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ON SUPRA QUOTIENT MAPPINGS

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ABSTRACT

The purpose of this paper is to introduce the concept of supra quotient mappings in supra topological spaces and study some fundamental properties of supra closed sets and supra open sets in supra topological spaces.

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1. INTRODUCTION:

Njastad [8] introduced the concept of an α-sets and Mashhour et al [6] introduced α-continuous mappings in topological spaces. The topological notions of semi-open sets and semi-continuity, and preopen sets and precontinuity were introduced by Levine [4] and Mashhour et al [5] respectively. After advent of these notions, Reilly [10] and Thivagar [3] obtained many interesting and important results on α-continuity and α-irresolute mappings in topological spaces. Lellis Thivagar [3] introduced the concepts of α-quotient mappings and α^*-quotient mappings in topological spaces. The notion of supra topological spaces was introduced by Mashhour et al [7] in 1983.

In this paper, we introduce a new class of supra topological mappings called supra α-quotient and supra-α^*-quotient mappings in supra topological spaces. At every places the new notions have been substantiated with suitable examples.

2. PRELIMINARIES:

Throughout this paper (X,τ), (Y, σ) and (Z, ν) (or simply, X, Y and Z) denote topological spaces on which no separation axioms are assumed unless explicitly stated. For a subset A of (X, τ), the closure and the interior of A are denoted by cl(A) and int(A) respectively. The complement of A is denoted by $X\backslash A$ or A^c.

Definition: 2.1 [7, 9] Let X be a non-empty set. The subfamily $\mu \subseteq P(X)$ where P(X) is the power set of X is said to be a supra topology on X if $X \in \mu$ and μ is closed under arbitrary unions.

The pair (X,μ) is called a supra topological space.

The elements of μ are said to be supra open in (X,μ).

Complements of supra open sets are called supra closed sets.

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Definition 2.2 [7, 9] Let (X, τ) be a topological space and μ be a supra topology on X. We call μ is a supra topology associated with τ if $\tau \subseteq \mu$.

Definition: 2.3 [9] Let A be a subset of X. Then
(i) the supra closure of A is, denoted by $cl^{\mu}(A)$, defined as $cl^{\mu}(A) = \cap \{ B : B$ is a supra closed and $A \subseteq B\}$.
(ii) the supra interior of A is, denoted by $int^{\mu}(A)$, defined as $int^{\mu}(A) = \cup\{G : G$ is a supra open and $A \supseteq G\}$.

Definition: 2.4 A subset A of X is called
(i) supra semi-open [9] if $A \subseteq cl^{\mu}(int^{\mu}(A))$;
(ii) supra α-open [1] if $A \subseteq int^{\mu}(cl^{\mu}(int^{\mu}(A)))$;
(iii) supra pre-open [11] if $A \subseteq int^{\mu}(cl^{\mu}(A))$.

The complements of the above mentioned open sets are called their respective closed sets. The family of all supra α-open [resp. supra semi-open, supra pre-open] sets of X is denoted by S-αO(X) [resp. S-SO(X), S-PO(X)].

Remark: 2.5
(i) If $\tau\subseteq\mu$, then every open set is supra open but not conversely.
(ii) Every supra open set is supra α-open but not conversely.
(iii) A supra semi-open [supra pre-open] set need not be supra α-open.

Example: 2.6 Let $Y = \{p, q, r\}$ and $\mu = \{\phi, Y, \{p\}, \{q\}, \{p, q\}\}$. We have
S-αO(Y) = $\{\phi, Y, \{p\}, \{q\}, \{p, q\}\}$;
S-SO(Y) = $\{\phi, Y, \{p\}, \{q\}, \{p, q\}, \{p, r\}, \{q, r\}\}$ and
S-PO(Y) = $\{\phi, Y, \{p\}, \{q\}, \{p, q\}\}$.

3. SUPRA α-CONTINUOUS MAPPINGS

Definition: 3.1 Let (X,τ) and (Y,σ) be two topological spaces and $\tau \subseteq \mu$. A map $f: (X,\tau) \to (Y,\sigma)$ is called
(i) supra continuous [9] if the inverse image of each open set of Y is a supra open set in X.
(ii) supra α-continuous [1] if the inverse image of each open set of Y is a supra α-open set in X.
(iii) supra semi-continuous [9] if the inverse image of each open set of Y is a supra semi-open set in X.
(iv) supra pre-continuous [11] if the inverse image of each open set of Y is a supra pre-open set in X.

Definition: 3.2 Let (X,τ) and (Y, σ) be two topological spaces and μ and λ be the supra topologies associated with τ and σ respectively. A map $f: (X, \tau) \to (Y, \sigma)$ is called
(i) supra M-open if the image of each supra open set of X is a supra open set in Y.
(ii) supra M-α-open if the image of each supra open set of X is a supra α-open set in Y.
(iii) supra M-semi-open if the image of each supra open set of X is a supra semi-open set in Y.
(iv) supra M-pre-open if the image of each supra open set of X is a supra pre-open set in Y.

Theorem: 3.3 A is supra semi-open set in X if and only if $cl^{\mu}(A) = cl^{\mu}(int^{\mu}(A))$.

Proof: Suppose A is supra semi-open set. Then $A \subseteq cl^{\mu}(int^{\mu}(A))$ and $cl^{\mu}(A) \subseteq cl^{\mu}(int^{\mu}(A))$. On the other hand, we have $int^{\mu}(A) \subseteq A$ and hence $cl^{\mu}(int^{\mu}(A)) \subseteq cl^{\mu}(A)$.

Conversely, we have $A \subseteq cl^{\mu}(A)$ and $cl^{\mu}(A) = cl^{\mu}(int^{\mu}(A))$. Therefore $A \subseteq cl^{\mu}(int^{\mu}(A))$. Hence A is supra semi-open set.

Theorem: 3.4 Let A be a subset of X. Then A is supra α-open set in X if and only if A is supra semi-open set and supra pre-open set in X.

Proof: Let $A \in$ S-αO(X). By the definition of supra α-open set, we have $A \subseteq int^{\mu}(cl^{\mu}(A))$ and $A \subseteq cl^{\mu}(int^{\mu}(A))$. Therefore $A \in$ S-PO(X) and $A \in$ S-SO(X). Hence $A \in$ S-SO(X) $\cap$ S-PO(X).

Conversely, let $A \in$ S-SO(X). Then by Theorem 3.3, $cl^{\mu}(A) = cl^{\mu}(int^{\mu}(A))$. Moreover let $A \in$ S-PO(X). Then $A \subseteq int^{\mu}(cl^{\mu}(A))$. Hence $A \subseteq int^{\mu}(cl^{\mu}(int^{\mu}(A)))$. It shows that $A \in$ S-αO(X).

Theorem: 3.5 The mapping $f: X \to Y$ is supra α-continuous if and only if it is supra semi-continuous and supra pre-continuous.

Proof: Let A be an open set in Y. Since f is supra α-continuous, $f^{-1}(A) \in$ S-αO(X) = S-SO(X) $\cap$ S-PO(X). Since $f^{-1}(A) \in$ S-SO(X) and $f^{-1}(A) \in$ S-PO(X), f is supra semi-continuous and supra pre-continuous.

Conversely, let f be supra semi-continuous and supra pre-continuous mapping. Let V be an open set in Y. Then $f^{-1}(V) \in$ S-SO(X) and $f^{-1}(V) \in$ S-PO(X). Therefore $f^{-1}(V) \in$ S-SO(X) $\cap$ S-PO(X) = S-αO(X).

Hence f is supra α-continuous.

4. SUPRA α-IRRESOLUTE MAPPINGS:

Definition: 4.1 Let S be a subset of X. Then S is said to be
(i) supra pre-closed if $cl^{\mu}(int^{\mu}(S)) \subseteq S$;
(ii) supra α-closed if $cl^{\mu}(int^{\mu}(cl^{\mu}(S))) \subseteq S$;
(iii)supra semi-closed if $int^{\mu}(cl^{\mu}(S)) \subseteq S$.

The family of all supra α-closed [resp. supra semi-closed, supra pre-closed] sets of X is denoted by S-αC(X) [resp. S-SC(X), S-PC(X)].

The complement of supra α-open [resp. supra semi-open, supra pre-open] set is supra α-closed [resp. supra semi-closed, supra pre-closed].

Definition: 4.2 Let (X, τ) and (Y, σ) be two topological spaces and μ and λ be supra topologies associated with τ and σ respectively. A map f: (X,τ) $\to$ (Y,σ) is called supra irresolute if $f^{-1}(V)$ is supra open set of X for every supra open set V in Y.

Definition: 4.3 Let (X, τ) and (Y, σ) be two topological spaces and μ and λ be supra topologies associated with τ and σ respectively. A map f: (X, τ) $\to$ (Y, σ) is called
(a) supra semi-irresolute if $f^{-1}(V)$ is supra semi-open set of X for every supra semi-open set V in Y.
(b) supra pre-irresolute if $f^{-1}(V)$ is supra pre-open set of X for every supra pre-open set V in Y.
(c) supra α-irresolute if $f^{-1}(V)$ is supra α-open set of X for every supra α-open set V in Y.

Theorem: 4.4 A mapping f: X $\to$ Y is supra semi-irresolute if and only if for every supra semi-closed subset A of Y, $f^{-1}(A)$ is supra semi-closed in X.

Proof: If f is supra semi-irresolute, then for every supra semi-open subset B of Y, $f^{-1}(B)$ is supra semi-open in X. If A is any supra semi-closed subset of Y, then Y\A is supra semi-open. Thus $f^{-1}(Y\backslash A)$ is supra semi-open but $f^{-1}(Y\backslash A) = X\backslash f^{-1}(A)$ so that $f^{-1}(A)$ is supra semi-closed in X.

Conversely, if, for all supra semi-closed subsets A of Y, $f^{-1}(A)$ is supra semi-closed in X and if B is any supra semi-open subset of Y, then Y\B is supra semi-closed. Also $f^{-1}(Y\backslash B) = X\backslash f^{-1}(B)$ is supra semi-closed. Thus $f^{-1}(B)$ is supra semi-open in X. Hence f is supra semi-irresolute.

Theorem: 4.5 Let f and g be two mappings. If f : X $\to$ Y and g : Y $\to$ Z are both supra semi-irresolute then gof : X $\to$ Z is supra semi-irresolute.

Proof: If A $\subseteq$ Z is supra semi-open, then $g^{-1}(A)$ is supra semi-open set in Y because g is supra semi-irresolute. Consequently since f is supra semi-irresolute, $f^{-1}(g^{-1}(A)) = (gof)^{-1}(A)$ is supra semi-open set in X. Hence gof is supra semi-irresolute mapping.

Corollary: 4.6 If f: X $\to$ Y and g: Y $\to$ Z are both supra α-irresolute mappings then gof: X $\to$ Z is supra α-irresolute.

Corollary: 4.7 If the mapping f : X $\to$ Y is supra α-irresolute and the mapping g : Y $\to$ Z is supra α-continuous then gof : X $\to$ Z is supra α-continuous mapping.

Corollary: 4.8 Let f: X $\to$ Y and g: Y $\to$ Z be two mappings. Then

(i) if f is supra semi-irresolute and g is supra semi-continuous, then gof is supra semi-continuous mapping.

(ii) if f is supra pre-irresolute and g is supra pre-continuous, then gof is supra pre-continuous mapping.

Theorem: 4.9 If the mapping f: X $\rightarrow$ Y is both supra semi-irresolute and supra pre-irresolute then f is supra α-irresolute mapping.

Proof: It is obvious.

5. SUPRA α-QUOTIENT MAPPINGS:

Definition: 5.1 Let (X,τ) and (Y, σ) be two topological spaces and μ and λ be supra topologies associated with τ and σ respectively. Let f: X $\rightarrow$ Y be a surjective mapping. Then f is said to be supra quotient provided a subset S of Y is supra open in Y if and only if $f^{-1}(S)$ is supra open in X.

Definition: 5.2 Let (X,τ) and (Y, σ) be two topological spaces and μ and λ be supra topologies associated with τ and σ respectively. Let f: X $\rightarrow$ Y be a surjective mapping. Then f is said to be

(a) a supra α-quotient if f is supra α-continuous and $f^{-1}(V)$ is supra open in X implies V is a supra α-open set in Y.

(b) a supra semi-quotient if f is supra semi-continuous and $f^{-1}(V)$ is supra open in X implies V is a supra semi-open set in Y.

(c) a supra pre-quotient if f is supra pre-continuous and $f^{-1}(V)$ is supra open in X implies V is a supra pre-open set in Y.

Example: 5.3 Let X = {a, b, c}; τ = {ϕ, X, {b}, {a, b}} and μ = {ϕ, X, {a}, {b}, {a, b}}. We have
S-αO(X) = {ϕ, X, {a}, {b}, {a, b}};
S-SO(X) = {ϕ, X, {a}, {b}, {a, b}, {a, c}, {b, c}} and
S-PO(X) = {ϕ, X, {a}, {b}, {a, b}}.

Let Y = {p, q, r}; σ = {ϕ, Y, {p}, {p, q}} and λ = {ϕ, Y, {p}, {q}, {p, q}}. We have

S-αO(Y) = {ϕ, Y, {p}, {q}, {p, q}};
S-SO(Y) = {ϕ, Y, {p}, {q}, {p, q}, {p, r}, {q, r}} and
S-PO(Y) = {ϕ, Y, {p}, {q}, {p, q}}.

Define f: X $\rightarrow$ Y by f (a) = p; f (b) = q; f(c) = r. Since the inverse image of each open in Y is supra α-open in X, clearly f is supra α-continuous and supra α-quotient mapping.

Theorem: 5.4 If the mapping f: X $\rightarrow$ Y is surjective, supra α-continuous and supra M-α-open then f is a supra α-quotient mapping.

Proof: Suppose $f^{-1}(V)$ is any supra open set in X. Then f $(f^{-1}(V))$ is a supra α-open set in Y as f is supra M-α-open. Since f is surjective, f $(f^{-1}(V))$ = V. Thus V is a supra α-open set in Y. Hence f is supra α-quotient mapping.

Theorem: 5.5 If the mapping f : X $\rightarrow$ Y is supra M-open surjective and supra α-irresolute, and the mapping g : Y $\rightarrow$ Z is a supra α-quotient then gof: X $\rightarrow$Z is a supra α-quotient mapping.

Proof: Let V be any open set and hence supra open set in Z. Since g is supra α-continuous,
$g^{-1}(V) \in$ S-αO(Y). Since f is supra α-irresolute, $f^{-1}(g^{-1}(V)) = (gof)^{-1}(V) \in$ S-αO(X). Thus gof is supra α-continuous.

Also suppose $f^{-1}(g^{-1}(V))$ is supra open set in X. Since f is supra M-open, f $(f^{-1}(g^{-1}(V)))$ is supra open set in Y. Since f is surjective, f $(f^{-1}(g^{-1}(V))) = g^{-1}(V)$ and since g is supra α-quotient, V $\in$ S-αO(Z). Hence gof is a supra α-quotient.

Corollary: 5.6 If the mapping f : X $\rightarrow$ Y is supra M-open surjective and supra semi-[supra pre-] irresolute and the mapping g : Y $\rightarrow$ Z is a supra semi- [supra pre-] quotient then gof : X $\rightarrow$ Z is a supra semi-[supra pre-] quotient mapping.

Theorem: 5.7 A mapping f : X $\rightarrow$ Y is a supra α-quotient if and only if it is supra semi-quotient mapping and supra pre-quotient mapping.

Proof: Let V be any open set and hence supra open set in Y. Since f is supra α-quotient, $f^{-1}(V) \in$ S-αO(X) = S-SO(X) $\cap$ S-PO(X). Thus f is both supra semi-continuous and supra pre-continuous. Also suppose $f^{-1}(V)$ is a supra open set in X. Since f is supra α-quotient, V $\in$ S-αO(Y) = S-SO(Y) $\cap$ S-PO(Y).

Thus V is both supra semi-open set and supra pre-open set in Y. Hence f is supra semi-quotient and supra pre-quotient.

Conversely, since f is supra semi-quotient and supra pre-quotient, f is supra semi-continuous and supra pre-continuous. Hence f is supra α-continuous. Also suppose $f^{-1}(V)$ is a supra open set in X. By Definition 5.2, V $\in$ S-SO(Y) $\cap$ S-PO(Y) = S-αO(Y). Thus f is supra α-quotient mapping.

Definition: 5.8
(i) Let f: X $\rightarrow$ Y be a surjective and supra α-continuous mapping. Then f is said to be strongly supra α-quotient provided a subset S of Y is supra open set in Y if and only if $f^{-1}(S)$ is a supra α-open set in X.

(ii) Let f: X $\rightarrow$ Y be a surjective and supra semi-continuous mapping. Then f is said to be strongly supra semi-quotient provided a subset S of Y is supra open set in Y if and only if $f^{-1}(S)$ is a supra semi-open set in X.

(iii) Let f: X $\rightarrow$ Y be a surjective and supra pre-continuous mapping. Then f is said to be strongly supra pre-quotient provided a subset S of Y is supra open set in Y if and only if $f^{-1}(S)$ is supra pre-open set in X.

Theorem: 5.9 If the mapping f : X $\rightarrow$ Y is strongly supra semi-quotient and strongly supra pre-quotient then f is strongly supra α-quotient mapping.

Proof: Since f is supra semi-continuous and supra pre-continuous, by Theorem 3.5,f is supra α-continuous. Also let V be an open set and hence supra open set in Y. By Definition 3.1 and Theorem 3.4, $f^{-1}(V) \in$ S-SO(X) $\cap$ S-PO(X) = S-αO(X).

Conversely, let $f^{-1}(V) \in$ S-αO(X). Then S-αO(X) = S-SO(X) $\cap$ S-PO(X). Since f is strongly supra semi-quotient and strongly supra pre-quotient, V is supra open set in Y. Hence f is strongly supra α-quotient mapping.

6. SUPRA α*-QUOTIENT MAPPINGS:

Definition: 6.1 Let f: X $\rightarrow$ Y be a surjective mapping. Then f is said to be
(i) supra α*-quotient if f is supra α-irresolute and $f^{-1}(S)$ is supra α-open set in X implies S is supra open set in Y.

(ii) supra semi*-quotient if f is supra semi-irresolute and $f^{-1}(S)$ is supra semi-open set in X implies S is supra open set in Y.

(iii) supra pre*-quotient if f is supra pre-irresolute and $f^{-1}(S)$ is supra pre-open set in X implies S is supra open set in Y.

Definition: 6.2 Let f: X $\rightarrow$ Y be a mapping. Then f is said to be strongly supra M-α-open if the image of every supra α-open set in X is a supra α-open set in Y.

Example: 6.3 Consider the Example 5.3. Clearly f is supra α-irresolute and supra α*-quotient mapping.

Example: 6.4 Consider the Example 5.3. Clearly f is strongly supra M-α-open mapping.

Theorem: 6.5 Let the mapping f : X $\rightarrow$ Y be surjective strongly supra M-α-open and supra α-irresolute, and the mapping g : Y $\rightarrow$ Z be a supra α*-quotient. Then gof: X $\rightarrow$ Z is a supra α*-quotient mapping.

Proof: Let V be any supra α-open set in Z. Then $g^{-1}(V)$ is a supra α-open set in Y as g is a supra α*-quotient mapping. Then $f^{-1}(g^{-1}(V)) = (gof)^{-1}(V)$ is a supra α-open set in X as f is supra α-irresolute. This shows that gof is supra α-irresolute. Also suppose $(gof)^{-1}(V) = f^{-1}(g^{-1}(V))$ is a supra α-open set in X.

Since f is strongly supra M-α-open, $f(f^{-1}(g^{-1}(V)))$ is a supra α-open set in Y.

Since f is surjective, f $(f^{-1}(g^{-1}(V))) = g^{-1}(V)$ is a supra α-open set in Y. Since g is a supra α*-quotient mapping, V is supra open set in Z. Hence the theorem.

Theorem: 6.6 If the mapping f: X → Y is supra semi*-quotient and supra pre*-quotient then f is supra α*-quotient mapping.

Proof: Since f is supra semi*-quotient and supra pre*quotient, f is supra semi-irresolute and supra pre-irresolute. By Theorem 4.9., f is supra α-irresolute. Also suppose $f^{-1}(V) \in$ S-αO(X). Then S-αO(X) = S-SO(X) ∩ S-PO(X). Therefore $f^{-1}(V)$ is supra semi-open in X and $f^{-1}(V)$ is supra pre-open in X. Since f is supra semi*-quotient and supra pre*-quotient, by Definition 6.1., V is supra open set in Y. Thus f is supra α*-quotient mapping.

Theorem: 6.7 Let f: X → Y be a strongly supra α-quotient and supra α-irresolute mapping and g: Y → Z be a supra α*-quotient mapping then gof: X → Z is a supra α*-quotient mapping.

Proof: Let V ∈ S-αO (Z). Since g is supra α-irresolute, $g^{-1}(V) \in$ S-αO(Y). Since f is supra α-irresolute, $f^{-1}(g^{-1}(V)) = (gof)^{-1}(V) \in$ S-αO(X). Thus gof is supra α-irresolute. Also suppose $(gof)^{-1}(V) = f^{-1}(g^{-1}(V)) \in$ S-αO(X). Since f is strongly supra α-quotient, $g^{-1}(V)$ is supra open set in Y. Then $g^{-1}(V) \in$ S-αO(Y). Since g is supra α*-quotient, V is supra open set in Z. Hence gof is supra α*-quotient mapping.

7. COMPARISON:

Theorem: 7.1 Let f: X → Y be a surjective mapping. Then f is supra α*-quotient if and only if it is strongly supra α-quotient mapping.

Proof: Let V be a supra open set in Y. Then V ∈ S-αO(Y). Since f is supra α*-quotient, $f^{-1}(V) \in$ S-αO(X). Conversely, let $f^{-1}(V) \in$ S-αO(X). Since f is supra α*-quotient, V is supra open set in Y. Hence f is strongly supra α-quotient mapping.

Conversely, let V be supra open set in Y. Then V ∈ S-αO(Y). Since f is strongly supra α-quotient, $f^{-1}(V) \in$ S-αO(X). Thus f is supra α-irresolute. Also since f is strongly supra α-quotient, $f^{-1}(V) \in$ S-αO(X) implies V is supra open set in Y. Hence f is supra α*-quotient mapping.

Theorem: 7.2 If the mapping f: X → Y is supra quotient then it is supra α-quotient mapping.

Proof: Let V be a supra open set in Y. Since f is supra quotient, $f^{-1}(V)$ is supra open set in X and $f^{-1}(V) \in$ S-αO(X). Hence f is supra α-continuous. Suppose $f^{-1}(V)$ is a supra open set in X. Since f is supra quotient, V is supra open set in Y. Then V ∈ S-αO(Y). Hence f is supra α-quotient mapping.

Theorem: 7.3 If the mapping f: X → Y is supra α-irresolute then it is supra α-continuous mapping.

Proof: Let A be open set and hence supra open set in Y. Then A ∈ S-αO(Y). Since f is supra α-irresolute, $f^{-1}(A) \in$ S-αO(X). It shows that f is supra α-continuous mapping.

Theorem: 7.4 If the mapping f: X → Y is supra α*-quotient then it is supra α-quotient mapping.

Proof: Let f be supra α*-quotient. Then f is supra α-irresolute. We have f is supra α-continuous. Also suppose $f^{-1}(V)$ is a supra open in X. Then $f^{-1}(V) \in$ S-αO(X). By assumption, V is supra open set in Y.

Therefore V ∈ S-αO(Y). Hence f is supra α-quotient mapping.

Theorem: 7.5 Every supra α*-quotient mapping is supra α-irresolute.

Proof: We obtain it from Definition 6.1.

Theorem: 7.6 Every supra α-quotient mapping is supra α-continuous.

Proof: We obtain it from Definition 5.2.

Remark: 7.7 The converses of Theorems 5.9 and 6.6 are not true as per the following example.

Example: 7.8 Let X= {a, b, c}, μ = {ϕ, X, {a}, {b}, {a, b}}, Y = {p, q, r} and λ = {ϕ, Y, {p}, {q}, {p, q}}. Define f: X → Y by f(a) = p; f(b) = q and f(c) = r. Clearly f is supra α-continuous and strongly supra α-quotient mapping. Since f^{-1}({p, r}) = {a, c} ∈ S-SO(X) and {p, r} is not supra open set in Y, f is not strongly supra semi-quotient mapping. Moreover f is supra α-irresolute, supra α*-quotient and supra semi-irresolute mapping.

Since f^{-1}({q, r}) = {b, c} ∈ S-SO(X) and {q, r} is not supra open set in Y, f is not supra semi*-quotient mapping.

Remark: 7.9 The converses of Theorems 7.4 and 7.5 are not true as per the following example.

Example: 7.10 Let X = {a, b, c}, μ = {ϕ, X, {b}, {b, c}}, Y = {p, q, r} and λ = {ϕ, Y, {q}, {q, r}}. Define f: X → Y by f (a) = p; f (b) = q and f(c) = r. Clearly f is supra α-irresolute and supra α-quotient mapping. Since f^{-1}({p, q}) = {a, b} ∈ S-αO(X) and {p, q} is not supra open set in Y, f is neither strongly supra α-quotient nor supra α*-quotient mapping.

Remark: 7.11 The converse of Theorem 7.2 is not true and A strongly supra α-quotient mapping need not be supra quotient as per the following example.

Example: 7.12 Let X = {a, b, c}, μ = {ϕ, X, {a}}, Y = {p, q, r} and λ = {ϕ, Y, {p}, {p, q}, {p, r}}. Define f: X → Y by f (a) = p, f (b) = q and f(c) = r. Clearly f is supra α-quotient and strongly supra α-quotient mapping. Since f^{-1}({p, q}) = {a, b} is not supra open where {p, q} is supra open, f is not supra quotient mapping.

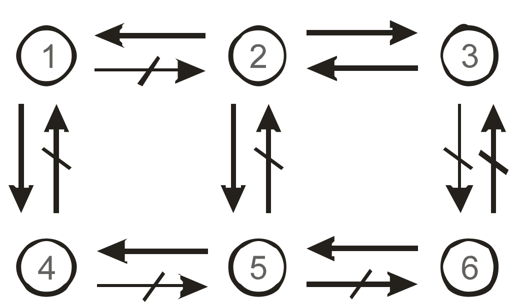
Remark: 7.13A supra quotient mapping need not be strongly supra α-quotient as per the following example.

Example: 7.14 Let X = {a, b, c}, μ = {ϕ, X, {a}, {a, b}}, Y = {p, q, r} and λ = {ϕ, Y, {p}, {p, q}}. Define f: X → Y by f(a) = p; f(b) = q and f(c) = r. Clearly f is supra quotient but not strongly supra α-quotient mapping.

Remark: 7.15 The converses of Theorems 7.3 and 7.6 are not true as per the following example.

Example: 7.16 Let X = {a, b, c}, μ = {ϕ, X, {a}, {b}, {a, b}}, Y = {p, q, r} and λ = {ϕ, Y, {p}}. Define f: X → Y by f (a) = p; f (b) = q and f(c) = r. Clearly f is supra α-continuous. Since f^{-1}({p, r}) = {a, c} ∉ S-αO(X) where {p, r}∈ S-αO(Y), f is not supra α-irresolute. Also, since f^{-1}({q}) = {b} is supra open in X where {q} ∉ S-αO(Y), f is not supra α-quotient mapping.

Remark: 7.17 We obtain the following diagram from the above discussions.



Where A $\not\rightarrow$ B means that A does not necessarily imply B and, moreover,

1) = supra α-irresolute mapping.
2) = supra α*-quotient mapping.
3) = strongly supra α-quotient mapping.
4) = supra α-continuous mapping.
5) = supra α-quotient mapping.
6) = supra quotient mapping.

REFERENCES:

[1] Devi, R., Sampathkumar, S. and Caldas, M. On supra α -open sets and sα-continuous maps, General Mathematics, 16(2) (2008), 77-84.

[2] Kamaraj, M., Ramkumar, G. and Ravi, O. Further properties of supra sg-closed sets in topological spaces (Submitted).

[3] Lellis Thivagar, M. A note on quotient mappings, Bull. Malaysian Math. Sci. Soc. (Second series), 14 (1991), 21-30.

[4] Levine, N. Semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly, 70 (1963), 36-41.

[5] Mashhour, A. S., Abd El-Monsef, M. E. and El-Deeb, S. N. On precontinuous mappings and weak pre-continuous mappings, Proc. Math. and Phys. Soc. Egypt, 53(1982), 47-53.

[6] Mashhour, A. S., Hasanein, I. A. and El-Deeb, S. N. α-continuous and α-open mappings, Acta Math. Hungar., 41(1983), 213-218.

[7] Mashhour, A. S., Allam, A. A., Mahmoud F. S. and Khedr, F. H. On supra topological spaces, Indian J. Pure and Appl. Math., 14(4) (1983), 502-510.

[8] Njastad, O. On some classes of nearly open sets, Pacific J. Math., 15 (1965), 961-970.

[9] Ravi, O., Ramkumar, G. and Kamaraj, M. On supra g-closed sets, International Journal of Advances in Pure and Applied Mathematics, 1(2) (2011), 52-66.

[10] Reilly, I. L and Vamanamurthy, M. K. On α-continuity in topological spaces, Acta Math. Hungar., 45 (1-2) (1985), 27-32.

[11] Sayed, O. R. Supra pre-open sets and supra pre-continuity on topological spaces, Vasile Alecsandri, University of Bacau, Faculty of Sciences, Scientific Studies and Research Series Mathematics and Informatics., 20(2)(2010), 79-88.

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