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ANALYSIS OF A BATCH ARRIVAL $M^X/G/1$ QUEUE WITH SINGLE WORKING VACATION

M. Jemila Parveen* and M. I. Afthab Begum

Department of Mathematics, Avinashilingam Deemed University, Coimbatore – 641 043
E-mail: mjemilap@gmail.com, afthabau@hotmail.com

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ABSTRACT

In this paper, a batch arrival $M^X/G/1$ queue with exponentially distributed working vacations is analyzed. Using supplementary variable technique, the probability generating function of the steady state system size probabilities is derived and the expected system size probabilities are presented in closed form. Further, the results obtained are illustrated numerically and the effect of system parameters on system performance measures is discussed. Some particular cases are also discussed.

2000 Mathematics subject classification: *60K25*

Keywords: *$M^X/G/1$ queue, single working vacation, supplementary variable technique.*

INTRODUCTION:

Queueing system with server vacations is useful to model a system in which the server has additional task during vacation is eminent. Thus, it has wide applicability in analyzing the performance of computer systems, data communication networks and production systems. During the last two decades, the queueing systems with vacations have been studied extensively. The details can be seen in the monographs of Takagi [15], the surveys of Doshi[4,5] and Tegham[16]. In these studies, it is assumed that the server stops primary service completely during the vacations. In 2002, Servi and Finn [14] according to which a customer is served at a lower rate during vacations. They have analyzed an M/M/1 working vacations queue in which the server works at a different rate rather than completely stopping the main service during vacation. They tried to approximate a multi-queue system whose service rate is one of two service speeds such that the fast speed mode cyclically moves from queue to queue with exhaustive service. They tried to apply the M/M/1 working vacation queue, to model a wave length division multiplexing (WDM) optical access network using multiple wave lengths which can be reconfigured. Subsequently, Kim et al [8] have analyzed the M/G/1 queue with exponentially distributed working vacations and obtained the steady-state queue length distribution through the decomposition approach. Later Wu and Takagi[17] extended Servi and Finn's model to an M/G/1 working vacation in which, both service times – regular service and the service in working vacation are assumed to be generally distributed. An imbedded Markov chain that describes the queue size process in an M/G/1 working vacation queue is introduced and the probability generating function for the steady state queue size is derived. Later, based on Servi and Finn's model, Liu et al [11] gave explicit expressions of distributions for the stationary queue length and waiting time which have intuitionistic probability sense for M/M/1 multiple working vacation. Tian et al [13] in their paper studied an M/M/1 queue with single working vacation. Using quasi birth and death process and matrix geometric method, they have given the distributions for the number of customers and the virtual time in system in steady state. GI/M/1 queue with working vacations was studied by Baba [1] using Matrix geometric method. Baink et al [2] analyzed the finite GI/M/1 N queue with working vacations. Later Li and Tian[9] considered two types of discrete time GI/Geo/1 queues with working vacations and vacation interruption. Li et al[10] in their paper considered the M/G/1 queue with exponentially distributed working vacations, which is a special case of that in Wu and Takagi[17]. Later Liu et al[12] extended the M/M/1 working vacation model to bulk input model $M^X/M/1$ working vacations.

Recently, Jemila parveen et al[6] analyzed M/M/1 queue with working vacations and derived the steady state solutions in a closed form by directly solving the difference differential equations. Later they have discussed the waiting time distribution of an arbitrary customer for the model and verified the classical relation between PGF of queueing system and L.S.T of the waiting time distribution. The steady state results of M/M/1 working vacation are also extended to $M^X/M/1$ working vacations queueing model for both multiple and single vacations by Julia Rose Mary and Afthab begum [7].

Corresponding author:** *M. Jemila Parveen*,E-mail:** *mjemilap@gmail.com*

In this paper, we analyze the batch arrival Non Markovian $M^X/G/1$ queue with exponentially distributed single working vacation. Authors so far discussed the results through matrix geometric method and that too no authors have discussed Non-Markovian single working vacation as for the best of authors' knowledge is concerned. But supplementary variable technique helps to obtain the results in a easy manner and also in a closed form. The steady state distribution of the system size at departure epochs and arbitrary epochs are derived using supplementary variable technique and the results obtained are presented in a closed form. Further the system performance measures including expected system length and various probabilities are derived and illustrated numerically. Few results exist in literature for working vacations models are obtained as particular cases.

MODEL DESCRIPTION:

We consider $M^X/G/1$ queueing systems in which the customers arrive according to the Compound Poisson process with the random arrival size X with arrival rate λ. The number of units arriving at an arrival instant is a random variable X, with the probabilities Pr(X=k) = g_k, k=1,2…The server begins a working vacation whenever the system becomes empty and the vacation duration V follows an exponential distribution with parameter η. During working vacation an arriving customer is served at a rate of μ_v, and at the end of the vacation, if the server finds customers waiting in the queue, the server changes his service rate from μ_v to regular service rate μ_b, and a regular busy period begins. On the other hand, at the end of vacation, if the server finds the system empty, then he joins the system and stays idle in the system until a batch of customers arrives. Thus, working vacation is an operation period in a lower rate. When the number of customers in the system is relatively few, we set a lower rate operation period in order to economize operation cost together with serving customers. Therefore, this single working vacation policy has practical significance in optimal design of the system. It is assumed that the remaining regular service time ($S_b^o(t)$) and remaining service time during vacation ($S_v^o(t)$) are supplementary variables following general distributions with finite mean and variance and they are independent of each other and also independent of the arrivals.

SYSTEM SIZE DISTRIBUTION:

In this section to derive the steady state system size equations, the following notations and probabilities are defined.

λ : group arrival rate
X: group arrival size random variable
g_k : Pr(X=k), k=1,2,3..
X (z): Probability generating function (PGF) of X

Let N(t) denote the system size including the one in service at time t.

Y(t) = {0, if the server is idle in vacation at time t ; 1, if the server is idle in the system at time t ; 2, if the server is busy in vacation with lower service rate at time t ; 3, if the server is busy with regular service rate at time t}.

Q_0 (t) = Pr (N (t) = 0, Y (t) = 0),
P_0 (t) = Pr (N (t) = 0, Y (t) = 1),
Q_n(x, t) = Pr (N (t) = n, Y (t) = 2, x $\le S_v^0(t) \le$ x + dt), n ≥ 1
P_n(x, t) = Pr (N (t) = n, Y (t) = 3, x $\le S_b^0(t) \le$ x + dt), n ≥ 1

Thus Q_0 and P_0 gives the probability that the server is idle in vacation and in system respectively at time t. Q_n(x, t) and P_n(x, t) gives the probability that there are n customers in the system while the server is serving at lower service rate and in regular service rate respectively at time t.

Assuming the steady state probabilities $Q_0 = \underset{t\to\infty}{lt}\, Q_0(t)$, $Q_n(x,t) = \underset{t\to\infty}{lt}\, Q_n(x,t)$, $P_n(x,t) = \underset{t\to\infty}{lt}\, P_n(x,t)$, n ≥ 1, $\frac{\partial}{\partial t}P_n(x,t) = \frac{\partial}{\partial t}Q_n(x,t) = 0$ exist and following the arguments of Cox[3] and observing the changes of states in the interval (t, t+Δ t) at any time t, we obtain the following steady state system size equations.

$$\lambda P_0 = \eta Q_0 \tag{1}$$

$$(\lambda+\eta)Q_0 = P_1(0) + Q_1(0) \tag{2}$$

$$-\frac{d}{dt}Q_1(x) = -(\lambda+\eta)Q_1(x) + Q_2(0)s_v(x) + \lambda g_1 Q_0 s_v(x)$$

$$-\frac{d}{dt}Q_n(x) = -(\lambda+\eta)Q_n(x) + Q_{n+1}(0)s_v(x) + \lambda g_n Q_0 s_v(x) + \lambda\sum_{k=1}^{n-1} Q_{n-k}(x)g_k, n \geq 2$$

$$-\frac{d}{dt}P_1(x) = -\lambda P_1(x) + P_2(0)s_b(x) + \lambda g_1 P_0 s_b(x) + \int_0^\infty Q_1(y)dy\,\eta s_b(x)$$

$$-\frac{d}{dt}P_n(x) = -\lambda P_n(x) + P_{n+1}(0)s_b(x) + \lambda g_n P_0 s_b(x) + \int_0^\infty Q_n(y)dy\,\eta s_b(x) + \lambda\sum_{k=1}^{n-1} P_{n-k}(x)g_k, n \geq 2$$

For further simplification, we define the following L.S.T

$Q_n^* = \int_0^\infty e^{-\theta x}Q_n(x)dx$ and $P_n^* = \int_0^\infty e^{-\theta x}P_n(x)dx$.

Taking the L.S.T on both sides of the above equations, we have

$$\theta Q_1^*(\theta) - Q_1(0) = (\lambda+\eta)Q_1^*(\theta) - Q_2(0)S_v^*(\theta) - \lambda Q_0 g_1 S_v^*(\theta) \tag{3}$$

$$\theta Q_n^*(\theta) - Q_n(0) = (\lambda+\eta)Q_n^*(\theta) - Q_{n+1}(0)S_v^*(\theta) - \lambda Q_0 g_n S_v^*(\theta) - \lambda\sum_{k=1}^{n-1} Q_{n-k}^*(\theta)g_k, n \geq 2 \tag{4}$$

$$\theta P_1^*(\theta) - P_1(0) = \lambda P_1^*(\theta) - P_2(0)S_b^*(\theta) - \lambda P_0 g_1 S_b^*(\theta) - \int_0^\infty Q_1(y)dy\,\eta S_b^*(\theta) \tag{5}$$

$$\theta P_n^*(\theta) - P_n(0) = \lambda P_n^*(\theta) - P_{n+1}(0)S_b^*(\theta) - \lambda P_0 g_n S_b^*(\theta) - \int_0^\infty Q_n(y)dy\,\eta S_b^*(\theta) - \lambda\sum_{k=1}^{n-1} P_{n-k}^*(\theta)g_k, n \geq 2 \tag{6}$$

STEADY STATE SOLUTIONS:

In order to derive the distribution of the system size probabilities, we define the following pgfs.

$Q_1^*(z,\theta) = \sum_{n=1}^\infty Q_n^*(\theta)z^n$, $Q_1(z,0) = \sum_{n=1}^\infty Q_n(0)z^n$, $P_V^*(z,\theta) = Q_1^*(z,\theta) + Q_0$, $P_B^*(z,\theta) = \sum_{n=1}^\infty P_n^*(\theta)z^n$ and $P_B(z,0) = \sum_{n=1}^\infty P_n(0)z^n$

Multiplying equations (3) and (4) by the proper powers of z and summing up over n= 1 to ∞, we get

$$\theta Q_1^*(z,\theta) - Q_1(z,0) = (\lambda+\eta)Q_1^*(z,\theta) - \frac{S_v^*(\theta)}{z}(Q_1(z,0) - Q_1(0)z) - \lambda X(z)Q_0 S_v^*(\theta) - \lambda\sum_{n=2}^\infty z^n\left(\sum_{k=1}^{n-1} Q_{n-k}^*(\theta)g_k\right)$$

Using the identity $\sum_{n=2}^\infty z^n\left(\sum_{k=1}^{n-1} Q_{n-k}^*(\theta)g_k\right) = X(z)Q_1^*(z,\theta)$, the above equation becomes

$$(\theta - h_X(z))Q_1^*(z,\theta) = Q_1(z,0)(1 - \frac{S_v^*(\theta)}{z}) - S_v^*(\theta)(\lambda X(z)Q_0 - Q_1(0))$$

At $\theta = h_X(z) = \eta + \lambda(1 - X(z))$, $Q_1(z,0) = \frac{zS_v^*(h_X(z))}{z - S_v^*(h_X(z))}(\lambda X(z)Q_0 - Q_1(0))$.

By similar argument of Li et al[10], the unique root z_1 of $z - S_v^*(h_X(z))$ lies inside (0,1).

Therefore, $Q_1(0) = \lambda X(z_1)Q_0$ (7)

Substituting for $Q_1(0)$ in $Q_1(z, 0)$, $Q_1(z,0) = \dfrac{\lambda z Q_0 S_v^*(h_X(z))(X(z) - X(z_1)}{z - S_v^*(h_X(z))}$ (8)

And $Q_1^*(z,\theta) = \dfrac{\lambda z Q_0 (X(z) - X(z_1)(S_v^*(h_X(z)) - S_v^*(\theta))}{(\theta - h_X(z))(z - S_v^*(h_X(z))))}$

At θ=0, $Q_1^*(z,0) = \dfrac{\lambda z Q_0 (X(z) - X(z_1)(1 - S_v^*(h_X(z)))}{h_X(z)(z - S_v^*(h_X(z))))}$ (9)

Similarly multiplying equations (5) and (6) by appropriate powers of z and then adding, we have

$$\theta P_B^*(z,\theta) - P_B(z,0) = \lambda P_B^*(z,\theta) - \frac{S_b^*(\theta)}{z}(P_B(z,0) - P_1(0)z) - \lambda X(z)P_0 S_b^*(\theta) \quad \sum_{n=1}^{\infty}\int_0^{\infty} Q_n(y)dyz^n = \sum_{n=1}^{\infty} Q_n^*(0)z^n = Q_1^*(z,0),$$

we have

$$(\theta - w_X(z))P_B^*(z,\theta) = P_B(z,0)(1 - \frac{S_b^*(\theta)}{z}) - S_b^*(\theta)(\eta Q_1^*(z,0) + \lambda X(z)P_0 - P_1(0))$$

At $\theta = w_X(z) = \lambda(1 - X(z))$,

$$P_B(z,0) = \frac{z S_b^*(w_X(z))}{z - S_b^*(w_X(z))}(\eta Q_1^*(z,0) + \lambda X(z)P_0 - P_1(0)) \tag{10}$$

And $P_B^*(z,\theta) = \dfrac{z(S_b^*(w_X(z)) - S_b^*(\theta))}{(\theta - w_X(z))(z - S_b^*(w_X(z)))}(\eta Q_1^*(z,0) + \lambda X(z)P_0 - P_1(0))$ (11)

Equation (2) implies $(\lambda + \eta)Q_0 = P_1(0) + Q_1(0) = P_1(0) + \lambda X(z)Q_0$ (from (7))

Therefore $P_1(0) = Q_0(\eta + \lambda(1 - X(z_1)))$ (12)

Substituting P_1 (0) and $Q_1^*(z,0)$ in equation (11) and on further simplification

$$P_B^*(z,\theta) = \frac{Q_0 z(S_b^*(w_X(z)) - S_b^*(\theta))}{(\theta - w_X(z))(z - S_b^*(w_X(z)))}[\frac{z\lambda\eta(X(z) - X(z_1))(1 - S_v^*(h_X(z)))}{h_X(z)(z - S_v^*(h_X(z)))} - \eta(1 - X(z)) - \lambda(1 - X(z_1))]$$

And at $\theta = 0$,

$$P_B^*(z,\theta) = \frac{Q_0 z(1 - S_b^*(w_X(z)))}{w_X(z)(z - S_b^*(w_X(z)))}[\frac{z\lambda\eta(X(z) - X(z_1))(1 - S_v^*(h_X(z)))}{h_X(z)(z - S_v^*(h_X(z)))} - \eta(1 - X(z)) - \lambda(1 - X(z_1))] \tag{13}$$

Thus the total PGF P(z) of the system size probabilities is given by

$$P(z) = P_B^*(z,0) + P_v^*(z,0) + P_0$$

Using the normalizing condition P(1)=1, $Q_0 = \dfrac{1 - \rho_b}{\dfrac{h_X(z_1)}{\eta} + \dfrac{\eta}{\lambda} - \dfrac{\rho_b(1 - X(z_1))S_v^*(\eta)}{E(X)(1 - S_v^*(\eta))}}$

MEAN SYSTEM LENGTH:

Let L_v and L_b denotes the mean system size during the working vacation and regular busy respectively. Then

$$L_v = \frac{d}{dz} P_v^*(z,0)_{z=1}$$
$$= \lambda Q_0 [\frac{E(X) h_X(z_1)}{\eta^2} - \frac{(1-X(z_1))S_v^*(\eta)}{\eta(1-S_v^*(\eta))}]$$

$$L_b = \frac{d}{dz}(P_b^*(z,0)+P_0)_{z=1}$$
$$= \left[\frac{\rho_b}{1-\rho_b} + \frac{\lambda E(X(X-1))E(S_b) + (\lambda E(X))^2 E(S_b^2)}{2(1-\rho_b)^2}\right]\left[\frac{h_X(z_1)}{\eta} + \frac{(X(z_1)-1)S_v^{\cdot}(\eta)}{1-S_v^{\cdot}(\eta)} + \frac{\eta}{\lambda}\right]$$
$$+ \frac{\rho_b}{1-\rho_b}\left[\frac{\lambda E(X(X-1))}{2(\lambda E(X))^2}\left(\frac{\lambda(1-X(z_1))S_v^{*}{}'(\eta)}{1-S_v^*(\eta)}\right) - \frac{1}{E(X)}\left(\frac{(1-X(z_1))(1+\lambda E(X)S_v^{*}{}'(\eta))}{(1-S_v^{\cdot}(\eta))^2}\right.\right.$$
$$\left.\left. - \frac{E(X)h_X(z_1)}{\eta(1-S_v^*(\eta))} - \frac{1-X(z_1)}{1-S_v^*(\eta)} + \frac{h_X(z_1)(\lambda E(X))^2}{\eta^2}\right)\right]$$

Hence the mean system size of the model L is given by $L=L_v+L_b$.

Other Performance measures:

- Probability that the server is on vacation (P_v) is given by $P_v = \underset{z\to 1}{lt}\, Q_1^*(z,0) = \frac{Q_0 h_X(z_1)}{\eta}$
- Probability that the server is busy (P_b) is given by

$$P_b = \underset{z\to 1}{lt}\, P_b^*(z,0) = \frac{Q_0}{\mu_b(1-\rho_b)}\left[\lambda E(X)\left(\frac{\eta}{\lambda} + \frac{h_X(z_1)}{\eta}\right) + \frac{\lambda(X(z_1)-1)S_v^*(\eta)}{1-S_v^*(\eta)}\right]$$

- Probability that the server is idle (P_I) is given by $P_I = \underset{z\to 1}{lt}\, P_0 = \frac{\eta Q_0}{\lambda}$

PARTICULAR CASES:

In this section, the steady state results of $M^X/M/1$ [7] and M/G/1 are deduced as particular cases of the model.

1. **M^X/M/1 single working vacation** [7]:

If both the services – regular service and service during working vacation follow exponential distribution, then $S_b^*(w_X(z)) = \frac{\mu_b}{\mu_b + w_X(z)}$, $S_v^*(h_X(z)) = \frac{\mu_v}{\mu_v + h_X(z)}$ and $P_{00}^s = Q_0$

$Q_1^*(z,0) = \frac{\mu_v(z-z_1)Q_0}{z_1[\mu_v(z-1)+zh_X(z)]}$ $=P_0(z)$ of M^X/M/1 single working vacation

$$P_b^*(z,0) = \frac{Q_0}{[\mu_b(z-1)+zw_X(z)]}[\frac{\mu_v\lambda z((z-1)z_1(X(z_1)-1)-(z_1-1)z(X(z)-1))}{z_1[\mu_v(z-1)+zh_X(z)]} - \eta z(1-X(z))]$$

$= P_1(z)$ of M^X/M/1 single working vacation

2. **M/G/1 Single working vacation:**

When both regular service time and service time during working vacation follow other than exponential distribution and by taking X(z)=z i.e, single arrival, the probability generating functions of the M/G/1 SWV model is deduced as follows.

$$Q_1^*(z,0) = \frac{\lambda z Q_0 (z-z_1)(1-S_v^*(h(z)))}{h(z)(z-S_v^*(h(z)))}$$

$$P_b^*(z,0) = \frac{zQ_0(1-S_b^*(w(z)))}{w(z)(z-S_b^*(w(z)))}\left[\frac{\eta\lambda(z-z_1)(1-S_v^*(h(z)))}{h(z)(z-S_v^*(h(z)))} - \eta(1-z) - \lambda(1-z_1)\right]$$

$$L_v = \underset{z\to 1}{lt}\, P_v^*(z,0) = \frac{h(z_1)}{\eta} \text{ and } L_b = \underset{z\to 1}{lt}\, P_b^*(z,0) = \frac{\rho_b Q_0}{1-\rho_b}\frac{\eta}{\lambda} + \frac{h(z_1)}{\eta} + \frac{(z_1-1)S_v^*(\eta)}{1-S_v^*(\eta)}$$

NUMERICAL ANALYSIS:

In this section, we present some numerical examples to explain the influence of various parameters such as mean vacation time ($1/\eta$), mean regular service ($1/\mu_b$) and mean service during working vacation ($1/\mu_v$) on mean system size (L) and on various probabilities. For the computation, the batch arrival is assumed to follow geometric distribution and service times follow Erlang k distributions.

In the classical vacation models, since the service is stopped completely during vacation, the system size increase notably as the mean vacation time increases. But in working vacation, since the service is done with a smaller rate $\mu_v (< \mu_b)$ during vacation, the vacation parameter η have less effect on the system size. The effects of η and μ_v on the expected mean system size under two situations ($\rho_b = 0.3$ and $\rho_b = 0.6$) are presented in table 1. The table 1 values also show that as μ_v or η increases, the mean system size decrease. Also we infer that as μ_v approaches to 0, the system size of single working vacation model (SWL) approaches the system size of the corresponding classical single vacation model (CSL). The data's in table 2 shows the effect of traffic intensity (ρ_b) on the probabilities including probability that the server is idle (P_I), on vacation (P_v) and on regular busy (P_b). In all the above discussion we fix μ_b =1. Figure 1 is the graphical representation of table 2.

Table: 1 L Vs μ_v Vs η

μ_v	η	$\rho_b = 0.3$		$\rho_b = 0.6$	
		L_{SWV}	L_{SV}	L_{SWV}	L_{SV}
0.25	0.5	2.4592	4.0917	14.7156	7.1871
	1	4.3912	1.5378	18.3425	3.5917
	1.5	6.3348	1.1312	23.3371	3.1052
	2	11.4063	0.9949	29.2266	2.9532
0.5	0.5	1.9542	4.0917	11.0570	7.1871
	1	4.1285	1.5378	16.6471	3.5917
	1.5	6.1732	1.1312	22.3264	3.1052
	2	8.1750	0.9949	28.0163	2.9532
0.75	0.5	1.6429	4.0917	8.6696	7.1871
	1	3.9115	1.5378	15.2405	3.5917
	1.5	6.0174	1.1312	21.3713	3.1052
	2	8.0587	0.9949	27.3205	2.9532

Table: 2 Probabilities Vs Traffic Intensity

ρ	P_b	P_v	P_I
0.1	0.0864	0.5353	0.3963
0.2	0.1283	0.6863	0.1854
0.3	0.2043	0.7021	0.0936
0.4	0.2970	0.6530	0.0500
0.5	0.4019	0.5703	0.0278
0.6	0.5146	0.4698	0.0157
0.7	0.6322	0.3592	0.0087
0.8	0.7529	0.2427	0.0044
0.9	0.8757	0.1225	0.0017

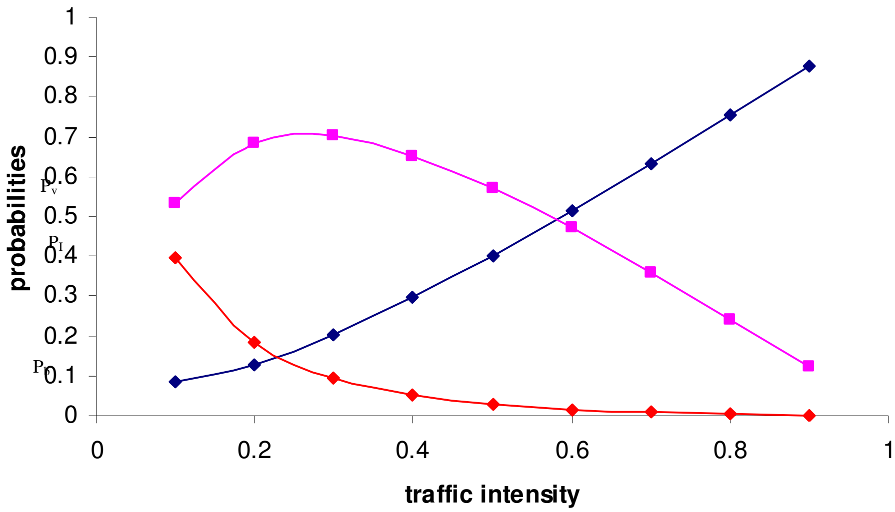

Figure: 1 Various probabilities with traffic intensity

CONCLUSION:

Over the past two decades, queueing systems with vacations have been studied by many researchers and have been applied to many situations. Working vacation is a new concept introduced by Servi and Finn [14] and the model discussed till now are M/M/1, M/G/1 multiple working vacations, GI/M/1 using matrix geometric method. In this paper we have made an attempt to discuss a batch arrival $M^X/G/1$ queue under single working vacation using supplementary variable technique and derived the steady state results in closed form. Various system performance measures are deduced from it. Further few results existing in literature are obtained as particular cases. Finally numerical examples are presented to justify the measure and to understand the model in a better way. The model analyzed in this paper may be extended to the model including the system with second optional service, with/without breakdowns, bulk service models, etc.

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