importance, the problem of unsteady forced convection flow has long been a major subject in fluid mechanics.

Boundary layer theory has been applied successfully to various non-Newtonian fluids models. Boundary layers of non-Newtonian fluids have received considerable attention in the last few decades. Almost a century ago Prandtl realized the key part that boundary layers play in determining accurately the flow of certain fluids. He showed for slightly viscous flows that although viscosity is negligible in the bulk of the flow, it assumes a vital role near boundaries. Lok et al. [14] studied the unsteady boundary layer flow of a micro polar fluid near the forward stagnation point of a plane surface by using the Newton's linearization technique of Kellr-box method. Seshadri et al. [21] used the implicit finite difference scheme to study the unsteady mixed convection flow in the stagnation region of a heated vertical plat due to impulsive motion. Abd-elaziz M. M and Ahmed S.E. [2] studied Group solution for unsteady boundary layer flow of a micro polar fluid near the rear stagnation point of a plane surface in a porous medium.

The difficulty of the study of such fluid problem is the paucity of boundary conditions and existence of deformable microelements as well as the time as the third independent variable. Many attempts were made to find analytical and numerical solution by applying certain special conditions and using different mathematical approaches. The class of solutions known as similarity solutions traditionally plays an important role in the analysis of boundary layer problems because it is the only class of exact solution for the boundary layer equations. The quest for exact solutions of the boundary-layer equations has a long history. Blasius [3] used a scaling to obtain the similarity solution of the steady boundary layer flow of a flat plate. Further work Burde'[4, 5, 6, 7] obtained similarity solution corresponding to stagnation point flows, flows past wedges, jets and flows near an oscillating plate. Jones and Watson [13] have given a comprehensive account of many of the classical exact solutions of the boundary-layer equations including Falkner-Skan forms and the asymptotic suction profile. A New Systematic Formalism for Similarity Analysis, with Applications to Boundary Layers Flows was introduced by Moran, Gaggioli and Scholten [18]. Similarity solution for

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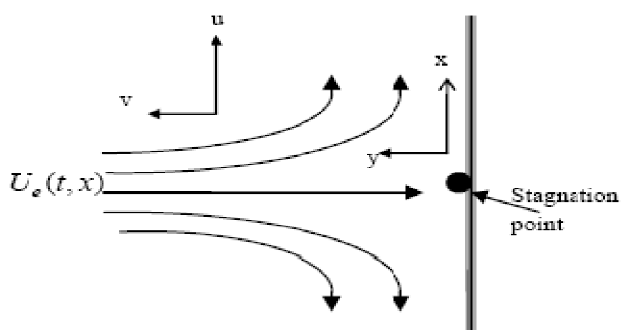
laminar, incompressible, three dimensional boundary layer flows was studied by Hansen and Herzig [12], they have been attempted to determine the conditions under which similarity solutions exist. Ma and Hui [16] used the method of Lie group transformations to derive all possible group-invariant similarity solutions to the problem of unsteady two-dimensional boundary-layer flow of an incompressible fluid. This method is based on non-linear superposition is then used to generate further similarity solution which are not group invariant. The symmetry analysis applied to boundary layer equations is, introduced by Cantwell [8]. M. Patel and M. G. Timol [20] derived similarity solution for three-dimensional boundary layer flows of non-Newtonian fluids. Using Lie group analysis unsteady laminar boundary layer equations of non-Newtonian fluids are studied by Yurusow and Pakdemirli [24, 25]. They have assumed that steer-stress is arbitrary function of velocity gradient. They have extended the previous work by Timol and Kalthia [22] who have assume the more general arbitrary functional relationship of stress and velocity gradient in their similarity analysis of three-dimensional boundary layer flow of general non-Newtonian fluids. The similarity analysis of governing equation is discussed using the group theoretic method which is already successfully applied to several non-linear problems. (Abd-el-Malek et al [1]; Parmar and Timol [19]).

Moran and Gaggioli [17] presented a New Systematic Formalism for Similarity Analysis. They utilized elementary group theory for the purpose of reducing the given system of partial differential equation to a system of ordinary differential equations in a single variable. The objective of this paper is to derive similarity equations for unsteady boundary layer flow of a micro polar non-Newtonian power law fluid near the rear stagnation point of a plane surface in a porous media. The systematic formalism introduced here, is based on Moran and Gaggioli [17] formalism, which reduce the system of partial differential equations to a system of ordinary differential equations with the appropriate boundary conditions.

2. MATHEMATICAL FORMULATION:

Let us consider the development of the two-dimensional boundary layer flow of a micro polar non-Newtonian power law fluid near the rear stagnation point of a plane surface in a porous medium. The fluid which occupies a semi-infinite domain bounded by an infinite plane and remains at rest for time $t < 0$ and starts to move impulsively away from the wall at $t = 0$. In our analysis, rectangular Cartesian coordinates (x, y) are used in which x and y are taken as the coordinates along the wall and normal to it, respectively. The flow configuration is shown schematically in Figure.

Figure:



Physical model of the problem

The boundary layer equations governing the unsteady flow of a micro polar non-Newtonian power law fluid through a porous medium with constant physical properties are (Lok et al., [15])

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U_e\frac{dU_e}{dx} + \left(\upsilon + \frac{k}{\rho}\right)\frac{\partial}{\partial y}\left\{\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}\right\} + \frac{k}{\rho}\frac{\partial N}{\partial y} + \frac{\upsilon}{k_1}(U_e - u) \tag{2}$$

$$\frac{\partial N}{\partial t} + u\frac{\partial N}{\partial x} + v\frac{\partial N}{\partial y} = \frac{\gamma}{\rho_j}\frac{\partial^2 N}{\partial y^2} - \frac{k}{\rho_j}\left(2N + \frac{\partial u}{\partial y}\right) \tag{3}$$

$$\frac{\partial j}{\partial t} + u\frac{\partial j}{\partial x} + v\frac{\partial j}{\partial y} = 0 \tag{4}$$

In the above equation,

u and v are the component of fluid velocity in the x and y directions respectively,
U_e is the uniform stream velocity,

N is the component of micro rotation,
ρ is the density,
k is the vortex viscosity,
γ is the spin-gradient viscosity,
j is the micro-inertia density,
v is the kinematic viscosity,
k_1 is the permeability of the porous medium.

Boundary and initial conditions:

$t < 0 : u(x, y, t) = 0 \,, v(x, y, t) = 0 \,, N(x, y, t) = 0$

$$t = 0 : u(t, x, \infty) = U_e(t, x),\;\; N(t, x, \infty) = 0 \tag{5}$$

$$t > 0 : \begin{cases} u(x, y, t) = v(x, y, t) = N + \beta \frac{\partial u}{\partial y} = 0 \quad at\; y = 0 \\ u \to U_e(t, x), \quad N \to 0 \qquad as \quad y \to \infty \end{cases}$$

Where, β is a constant and $0 \le \beta \le 1$

Case 1: $\boldsymbol{\beta = 0}$

Which indicates N=0, represent concentrated particle flow in which the microelement close to the wall surface are unable to rotate.

Case 2: $\boldsymbol{\beta = \frac{1}{2}}$

This indicates to the vanishing of anti-symmetric part of the stress tensor and denoted week concentration.

Case 3: $\boldsymbol{\beta = 1}$

It is used for the modeling of turbulent boundary layer flows.

We shall consider here only the value of $\beta = 0$ and $\beta = \frac{1}{2}$

At this point, we introduce the dimensionless stream function $\psi(x, y, t)$ such that $u = \frac{\partial \psi}{\partial y}$,

$v = -\frac{\partial \psi}{\partial x}$.

We can find that equation (1) and (4) is identically satisfied and equation (2) and (3) reduce to,

$$\frac{\partial^2 \psi}{\partial y \partial t} + \frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial y \partial x} - \frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial y^2} = U_e \frac{dU_e}{dx} + v\left[(1 + \Delta)\frac{\partial}{\partial y}\left\{\left|\frac{\partial^2 \psi}{\partial y^2}\right|^{n-1}\frac{\partial^2 \psi}{\partial y^2}\right\} + \Delta \frac{\partial N}{\partial y} + \frac{1}{k_1}\left(U_e - \frac{\partial \psi}{\partial y}\right)\right] \tag{6}$$

Where $\Delta = \frac{k}{\rho v}$ (coupling constant)

$$\frac{\partial N}{\partial t} + \frac{\partial \psi}{\partial y}\frac{\partial N}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial N}{\partial y} = \lambda \frac{\partial^2 N}{\partial y^2} - \sigma\left(2N + \frac{\partial^2 \psi}{\partial y^2}\right) \tag{7}$$

Where, $\lambda = \frac{\gamma}{\rho_j}$ (microrotation parameter)
$\sigma = \frac{k}{\rho_j}$ (Dimensionless material parameter)

With The boundary condition:

$y = 0: \frac{\partial \psi}{\partial y} = \frac{\partial \psi}{\partial x} = N + \beta \frac{\partial^2 \psi}{\partial y^2} = 0$

$$y \to \infty: \frac{\partial \psi}{\partial y} = U_e,\; \frac{\partial \psi}{\partial x} = N = 0 \tag{8}$$

3. METHOD OF SOLUTION:

In this section, two parameter transformation group is applied to the system of equations (6) and (7) with the boundary conditions (8). The system of equations reduces to a system of ordinary differential equation in a single independent variable with appropriate boundary conditions.

A. Group Formulation of the Problem:

The procedure is initiated with the group G, A class of two parameter group of the form

$$G = \begin{cases} s: \begin{cases} \bar{x} = c^{x}(a_1, a_2)x + k^{x}(a_1, a_2) \\ \bar{y} = c^{y}(a_1, a_2)y + k^{y}(a_1, a_2) \\ \bar{t} = c^{t}(a_1, a_2)t + k^{t}(a_1, a_2) \end{cases} \\ \bar{\psi} = c^{\psi}(a_1, a_2)\psi + k^{\psi}(a_1, a_2) \\ \bar{N} = c^{N}(a_1, a_2)N + k^{N}(a_1, a_2) \\ \overline{U_e} = c^{U_e}(a_1, a_2)U_e + k^{U_e}(a_1, a_2) \end{cases} \tag{9}$$

where C^s and k^s are real valued and at least differentiable in their real argument (a_1, a_2), which possesses complete sets of absolute invariant $\eta(x, y, t)$ and $g_j(x, y, t, N, U_e, \psi)$, i=1,2,3 where g_j are the three absolute invariant corresponding to N, ψ, U_e. If $\eta(x, y, t)$ is the absolute invariant of independent variables then

$$g_j(x, y, t, N, U_e, \psi) = F_j(\eta(x, y, t)) \qquad \text{i=1, 2, 3}$$

B. The Invariance Analysis:

The transformation of the dependent variables and their partial derivatives are obtain from G via chain-rule operations

$$\frac{\partial \bar{p}}{\partial \bar{\imath}} = \left(\frac{c^{\,p}}{c^{i}} \frac{\partial p}{\partial i}\right)$$

$$\frac{\partial^2 \bar{p}}{\partial \bar{\imath}^2} = \left(\frac{c^{\,p}}{\left(c^{i}\right)^2} \frac{\partial^2 p}{\partial i^2}\right)$$

$$\frac{\partial^3 \bar{p}}{\partial \bar{\imath}^3} = \left(\frac{c^{\,p}}{\left(c^{i}\right)^3} \frac{\partial^3 p}{\partial i^3}\right) \qquad i = x,\ y,\ t \tag{10}$$

Where, p stands for $N, U_e, \psi;$

Equation (6) is said to be invariantly transformed under (9) and (10) whenever

$$\left(\frac{c^{\,\psi}}{c^{t}c^{y}}\right)\frac{\partial^2 \psi}{\partial y \partial t} + \left(\frac{c^{\,2\psi}}{c^{x}c^{2y}}\right)\frac{\partial \psi}{\partial y}\,\frac{\partial^2 \psi}{\partial y \partial x} - \left(\frac{c^{\,2\psi}}{c^{x}c^{2y}}\right)\frac{\partial \psi}{\partial x}\,\frac{\partial^2 \psi}{\partial y^2} - \left(\frac{c^{\,2U_e}}{c^{x}}\right)\,U_e\,\frac{dU_e}{dx}$$

$$-v\left[n(1+\Delta)\frac{\left(c^{\,\psi}\right)^n}{\left(c^{y}\right)^{2n+1}}n\left(\frac{\partial^2 \psi}{\partial y^2}\right)^{n-1}\frac{\partial^3 \psi}{\partial y^3} + \Delta\left(\frac{c^{\,N}}{c^{y}}\right)\frac{\partial N}{\partial y} + \frac{1}{k_1}\left(c^{U_e}U_e - \left(\frac{c^{\,\psi}}{c^{y}}\right)\frac{\partial \psi}{\partial y}\right)\right] - R$$

$$= H_1(a_1, a_2)\left[\frac{\partial^2 \psi}{\partial y \partial t} + \frac{\partial \psi}{\partial y}\,\frac{\partial^2 \psi}{\partial y \partial x} - \frac{\partial \psi}{\partial x}\,\frac{\partial^2 \psi}{\partial y^2} = U_e\frac{dU_e}{dx} + v\left[(1+\Delta)\frac{\partial}{\partial y}\left\{\left|\frac{\partial^2 \psi}{\partial y^2}\right|^{n-1}\frac{\partial^2 \psi}{\partial y^2}\right\} + \Delta\frac{\partial N}{\partial y} + \frac{1}{k_1}\left(U_e - \frac{\partial \psi}{\partial y}\right)\right]\right] \tag{11}$$

Where, $R = \frac{v}{k_1}k^{U_e} + k^{U_e}\left(\frac{c^{\,U_e}}{c^{x}}\right)\frac{dU_e}{dx}$

(12)

For invariant transformation R is equated to zero. This is satisfied by setting

$R = 0: k^{U_e} = 0$,

(13)

And comparing the coefficient in both sides of (11) and with $H_1(a_1, a_2)$, we obtain

$$\left(\frac{c^{\,\psi}}{c^{x}}\frac{\partial \psi}{\partial x}\right) = \left(\frac{c^{\,2\psi}}{c^{x}c^{2y}}\right) = \left(\frac{c^{\,\psi}}{c^{3y}}\right) = \left(\frac{c^{\,\psi}}{c^{y}}\right) = \left(c^{U_e}\right) = \left(\frac{c^{\,2U_e}}{c^{x}}\right) = \left(\frac{c^{\,N}}{c^{y}}\right) = H_1(a_1, a_2)$$

(14)

Where, $H_1(a_1, a_2) = constant$

In similar manner the invariant transform of (7) under (9) and (10). Whenever there is a function $H_2(a_1, a_2)$ such that

$$\left(\frac{c^N}{c^t}\right)\frac{\partial N}{\partial t} + \left(\frac{c^\psi}{c^y}\frac{c^N}{c^x}\right)\frac{\partial \psi}{\partial y}\frac{\partial N}{\partial x} - \left(\frac{c^\psi}{c^x}\frac{c^N}{c^y}\right)\frac{\partial \psi}{\partial x}\frac{\partial N}{\partial y} - \lambda\left(\frac{c^N}{c^{2y}}\right)\frac{\partial^2 N}{\partial y^2} + \sigma\left(2c^N N + \left(\frac{c^\psi}{c^{2y}}\right)\frac{\partial^2 \psi}{\partial y^2}\right) + R_1$$

$$= H_2(a_1, a_2)\left[\frac{\partial N}{\partial t} + \frac{\partial \psi}{\partial y}\frac{\partial N}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial N}{\partial y} - \lambda\frac{\partial^2 N}{\partial y^2} + \sigma\left(2N + \frac{\partial^2 \psi}{\partial y^2}\right)\right] \tag{15}$$

Where, $R_1 = 2\sigma k^N$ (16)

The invariance condition implies that,

$$\left(\frac{c^N}{c^t}\right) = (c^N) = \left(\frac{c^\psi}{c^y}\frac{c^N}{c^x}\right) = \left(\frac{c^\psi}{c^{2y}}\right) = \left(\frac{c^N}{c^{2y}}\right) = H_2(a_1, a_2) \tag{17}$$

And $R_1 = 0$: $k^N = 0$ (18)

Moreover, the boundary conditions (8) is also invariant in form whenever the condition $k^y = 0$ is appended to (13), (14), (17), (18); that is

$$\bar{y} = 0: \frac{\partial \bar{\psi}}{\partial \bar{y}} = \frac{\partial \bar{\psi}}{\partial \bar{x}} = \bar{N} + \beta\frac{\partial^2 \bar{\psi}}{\partial \bar{y}^2} = 0$$

$$: \frac{\partial \bar{\psi}}{\partial \bar{y}} = \frac{\partial \bar{\psi}}{\partial \bar{x}} = 0, \bar{N} = -\beta\frac{\partial^2 \bar{\psi}}{\partial \bar{y}^2} \tag{19}$$

$$\bar{y} \to \infty: \frac{\partial \bar{\psi}}{\partial \bar{y}} \to \bar{U}_e,\ \bar{N} \to 0$$

Combining equation (13), (14), (17) and (18); We get,

$c^y = c^t = 1$ And $c^N = c^\psi = c^{U_e} = c^x$ (20)

Thus, Group G is of the form

$$G = \begin{cases} S: \begin{cases} \bar{x} = c^x(a_1, a_2)x + k^x(a_1, a_2) \\ \bar{y} = y \\ \bar{t} = t + k^t(a_1, a_2) \end{cases} \\ \bar{\psi} = c^\psi(a_1, a_2)\psi \\ \bar{N} = c^N(a_1, a_2)N \\ \overline{U_e} = c^{U_e}(a_1, a_2)U_e \end{cases} \tag{21}$$

This group transforms invariantly the differential equation (6) and (7) and the boundary conditions (8).

C. Complete sets of Absolute Invariants:

The complete set of absolute invariants is

(1) The absolute invariants of the independent variables (x, y, t) is $\eta = \eta(x, y, t)$,

(2) The absolute invariants of the dependent variables $(\psi, N. U_e)$, then

$$g_j(x, y, t; \psi, N, U_e) = F_j(\eta(x, y, t)) \qquad \text{i=1, 2, 3} \tag{22}$$

The application of a basic theorem in group theory, see Moran and Gaggioli [17]

States that: A function $g_j(x, y, t; \psi, N, U_e)$ is an absolute invariant of a two-parameter group if it satisfies the following two first-order linear differential equations:

$$\sum_{i=1}^{11}(\alpha_i S_i + \alpha_{i+1})\frac{\partial g}{\partial S_i} = 0 \tag{23}$$

$$\sum_{i=1}^{11}(\beta_i S_i + \beta_{i+1})\frac{\partial g}{\partial S_i} = 0 \tag{24}$$

Where, $S_i = x, y, t; \psi, N, U_e$ And

$\alpha_1 = \frac{\partial C^x}{\partial a_1}(a_1{}^0, a_2{}^0)$ $\qquad$ $\beta_1 = \frac{\partial C^x}{\partial a_2}(a_1{}^0, a_2{}^0)$

$\alpha_2 = \frac{\partial k^x}{\partial a_1}(a_1{}^0, a_2{}^0)$ $\qquad$ $\beta_2 = \frac{\partial k^x}{\partial a_2}(a_1{}^0, a_2{}^0)$

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$\alpha_{10} = \frac{\partial c^{U_e}}{\partial a_1}(a_1{}^0, a_2{}^0)$ $\qquad$ $\beta_{10} = \frac{\partial C^{U_e}}{\partial a_2}(a_1{}^0, a_2{}^0)$

$\alpha_{11} = \frac{\partial k^{U_e}}{\partial a_1}(a_1{}^0, a_2{}^0)$ $\qquad$ $\beta_{11} = \frac{\partial k^{u_e}}{\partial a_2}(a_1{}^0, a_2{}^0)$

Where, $(a_1{}^0, a_2{}^0)$ are the identity elements of the group.

D. Absolute Invariants of Independent Variables:

The absolute invariant $\eta = \eta(x, y, t)$ of the independent variables (x, y, t) is determined using equations (23) and (24)

$$(\alpha_1 x + \alpha_2)\frac{\partial \eta}{\partial x} + (\alpha_3 y + \alpha_4)\frac{\partial \eta}{\partial y} + (\alpha_5 t + \alpha_6)\frac{\partial \eta}{\partial t} = 0$$

$$(\beta_1 x + \beta_2)\frac{\partial \eta}{\partial x} + (\beta_3 y + \beta_4)\frac{\partial \eta}{\partial y} + (\beta_5 t + \beta_6)\frac{\partial \eta}{\partial t} = 0 \qquad (25)$$

Since $k^y = 0$,then $\alpha_4 = \beta_4 = 0$, then we have equation (25) becomes

$$(\alpha_1 x + \alpha_2)\frac{\partial \eta}{\partial x} + (\alpha_3 y)\frac{\partial \eta}{\partial y} + (\alpha_5 t + \alpha_6)\frac{\partial \eta}{\partial t} = 0$$

$$(\beta_1 x + \beta_2)\frac{\partial \eta}{\partial x} + (\beta_3 y)\frac{\partial \eta}{\partial y} + (\beta_5 t + \beta_6)\frac{\partial \eta}{\partial t} = 0 \qquad (26)$$

The similarity analysis of (6) to (8) now proceeds for the particular case of two parameter groups of the form (21) according to basic theorem from group theory; equation (26) has only one solution if at least one of the following conditions is satisfied

$$\begin{aligned} &\lambda_{13} x + \lambda_{23} \neq 0 \\ &\lambda_{35} t + \lambda_{36} \neq 0 \\ &\lambda_{15} xt + \lambda_{16} x + \lambda_{25} t + \lambda_{26} \neq 0 \end{aligned} \qquad (27)$$

For convenience, then, the system (26) will be rewritten in terms of the quantities given by (27), we get

$$(\lambda_{13}\mathrm{x} + \lambda_{23})\frac{\partial \eta}{\partial x} + (\lambda_{35}\mathrm{t} + \lambda_{36})\frac{\partial \eta}{\partial t} = 0$$

$$(\lambda_{13}\mathrm{x} + \lambda_{23})\frac{\partial \eta}{\partial y} - (\lambda_{15}\mathrm{xt} + \lambda_{16}\mathrm{x} + \lambda_{25}\mathrm{t} + \lambda_{26})\frac{\partial \eta}{\partial t} = 0 \qquad (28)$$

From the transformation (21) and the definition of $\alpha's, \beta's$ and $\lambda's$, we have

$$\lambda_{13} = \lambda_{23} = \lambda_{35} = \lambda_{36} = \lambda_{15} = \lambda_{25} = 0$$

This implies

$$\lambda_{13}\mathrm{x} + \lambda_{23} = 0$$

$$\lambda_{35}\mathrm{t} + \lambda_{36} = 0 \qquad (29)$$

Then condition (27) reduce to

$$\lambda_{16}x + \lambda_{26} \neq 0 \tag{30}$$

Applying equation (29) and (30) to equation (28) gives

(1) The first equation is identically satisfied.

(2) The second equation of (28) reduce to

$$\frac{\partial\eta}{\partial t} = 0 \tag{31}$$

For convenience, equations (28) can be rewritten in the form

$$(\lambda_{16}x + \lambda_{26})\frac{\partial\eta}{\partial x} = 0 \tag{32}$$

Again,

$$\frac{\partial\eta}{\partial x} = 0 \tag{33}$$

From equation (31) and (33) we have

$$\eta = f(y) \tag{34}$$

Without loss of generality the independent absolute invariant $\eta(y)$ in equation (34) may assume of the form

$$\eta = Ay \tag{35}$$

E. Absolute Invariant of Dependent Variables:

For the absolute invariant corresponding to the dependent variable ψ, N *and* U_e. A function $g_1(x,t,\psi)$ is absolute invariant of a two parameter group it satisfies the first order linear differential equations

$$(\alpha_1 x + \alpha_2)\frac{\partial g_1}{\partial x} + (\alpha_3 t + \alpha_4)\frac{\partial g_1}{\partial t} + (\alpha_5\psi + \alpha_6)\frac{\partial g_1}{\partial \psi} = 0$$

$$(\beta_1 x + \beta_2)\frac{\partial g_1}{\partial x} + (\beta_3 t + \beta_4)\frac{\partial g_1}{\partial t} + (\beta_5\psi + \beta_6)\frac{\partial g_1}{\partial \psi} = 0 \tag{36}$$

The solution of equation (5.36) gives

$$g_1(x,t,\psi) = \phi_1(\psi/\omega_1(x,t)) = F(\eta) \tag{37}$$

In similar manner, we get

$$g_2(x,t,N) = \phi_2(N/\omega_2(x,t)) = G(\eta) \tag{38}$$

$$g_3(x,t,U_e) = \phi_3(U_e/\omega_3(x,t)) = E(\eta) \tag{39}$$

Where $\omega_1(x,t), \omega_2(x,t)$ *and* $\omega_3(x,t)$ are functions to be determined.

Without loss of generality, the $\phi's$ in (37), (.38) and (39) are selected to be the identity functions. ηThen we can express the function $\psi(x,y,t), N(x,y,t)$*and* $U_e(x,y,t)$ in terms of the absolute invariants $F(\eta)$*and* $G(\eta)$ in the form

$$\psi(x,y,t) = \omega_1(x,t)F(\eta) \tag{40}$$

$$N(x,y,t) = \omega_2(x,t)G(\eta) \tag{41}$$

$$U_e(x,t) = \omega_3(x,t)E(\eta) \tag{42}$$

Since $\omega_3$is independent of y, whereas η depends on y, it follow that E in (42) must be equal to constant. Then (42) becomes

$$U_e(x,t) = U_0\omega_3(x,t) \tag{43(a)}$$

Here, we assuming that, $U_e(x,t) = U_e(x) = ax$ (43(b))

The forms of the functions $\omega_1\ and\ \omega_2$are those for which the governing equations (6) and (8) reduce to ordinary differential equations.

4. THE REDUCTION TO ORDINARY DIFFERENTIAL EQUATION:

Here, we will consider two different cases to reduced equation (6) and (7) into ordinary differential equations.

(1) Steady case:

In this case, the velocity does not depend on time. So, as a special case of present study, we can take $\omega_1(x), \omega_2(x)$ and A in the form

$$\omega_1(x) = ax\sqrt{\upsilon/a}$$

$$\omega_2(x) = ax\sqrt{a/\upsilon} \tag{44}$$

$$A = \sqrt{a/\upsilon}$$

Then $\eta, \psi\ and\ N$ become

$$\eta = \sqrt{a/\upsilon}\ y$$

$$\psi = ax\sqrt{\upsilon/a}\ F(\eta)$$

$$N = ax\sqrt{a/\upsilon}\ G(\eta) \tag{45}$$

The equation (6) and (7) gives,

$$n(1+\Delta)(\omega_1)^{n-1}(F'')^{n-1}F''' + \Delta G' + FF'' - (F')^2 + m(1-F') + 1 = 0 \tag{46}$$

Where, $m = \frac{\upsilon}{ak_1}$ is permeability parameter.

$$F'\mathrm{G} - FG' - \frac{\lambda}{\upsilon}G'' + \frac{\sigma}{a}(2G + F'') = 0 \tag{47}$$

Where, $\frac{\lambda}{\upsilon} = p\ and\ \frac{\sigma}{a} = q$ (Dimensionless nos.)

$$pG'' + [FG' - F'\mathrm{G}] - q(2G + F'') = 0 \tag{48}$$

With boundary conditions

$$F(0) = 0,\ F'(0) = 0,\ G(0) = -\beta F''(0)$$

$$F \to 1,\ G \to 0\ as\ n \to \infty$$

(2) Unsteady case:

In this case, the velocity varies with respect to time.

So, we can take $\omega_1(x,t), \omega_2(x,t)$ and A in the form

$$\omega_1(x,t) = 2\sqrt{\upsilon t}\ ax$$

$$\omega_2(x,t) = \frac{ax}{2\sqrt{\upsilon t}} \tag{49}$$

$A = \frac{1}{2\sqrt{\upsilon t}}$

From equation (49), we will introduced the transformation for the dimension less stream function F, the dimensionless micro-rotation function G and a pseudo-similarity η which are given by

$$\psi = 2\sqrt{\upsilon t}\; ax\, F(\eta, t)$$

$$N = \frac{ax}{2\sqrt{\upsilon t}}\; G(\eta, t)$$

$$\eta = \frac{y}{2\sqrt{\upsilon t}} \tag{50}$$

$$\tau = 2\sqrt{at}$$

The outcome of this transformation is that the equations (6) and (7) become

$$(1+\Delta)n(w_2)^{n-1}\left(\frac{\partial^2 F}{\partial \eta^2}\right)^{n-1}\frac{\partial^3 F}{\partial \eta^3} + \Delta\,\frac{\partial G}{\partial \eta} + 2\eta\frac{\partial^2 F}{\partial \eta^2} - 2\tau\frac{\partial^2 F}{\partial \eta \partial \tau} + \tau^2\left\{1 + F\,\frac{\partial^2 F}{\partial \eta^2} - \left(\frac{\partial F}{\partial \eta}\right)^2 + m\left(1 - \frac{\partial F}{\partial \eta}\right)\right\} = 0 \tag{51}$$

$$P\,\frac{\partial^2 G}{\partial \eta^2} + 2\eta\,\frac{\partial G}{\partial \eta} - 2\tau\,\frac{\partial G}{\partial \tau} + \tau^2\left(F\,\frac{\partial G}{\partial \eta} - G\,\frac{\partial F}{\partial \eta} - Q\left(2G + \frac{\partial^2 F}{\partial \eta^2}\right)\right) = 0 \tag{52}$$

Where, $\frac{\lambda}{\upsilon} = P\ and\ \frac{\sigma}{a} = Q$ (Dimensionless nos.) And the boundary conditions (8) transforms to

$$F = \frac{\partial F}{\partial \eta} = 0, G = -\beta\,\frac{\partial^2 F}{\partial \eta^2}\ on\ \eta = 0$$

$$\frac{\partial F}{\partial \eta} \to 1, G \to 0\ as\ \eta \to \infty \tag{53}$$

5. RESULTS AND CONCLUSION:

In this work, we have presented similarity equations that can arise from the boundary layer flow of a micro polar non-Newtonian power law fluid through a porous medium. The carried out analysis, shows the effectiveness of the method in obtaining invariant solutions for the system of partial differential equations. The main difficulty of the problem is due the nonlinear boundary conditions, and consequently the analysis of the problem faces many difficulties that were an essential obstacle in many other analytical methods.

In this problem, we have used via-chain rule under two parameter group of transformations to reduce the number of independent variables of the problem by two and consequently the governing partial differential equations with the boundary conditions to ordinary differential equations with the appropriate corresponding boundary conditions. The ordinary differential equations so derived are the aim of this work.

It is hoped that the present work finds it significant role in many engineering applications. The possible forms of ordinary differential equation cannot be solved analytically, but one can easily obtained numerical solution using a fourth-order Runge-Kutta scheme.

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