



EFFECT OF HEAT GENERATION AND RADIATION ON MHD BOUNDARY LAYER FLOW IN A POROUS VERTICAL FLAT PLATE

B. Rushi Kumar*

Fluid Dynamics Division, School of Advanced Sciences, VIT University, Vellore, INDIA

E-mail: rushikumar@vit.ac.in

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ABSTRACT

In this paper, the effect of heat generation and radiation parameters on momentum, heat and mass transfer flow in a porous vertical flat plate in the presence of a magnetic field has been investigated numerically. By using the similarity transformation, the governing partial differential equations of the problem are transformed to nonlinear ordinary differential equations. Then the numerical solution of the problem is obtained using Shooting method with the fourth order Runge-Kutta integration scheme. The physical behavior of different parameters for velocity, temperature and concentration has been examined graphically and analyzed quantitatively. Comparisons with previously published work are performed and the results are found to be in very good agreement.

Key words: Thermal Radiation, Heat generation, Magnetic field, Similarity Solution

INTRODUCTION:

The impotence of radiation effect on MHD flow and heat transfer problems has found increasing attention in industries. At high operating temperature, radiation effect can be quite significant. Many process in engineering areas occur high temperatures and knowledge of radiation heat transfer becomes very important for the design of pertinent equipment. Nuclear power plants, gas turbines and the various propulsion devices for aircraft, missiles, satellites and space vehicles are examples of such engineering areas. In view of this Raptis et al. [18] analyzed the radiative flow in the presence of a magnetic field while Cortell [5, 6] studied the effects of thermal radiation on several distinct boundary layers. Cortell [7, 8] further studied radiation effects on Blasius and Sakiadis flows when plate is maintained at a constant temperature. He determined the effects of physical parameters like Prandtl number (Pr) and radiation parameter (N_R) on heat transfer characteristics. Makinde et al. [16] analyzed the effect of thermal radiation on the heat and mass transfer flow of a variable viscosity fluid past a vertical porous plate permeated by a transverse magnetic field.

The heat source/sink effects in thermal convection are significant where there may exist high temperature differences between the surface (e.g. space craft body) and the ambient fluid. Heat generation is also important in the context of exothermic or endothermic chemical reaction. Tania et al (19) has investigated the Effects of radiation, heat generation and viscous dissipation on MHD free convection flow along a stretching sheet. Furthermore, Moalem [17] studied the effect of temperature dependent heat sources taking place in electrically heating on the heat transfer within a porous medium. Vajravelu and Nayfeh [20] reported on the hydro magnetic convection at a cone and a wedge in the presence of temperature dependent heat generation or absorption effects. Moreover, Chamkha [4] studied the effect of heat generation or absorption on hydro magnetic three-dimensional free convection flow over a vertical stretching surface.

Boundary layer flows induced over flat-plates by uniform free streams are well known in literature as Blasius problems. Howarth [10] first conducted hand computations using Runge-Kutta numerical methods for flat plate flows. Thereafter, many authors investigated various aspect of the problem. Blasius solution for flow past a flat plate was investigated by Abussita [1] and the existence of a solution was established. Asaithambi [2] presented a finite-difference method for the solution of the Falkner-Skan equation and recently, Wang [21] obtained an approximate solution for classical Blasius equation using Adomian decomposition method. Kuo [14] studied the solutions of thermal boundary layer problems for flow past flat-plates using differential transformation method.

***Corresponding author: B. Rushi Kumar*, *E-mail: rushikumar@vit.ac.in**

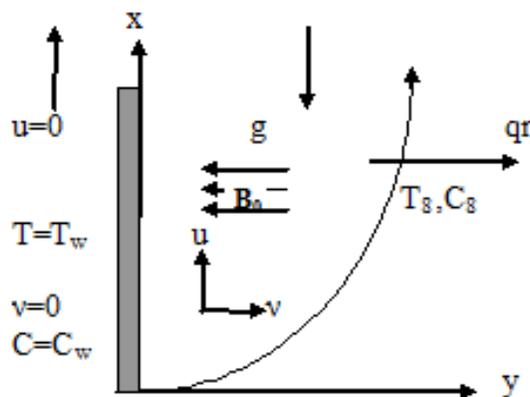


Figure 1. Physical configuration and coordinate system.

Makinde [15] investigated a convective flow with thermal radiation and mass transfer past a moving vertical porous plate and assumed a time-dependency for the vertical velocity. The resultant similarity equations were solved numerically using a superposition method. Ibrahim *et al* [11] analytically derived the heat and mass transfer of a chemical convective process assuming an exponentially decreasing suction velocity at the surface of a porous plate and a two terms harmonic function for the rest of the variables. Recently Ibrahim and Makinde [12] analyzed the radiation effect on chemically reacting MHD boundary layer flow of heat and mass transfer through a porous vertical flat plate with suction. More recently Elbashbeshy and Aldawody studied (9) Effects of thermal radiation and magnetic field on unsteady mixed convection flow and heat transfer over a porous stretching surface in the presence of internal heat generation/absorption. Makinde *et al* (13) studied effect of Radiation on MHD boundary layer flow of heat and mass transfer in a porous vertical flat plate.

The objective of the present investigation is to study the effects heat generation and radiation on MHD free convective flow in porous vertical plate. In the problem formulation, the continuity, momentum, energy and concentration equations are reduced to some parameter problem by introducing suitable transformation variables. The equations that govern the flow are coupled and solve numerically using shooting techniques with the fourth order Runga-Kutta integration scheme. The effects of various governing parameters on the velocity, temperature, concentration are presented graphically and discussed quantitatively

MATHEMATICAL ANALYSIS:

Consider a two-dimensional free convection effects on the steady incompressible laminar MHD heat and mass transfer characteristics of a radiated vertical plate, the velocity of the fluid far away from the plate surface is considered as free stream value. The variations of surface temperature and concentration are linear. The flow configuration and coordinate system are shown in Figure 1. All the fluid properties are assumed to be constant except for the density variations in the buoyancy force term of the linear momentum equation. The magnetic Reynolds number is assumed to be small, so that the induced magnetic field is neglected. No electrical field is assumed to exist. The Hall effects and the joule heating terms are also neglected. Under these assumptions, along with Boussinesq approximations, the boundary layer equations describing this flow as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} u + g \beta_r (T - T_\infty) + g \beta_c (C - C_\infty) \tag{2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y} + \frac{Q}{\rho c_p} (T - T_\infty) \tag{3}$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_m \frac{\partial^2 C}{\partial y^2} \tag{4}$$

Where If u, v, T and C are the fluid x -component of velocity, y -component of velocity, temperature and concentration respectively, ν is the fluid kinematics viscosity, ρ - the density, σ - the electric conductivity of the fluid, β_T and β_c - the coefficients of thermal and concentration expansions respectively, k - the thermal conductivity, C_∞ - the free stream concentration, B_0 - the magnetic induction, D_m - the mass diffusivity and g is the gravitational acceleration, α - the thermal diffusivity.

The boundary conditions at the plate surface and for into the cold fluid may be written as

$$v = 0, u = 0, T = T_w = T_\infty + ax, C = C_w = C_\infty + bx \text{ at } y = 0, \\ u \rightarrow u_\infty, T \rightarrow T_\infty, C \rightarrow C_\infty \text{ as } y \rightarrow \infty . \tag{5}$$

Where “a” and “b” denotes the stratification rate of the gradient of ambient temperature and concentration profiles.

We use the Rosseland approximation for radiation of an optically thick boundary layer given by Raptis et al. [18] and Cortel [7] in a simplified radiative heat flux form as

$$q_r = -\frac{4 \sigma^* \partial T^4}{3 k^* \partial y} \tag{6}$$

Where σ^* and k^* are the Stefan-Boltzmann constant and Rosseland mean absorption coefficient, respectively.

We assume that the temperature difference within the flow such as the term T^4 may be expressed as a linear function of temperature. Hence, expanding T^4 in a Taylor series about T_∞ (the fluid temperature at the free stream) and neglecting higher-order terms as in Bataller (3), we obtain

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \tag{7}$$

Substituting Equations (6) and (7) into Equation (3) in the appropriate form leads to

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \left(\alpha + \frac{16\sigma^* T_\infty^3}{3\rho c_p k^*} \right) \frac{\partial^2 T}{\partial y^2} + \frac{Q}{\rho c_p} (T - T_\infty) \tag{8}$$

where $\alpha = k / \rho c_p$ is the thermal diffusivity.

It is clear from Equation (8) that the effect of radiation is to enhance the thermal diffusivity. If we take,

$$N_R = \frac{k k^*}{4\sigma^* T_\infty^3} \text{ as the radiation parameter as in Bataller [3], Equation (8) can be rewritten as}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\alpha}{k_0} \frac{\partial^2 T}{\partial y^2} + \frac{Q}{\rho c_p} (T - T_\infty) \tag{9}$$

Where

$$k_0 = \frac{3N_R}{3N_R + 4} \tag{10}$$

We introduce the similarity variable η and the dimensionless stream function $f(\eta)$ as

$$\eta = y \sqrt{\frac{U_\infty}{\nu x}} = \frac{y}{x} \sqrt{\text{Re}_x}, f(\eta) = \frac{\psi(\eta)}{\sqrt{x\nu U_\infty}}, \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \tag{11}$$

Where $\theta(\eta), \phi(\eta)$ are dimensionless temperature and concentration of the fluid, η is the similarity variable. The velocity components u and v are respectively obtained as follows:

$$u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x}. \tag{12}$$

Equation (1) is satisfied simultaneously while equations (2), (3), and (4) reduced to equations (13), (14) and (15) respectively:

$$f''' + \frac{1}{2}ff'' + Gr\theta + Gc\phi - M(f' - 1) = 0 \tag{13}$$

$$\theta'' + \frac{Pr k_0}{2} f\theta' + Pr k_0 S\theta = 0 \tag{14}$$

$$\phi'' + \frac{Sc}{2} f\phi' = 0 \tag{15}$$

Where the prime symbol represents the derivative with respect to η . The boundary conditions are also transformed into the following:

$$\begin{aligned} f'(0) = 0, f(0) = 0, \theta(0) = 1, \phi(0) = 1 \\ f'(\infty) = 1, \theta(\infty) = \phi(\infty) = 0. \end{aligned} \tag{16}$$

Where

$$\begin{aligned} M = \frac{\sigma B_0^2 x}{\rho U_\infty} \text{ (The Magnetic parameter), } Gr = \frac{g \beta_r x (T_w - T_\infty)}{U_\infty^2} \text{ (The local temperature Grashof number),} \\ Gc = \frac{g \beta_c x (C_w - C_\infty)}{U_\infty^2} \text{ (The local casentration Grashof number), } Pr = \frac{\nu}{\alpha} \text{ (The Prandtl number),} \\ Sc = \frac{\nu}{D_m} \text{ (The Schmidt number), } S = \frac{xQ}{U_\infty} \text{ is the Heat generation (>0) or absorption parameter.} \end{aligned}$$

NUMERICAL PROCEDURE:

The set of equations (11) to (15) under the boundary conditions (16) have been solved numerically using the Runge-Kutta parameter integration scheme with a modified version of the Newton-Raphson shooting method. We let:

$$f = x_1, f' = x_2, f'' = x_3, \theta = x_4, \theta' = x_5, \phi = x_6, \phi' = x_7. \tag{17}$$

Equations (11) to (15) are transformed into systems of first order differential equations as follows:

$$\begin{aligned} x_1' &= x_2, \\ x_2' &= x_3, \\ x_3' &= -\frac{1}{2}x_1x_3 + M(x_2 - 1) - Grx_4 - Gcx_6, \\ x_4' &= x_5, \\ x_5' &= -\frac{1}{2}Pr k_0 x_1 x_5 - Pr k_0 S x_4, \\ x_6' &= x_7, \\ x_7' &= -\frac{1}{2}Sc x_1 x_7, \end{aligned} \tag{18}$$

Subject to the following initial conditions:

$$x_1(0) = 0, x_2(0) = 0, x_3 = s_1, x_4(0) = 1, x_5(0) = s_2, x_6(0) = 1, x_7(0) = s_3. \tag{19}$$

In a shooting method, the unspecified initial conditions: s_1, s_2 and s_3 in Equation (19) are assumed. Equation (18) is then integrated numerically as an initial valued problem to a given terminal point. The accuracy of the assumed missing initial condition is then checked by comparing the calculated value of the dependent variable at the terminal point with its given value there. If a difference exists, improved values of the missing initial conditions must be obtained and the process is repeated. The computations were done by a written program which uses a symbolic and computational computer language MATHEMATICA. A step size of $\Delta\eta = 0.001$ was selected to be satisfactory for a convergence criterion of 10^{-7} in nearly all cases. The maximum value of η_∞ , to each group of parameters Sc, M, Gr, Gc, Pr and S is determined when the values of unknown boundary conditions at $\eta = 0$ not change to successful loop with error less than 10^{-7} . From the process of numerical computation, the local skin friction coefficient, the local Nusselt number and the local Sherwood number, which are respectively proportional to $f''(0), -\theta'(0)$ and $-\phi'(0)$ are worked out and their numerical values presented in table -1 and table - 2.

RESULTS AND DISCUSSIONS:

Numerical results are reported in the tables 1-2. The Prandtl number was taken to be $Pr=0.72$ which corresponds to air, the value of Schmidt number (Sc) were chosen to be $Sc=0.24, 0.62, 0.78, 2.62$, representing diffusing chemical species of most common interest in air like H_2, H_2O, NH_3 and Propyl Benzene respectively. Attention is focused on positive value of the buoyancy parameters that is, local temperature Grashof number $Gr>0$ (which corresponds to the cooling problem)and local concentration Grashof number $Gc>0$ (which indicates that the chemical species concentration in the free stream region is less then the concentration at the boundary surface).In order to benchmark our numerical results, we have compared with $f''(0), -\theta'(0)$ and $-\phi'(0)$ in the absence of the Heat generation (>0) or absorption parameter with those of Ibrahim and Makinde [20] and found them in excellent agreement as demonstrated in table 1.From table 2, it is important to note that the local skin friction together with the local heat and mass transfer rate at the moving plate surface increases with increasing in the Heat generation (>0) or absorption parameter.

Figure 1 and 2 shows that when the Prandtl number, the Schmidt number increases, the velocity profile increases and decreases. However, increasing the thermal and mass Grashof number increases the velocity near the plate (Figure 3 and 4). This is as expected because near a heat radiating plate, molecules of the fluid have higher activity due to the energy absorbed. Increasing the radiation parameter also increases the velocity as illustrated in Figure 5. In all cases, the velocity at the plate surface is zero due to the ‘No slip’ condition. This increases to beyond the free stream value due to the radiated plate but settles down to the free stream value after some time. Further more Figure 6 shows when the heat generation parameter increases, the velocity profile increases.

Figures 7 to 11 illustrate the effect of various parameter values on the thermal boundary layer thickness. In Figure 7, increasing Prandtl number there is a pronounced increase in the thermal boundary layer, however, increase in thermal and mass Grashof number (Figures 8 & 9) respectively, causes a reduces in the thermal boundary layer thickness. Further more Figures 10 & 11 shows when the Radiation parameter and heat generation parameter increases, the temperature profile increases.

Figures 12 to 15 depict chemical species concentration profiles against span wise coordinate η for varying physical parameter values in the boundary layer region. The species concentration is highest at the plate surface and decreases to zero far away from the plate satisfying the boundary condition. It can be observed further that increasing the values of Heat generation parameter and the Schmidt number, the thermal and mass Grashof number reduce the concentration boundary layer (Figures 12 to 15).

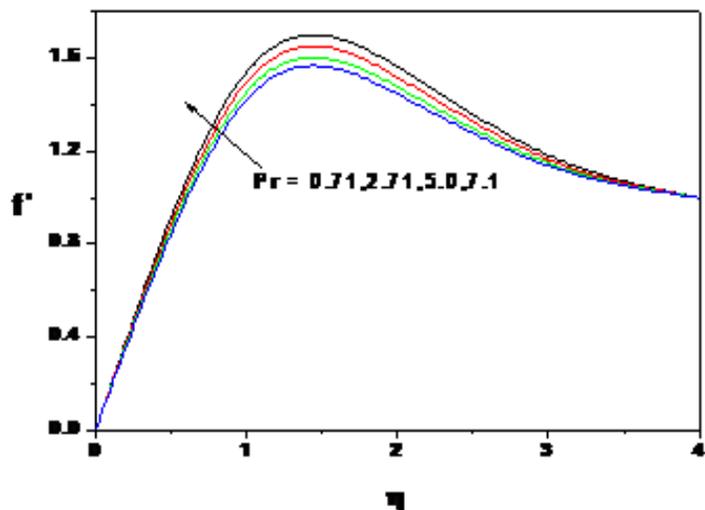


Fig.1: Variation of the velocity component f' with Pr for $M=0.1$, $Sc=0.24$, $Gr=Gc=1$, $S=1$, $N_R=0.1$.

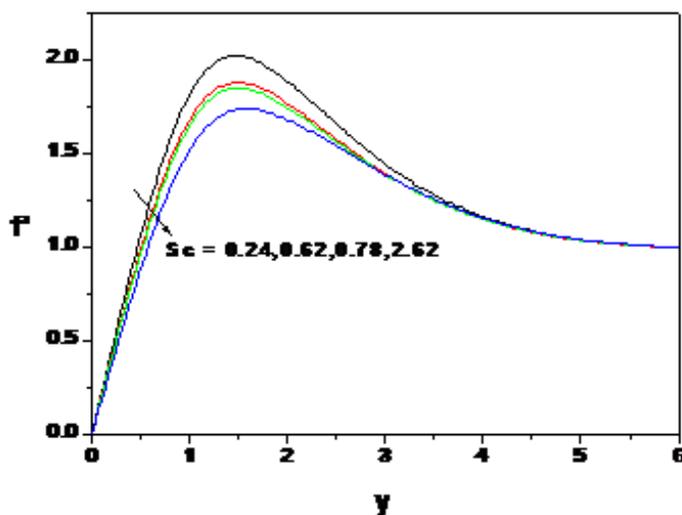


Fig.2: Variation of the velocity component f' with Sc For $Pr=0.71$, $Gr=Gc=1$, $S=1$, $N_R=0.1$, $M=0.1$.

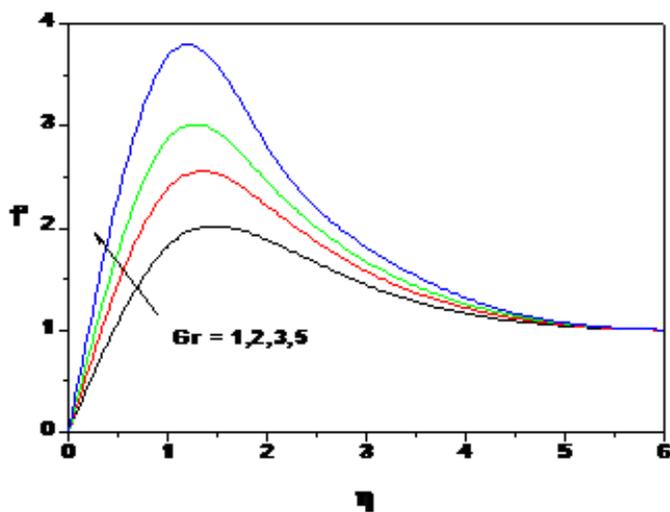


Fig.3: Variation of the velocity component f' with Gr For $Pr=0.71$, $Sc=0.24$, $Gc=1$, $N_R=0.1$, $S=1$, $M=0.1$.

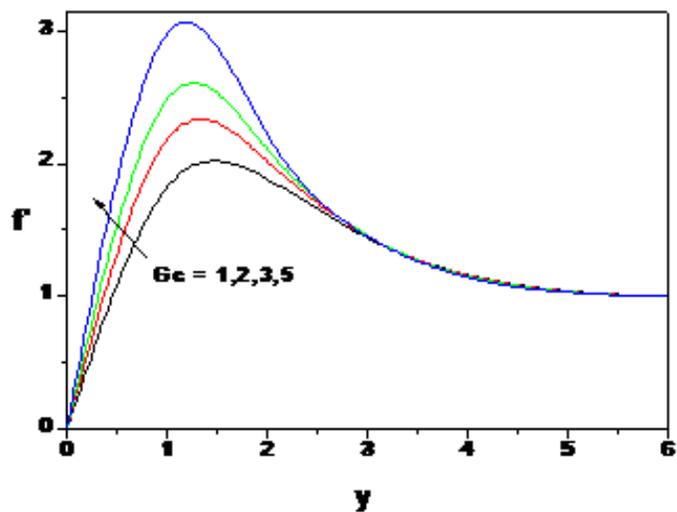


Fig.4: Variation of the velocity component f' with Gc for $Pr=0.71$, $Sc=0.24$, $Gr=1$, $S=1$, $N_R=0.1$, $M=0.1$.

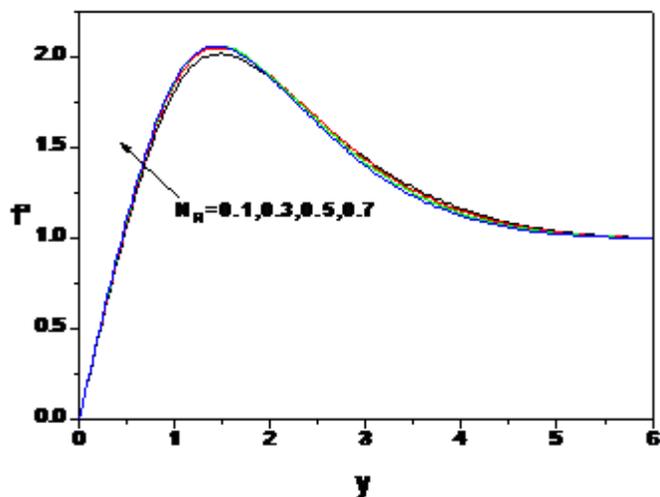


Fig .5: Variation of the velocity component f' with N_R for $Pr=0.71$, $Sc=0.24$, $Gr=Gc=1$, $S=1$, $M=0.1$.

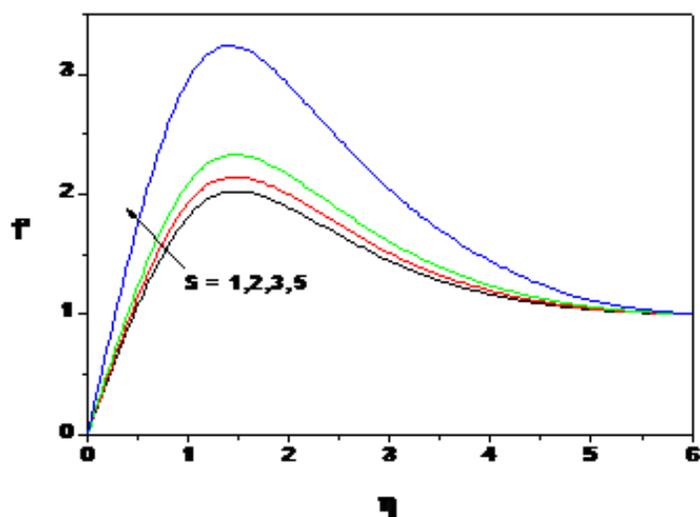


Fig .6: Variation of the velocity component f' with S for $Pr=0.71$, $Sc=0.24$, $Gr=Gc=1$, $N_R=0.1$, $M=0.1$.

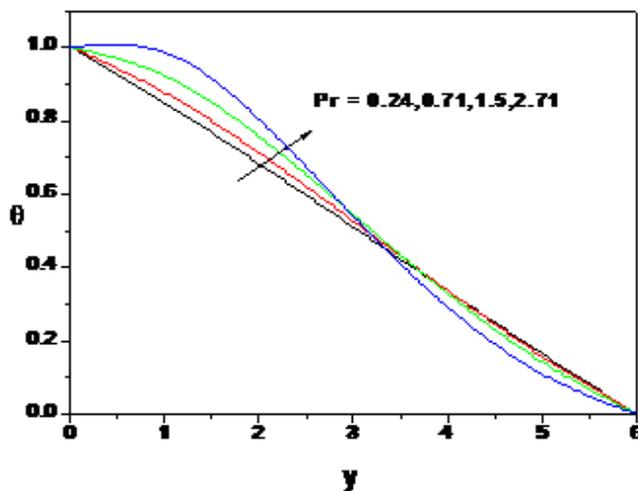


Fig.7: Variation of the temperature θ with Pr for Sc=0.24, Gc =Gr =1, S=1, N_R =0.1, M=0.1.

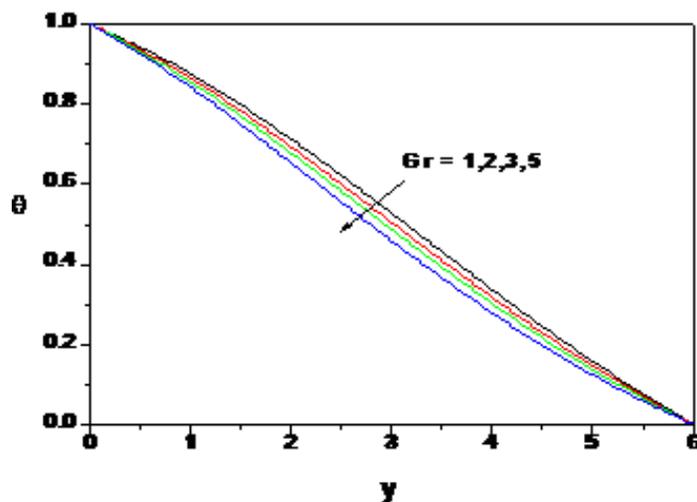


Fig.8: Variation of the temperature θ with Gr for Pr=0.71, Sc=0.24, Gc =1, S=1, N_R =0.1, M=0.1.

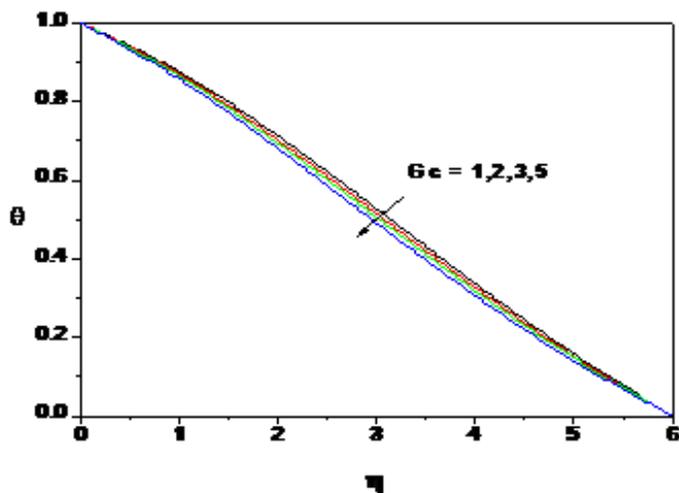


Fig.9: Variation of the temperature θ with Gc for Pr=0.71, Sc=0.24, Gr=1, S=1, N_R =0.1, M=0.1.

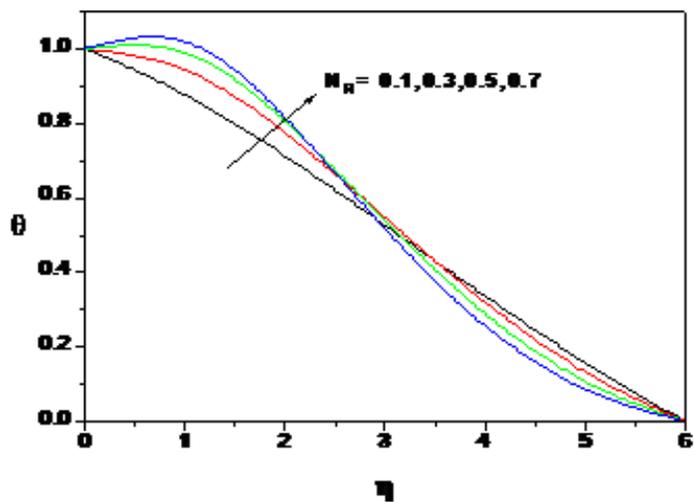


Fig.10: Variation of the temperature θ with N_R for $Pr=0.71, Sc=0.24, Gr =Gc=2, S=1, M=0.1$.

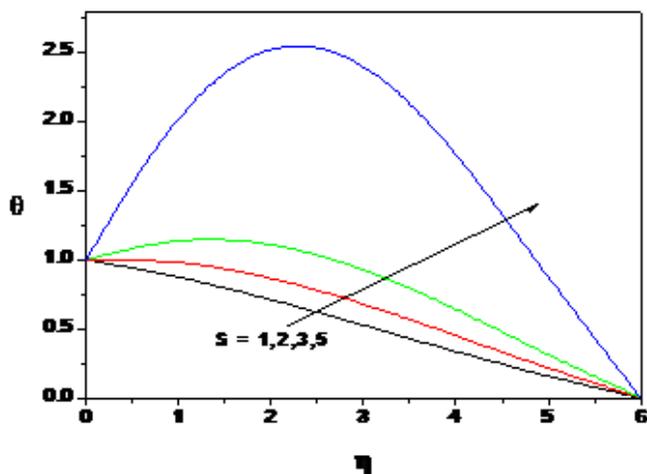


Fig.11: Variation of the temperature θ with S for $Pr=0.71, Sc=0.24, Gr =Gc=1, N_R=0.1, M=0.1$.

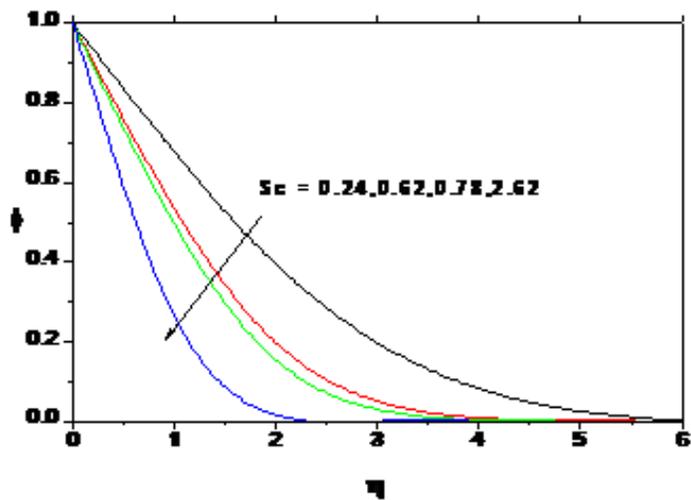


Fig.12: Variation of the concentration ϕ with Sc for $Pr=0.72, Gr=Gc=2, S=1, N_R=0.1, M=0.1$.

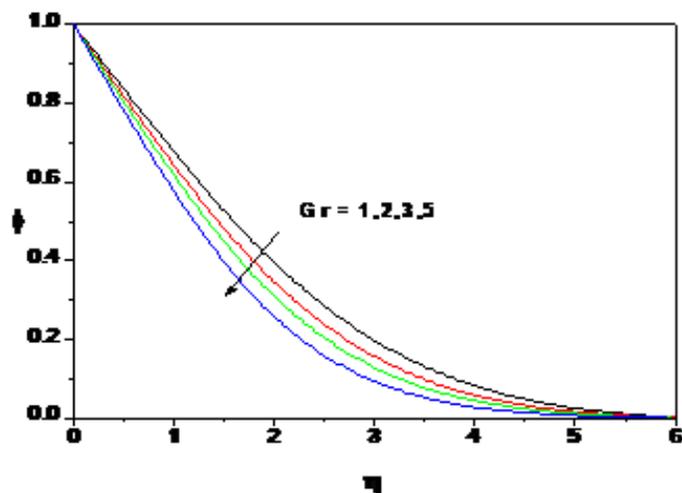


Fig.13: Variation of the concentration ϕ with Gr for $Pr=0.71, Sc=0.24, M=0.1, Gc=1, S=1, N_R=0.1$.

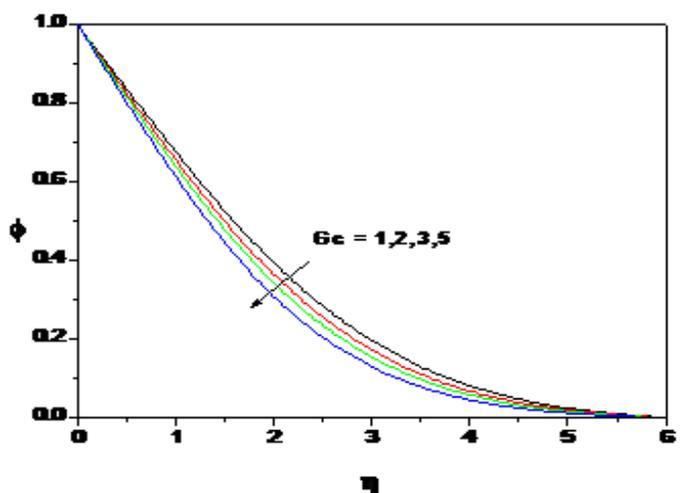


Fig.14: Variation of the concentration ϕ with Gc for $Pr=0.71, Sc=0.24, Gr=1, S=1, N_R=0.1, M=0.1$.

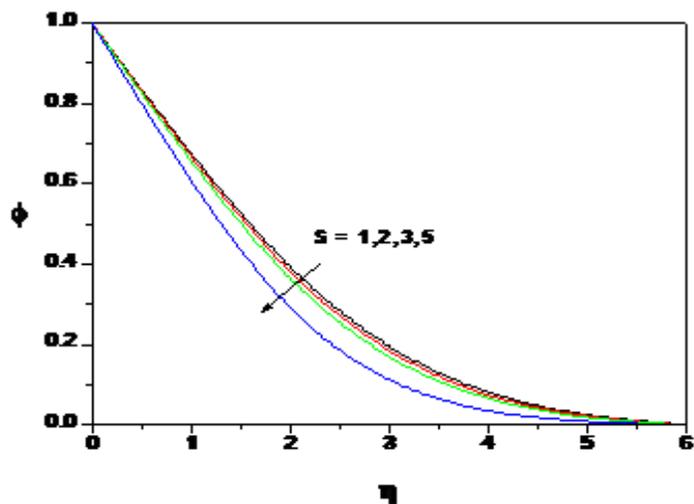


Fig.15: Variation of the concentration ϕ with S for $Pr=0.71, Sc=0.24, Gr=Gc=1, N_R=0.1, M=0.1$

Table 1 computations showing comparison with Ibrahim and Makinde [13] results for $f''(0)$, $-\theta'(0)$, $-\phi'(0)$ at the plate with Gr, Gc, M, N_R, Sc for $Pr=0.72, S=0$

M	Sc	Gr	Gc	N_R	Ibrahim and Mikinde [13]			Present work		
					$f''(0)$	$-\theta'(0)$	$-\phi'(0)$	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
0.1	0.24	1	1	0.1	2.866119713808	0.161700863207	0.328152353412	2.51528	0.269269	0.340707
0.5	0.24	1	1	0.1	2.715774578241	0.157411248176	0.314801374214	2.47164	0.268105	0.335381
1.0	0.24	1	1	0.1	2.643497318099	0.153586285415	0.30305664137	2.14207	0.505244	0.52501
0.1	0.62	1	1	0.1	2.695192292243	0.158655504441	0.483332708610	2.56434	0.198916	0.468443
0.1	0.78	1	1	0.1	2.657009248999	0.158108875638	0.528123751194	2.52438	0.198435	0.512031
0.1	2.62	1	1	0.1	2.475889567788	0.156053727069	0.825402340528	2.33561	0.196564	0.801736
0.1	0.24	2	1	0.1	3.951783250461	0.174262325794	0.365670905056	3.74548	0.2.9.66	0.353895
0.1	0.24	3	1	0.1	4.956292787925	0.184369404514	0.394692607664	4.69519	0.21571	0.380337
0.1	0.24	1	2	0.1	3.688807313223	0.167771186014	0.349825387426	3.57119	0.206461	0.344148
0.1	0.24	1	3	0.1	4.452867338900	0.172780621291	0.367601468364	4.34572	0.210754	0.362506
0.1	0.24	1	1	0.5	2.632252841126	0.281993102788	0.310074237495	2.59913	0.288767	0.312851
0.1	0.24	1	1	1	2.546307639267	0.341497727175	0.303814500108	2.527	0.342435	0.308277
0.1	0.24	1	1	2	2.484472309332	0.392179538023	0.299598770273	2.46843	0.391142	0.30469

Table 2 Variation of $f''(0)$, $-\theta'(0)$, $-\phi'(0)$ at the porous plate with S for $Gr = Gc = 1, M = N_R = 0.1, Sc = 0.24, Pr = 0.72$.

S	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
1	2.83219	0.101169	0.3272805
2	2.96875	0.0292412	0.33499
3	3.17629	0.216792	0.346342
5	4.2787	1.19517	0.398748

CONCLUSIONS:

In this paper, we have investigated numerically the effect of radiation and heat generation on a two-dimensional, steady, viscous, incompressible, electrically conducting and laminar free convection boundary layer flow from a flat plate in the presence of a transverse magnetic field. Using similarity transformations the governing equations have been transformed into non-linear ordinary differential equations and were solved for similar solutions by employing a shooting 4th order Runge-Kutta algorithm. Influences of thermal radiation, internal heat generation were examined. From our numerical results the following conclusions may be drawn:

1. Thermal boundary layer thickness increases with increasing thermal radiation.
2. Velocity profiles increase with the increase of thermal and mass Grashof numbers where as the trend is reversed in temperature and concentration profiles.
3. As the heat generation parameter increases, both the velocity and thermal boundary layer increases where as concentration boundary layer decreases.
4. The presence of the thermal radiation in the energy equation yields an augment in the fluid's temperature.
5. An increase in Schmidt number leads to a decrease in the velocity and concentration.

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