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CONTROLLING THE SPREAD OF HIV THROUGH TWO COMPONENTS

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ABSTRACT

Hundreds of HIV/AIDS studies have been implemented throughout the world. The time to cross the threshold of the infected person is a vital event in seroconversion. Mathematical model is obtained for the expected time of breakdown point to reach the seroconversion threshold level. In this paper two component namely sexual contact and needle sharing are the modes of transmission. Numerical examples are given to illustrate various aspects of the model considered for the expected time (mean) and Variance to seroconversion.

Keywords: *Component, Expected Time, Threshold, Seroconversion*

INTRODUCTION:

Since the beginning of HIV/AIDS, epidemic mathematicians and statisticians have developed models to describe and predict the course of the infection. Expected time of breakdown point to reach the seroconversion threshold level, in the context of HIV/AIDS with the assumptions that the times between decision period are independent and identically distributed (i.i.d) random variable, the number of outlet at each period of time are i.i.d. random variables and the threshold level is a random variable following Alpha-Poisson distribution.

Esary et al. (1973), consider a component, which can be either an engineering system or a bio-component, subjected to shocks occurring randomly in time. One can see for more detail related to the study of expected time through shock model in Palanivel et al. (2009), threshold level using Multisource of HIV Transmission by Pandiyan et al. (2010). Rajivgandhi et al. (2010) discussed about the expected time to cross the threshold level of the component. Mathematical model is obtained for the expected time of breakdown point to reach the threshold level through alpha-Poisson distribution.

Sexual contacts and Needle sharing are the two source of HIV infection. The threshold of any individual is a random variable. If the total damage crosses a threshold level Y which itself is a random variable, the seroconversion occurs and a person is recognized as infection. The inter-arrival times between successive contacts, the sequence of damage and the threshold are mutually independent.

NOTATIONS:

X_i : a discrete random variable denoting the amount of contribution to the threshold due to the HIV transmitted in the i^{th} contact, in other words the damage caused to the immune system in the i^{th} contact, with p.d.f g (.) and c.d.f G (.).

Y_1, Y_2 : discrete random variable denoting the threshold levels for the two components which follows alpha-Poisson distribution.

U_i : a random variable denoting the inter-arrival times between contact with c.d.f. $F_i(.)$, $i = 1,2,3 \ldots k$.

$g(.)$: The probability density function of X_i.

$g^*(.)$: Laplace transform of $g(.)$. $g_k(.)$: the k- fold convolution of $g(.)$ i.e., p.d.f. of $\sum_{j=1}^{k} X_i$

$F_k(.)$: k-fold convolution of $F(.)$.

$f(.)$: p. d. f. of random variable denoting between successive contact announcement with the corresponding c. d. f. $F(.)$.

$S(.)$: Survival function. $V_k(t)$: Probability of exactly k component. $L(t)$: $1 - S(t)$.

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RESULTS:

Any component exposed to shocks which cause damage to the immune system is likely to fail when the total cumulated damage exceed a level called threshold.

$$\bar{H}(x) = (a_1 - a_1 r)(a_2 - a_2 r) \tag{1}$$

In general, assuming that the threshold Y follows an alpha-Poisson Distribution with parameter r, it can be proved that

$$(PX_i < Y) = \int_0^\infty g_k(x)\ \bar{H}(x)dx$$

Transfer of system from Y_1 to Y_2 is also possible. We have the breakdown of the component is at $Y = max(Y_1, Y_2\ )$.

$$P[max(Y_1, Y_2\ )] = P[(Y_1 < y) \cap (Y_2 < y)]$$

$$= P[Y_1 < y]P[Y_2 < y]$$

Now that, Y_1 and Y_2 follow alpha-Poisson distribution with parameter λ_1, λ_2

$$= \int_0^\infty g_k(x)[a_1 a_2 - 2a_1 a_2 r + a_1 a_2 r^2]\,dx$$

$$= [g^*(a_1 a_2)]^k - [g^*(2a_1 a_2 r)]^k + [g^*(a_1 a_2 r^2)]^k$$

Survival analysis is a class of statistical methods for studying the occurrence and timing of events. The survival function $S(t)$ is

$$P(T > t) = \sum_{k=0}^{\infty} P[\text{there are exactly k contacts in}(0, t)] * P[\text{the total cumulative threshold } (0, t)]$$

$$S(t) = P(T > t) = \sum_{k=0}^{\infty} V_k(t) P\big(X_i < max(Y_1, Y_2)\big)$$

It may happen that successive shocks become increasingly effective in causing damage, even though they are independent. This means that $V_k(t)$, the distribution function of the k^{th} damage is decreasing in $k = 1,2, \ldots$ for each t. It is also known from renewal process that

$$P(\text{exactly k policy decesions in } (0, t]) = F_k(t) - F_{K+1}(t) \qquad \text{with} \quad F_0(t) = 1$$

$$= \sum_{k=0}^{\infty} V_k(t) P(X_i < Y)$$

$$= \sum_{k=0}^{\infty} V_k(t)\ [[g^*(a_1 a_2)]^k - [g^*(2a_1 a_2 r)]^k + [g^*(a_1 a_2 r^2)]^k]$$

$$S(t) = \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)]\ \big[g^*[(a_1 a_2)]\big]^k - \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)]\ \big[g^*[2a_1 a_2 r]\big]^k + \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)]\,[g^* a_1 a_2 r^2]^k \tag{2}$$

Now, $L(T) = 1 - S(t)$

Taking Laplace transform of $L(T)$, we get

$$L(T) = 1 - S(t)$$

$$= 1 - \left\{ \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)]\,[g^*(a_1a_2)]^k - \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)][g^* 2a_1a_2r]^k + \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)][g^*(a_1a_2r^2)]^k \right\}$$

$$l^*(s) = \frac{[1-g^*(a_1a_2)]f^*(s)}{[1-g^*(a_1a_2)]f^*(s)} - \frac{[1-g^*(2a_1a_2r)]f^*(s)}{[1-g^*(2a_1a_2r)]f^*(s)} + \frac{[1-g^*(a_1a_2r^2)]f^*(s)}{[1-g^*(a_1a_2r^2)]f^*(s)} \tag{3}$$

Let the random variable U denoting inter arrival time which follows exponential with parameter c. Now $f^*(s) = \left(\frac{c}{c+s}\right)$, substituting in the above equation (3) we get

$$= \frac{[1-g^*(a_1a_2)]\frac{c}{c+s}}{[1-g^*(a_1a_2)]\frac{c}{c+s}} - \frac{[1-g^*(2a_1a_2r)]\frac{c}{c+s}}{[1-g^*(2a_1a_2r)]\frac{c}{c+s}} + \frac{[1-g^*(a_1a_2r^2)]\frac{c}{c+s}}{[1-g^*(a_1a_2r^2)]\frac{c}{c+s}}$$

$$E(T) = -\frac{d}{ds} l^*(s), given\ s = 0$$

$$E(T) = \frac{1}{c\,[1-g^*(a_1a_2)]} - \frac{1}{c\,[1-g^*(2a_1a_2r)]} + \frac{1}{c\,[1-g^*(a_1a_2r^2)]} \tag{4}$$

$$g^*(.) \sim Mittag\ Leffler\ Distribution\ \frac{1}{1+\lambda^\alpha}$$

$$E(T) = \frac{1+(a_1a_2)^\alpha}{c\,(a_1a_2)^\alpha} - \frac{1+(2a_1a_2r)^\alpha}{c\,(2a_1a_2r)^\alpha} + \frac{1+\left(a_1a_2r^2\right)^\alpha}{c\,(a_1a_2r^2)^\alpha} \qquad On\ Simplification \tag{5}$$

$$E(T^2) = \frac{d^2}{ds^2} l^*(s) \quad given \quad s = 0$$

$$= -\frac{[1-g^*(a_1a_1)]c}{[c+s-g^*(a_1a_1)c]^2} + \frac{[1-g^*(2a_1a_1r)]c}{[c+s-g^*(2a_1a_1r)c]^2} - \frac{[1-g^*(a_1a_1r^2)]c}{[c+s-g^*(a_1a_1r^2)c]^2}$$

$$E(T^2) = \frac{2}{c^2\left[1-\frac{1}{1+(a_1a_2)^\alpha}\right]^2} - \frac{2}{c^2\left[1-\frac{1}{1+(2a_1a_2r)^\alpha}\right]^2} + \frac{2}{c^2\left[1-\frac{1}{1+(a_1a_2r^2)^\alpha}\right]^2}$$

$$E(T^2) = \frac{2\,[1+(a_1a_2)^\alpha]^2}{c^2[(a_1a_2)^\alpha]^2} - \frac{2\,[1+(2a_1a_2r)^\alpha]^2}{c^2[(2a_1a_2r)^\alpha]^2} + \frac{2\left[1+\left(a_1a_2r^2\right)^\alpha\right]^2}{c^2[(a_1a_2r^2)^\alpha]^2} \tag{6}$$

From which $V(T)$ can be obtained through equation (5) and (6), $V(T) = E(T^2) - [E(T)]^2$

$$V(T) = 2\left[\left(\frac{1+(a_1a_2)^\alpha}{c\,(a_1a_2)^\alpha}\right)\left(\frac{1+(2a_1a_2r)^\alpha}{c\,(2a_1a_2r)^\alpha}\right) - \left(\frac{1+\left(a_1a_2r^2\right)^\alpha}{c\,(a_1a_2r^2)^\alpha}\right)\left(\frac{1+(a_1a_2)^\alpha}{c\,(a_1a_2)^\alpha}\right) + \left(\frac{1+(2a_1a_2r)^\alpha}{(2a_1a_2r)^\alpha}\right)\left(\frac{1+\left(a_1a_2r^2\right)^\alpha}{c\,(a_1a_2r^2)^\alpha}\right)\right] \tag{7}$$

$$On\ Simplification$$

NUMERICAL ILLUSTRATION:

On the basis of the numerical illustration the following conclusions regarding expected time and variance consequent to the changes in the different parameters can be observed in Figures 1 to 4 that follow.

CONCLUSIONS:

When r is kept fixed with other parameters α, a_1, a_2, the inter-arrival time $'c'$, which follows exponential distribution, is an increasing parameter. Therefore, the value of the expected time $E\ (T)$ of the component to cross the threshold level decreases, for all cases of the parameter value $r = 0.5, 1, 1.5, 2$. When the value of the parameter r increases, the expected time is also found decreasing, this is observed in Figure 1a. The same case is found in Variance V (T) which is observed in Figure 1b.

When α is kept fixed with other parameters r, a_1, a_2, the inter-arrival time $'c'$ increases, the value of the expected time E (T) to cross the threshold level is found to be decreasing, in all the cases of the parameter value $\alpha = 1,1.5, 2,2.5$. When the value of the parameter α increases, the expected time is found increasing. This is indicated in Figure 2a. The same case is observed for Variance V (T) which is seen in Figure 2b.

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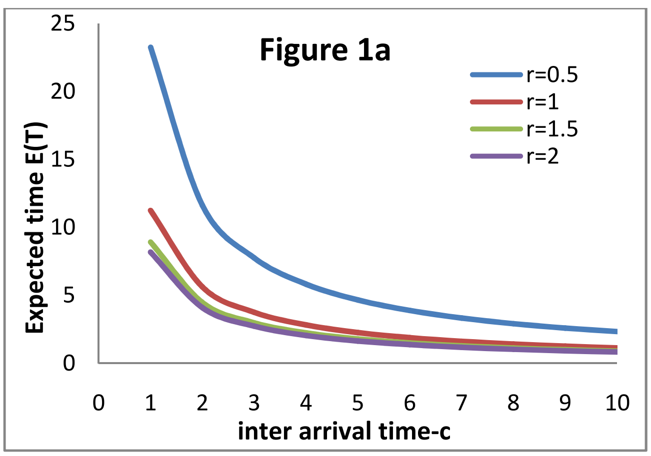

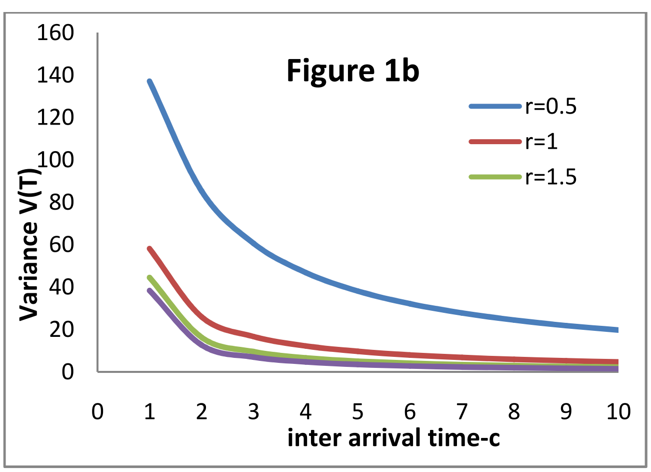

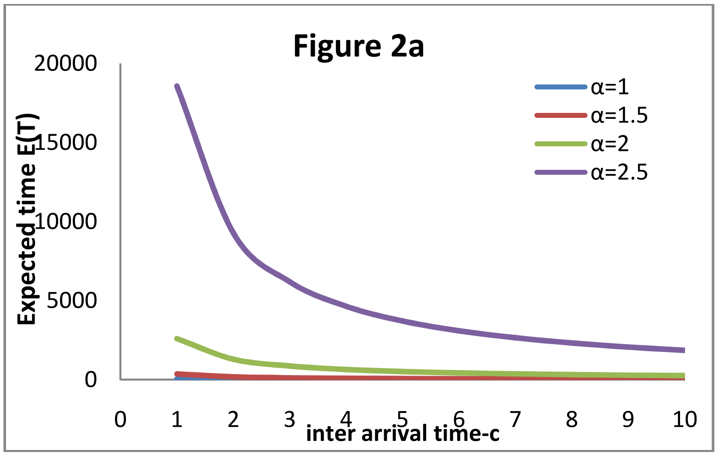
Figure 2a
20000
15000
10000
5000
0
Expected time E(T)
0
1
2
3
4
5
6
7
8
9
10
inter arrival time-c
α=1
α=1.5
α=2
α=2.5

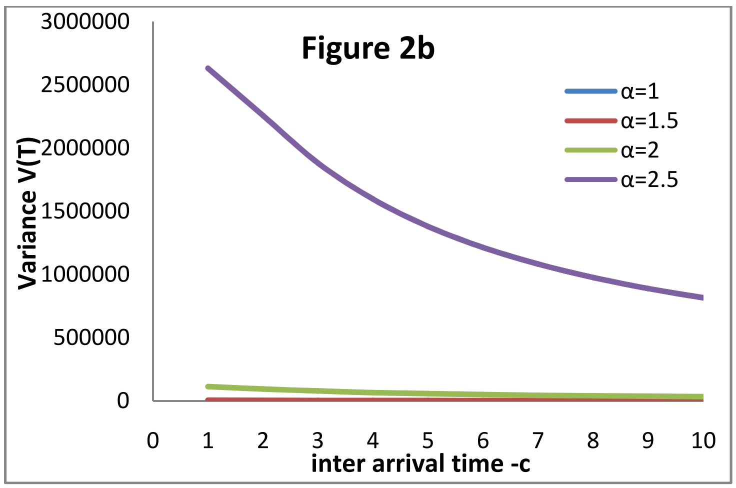
Figure 2b
3000000
2500000
2000000
1500000
1000000
500000
0
Variance V(T)
0
1
2
3
4
5
6
7
8
9
10
inter arrival time -c
α=1
α=1.5
α=2
α=2.5

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