International Journal of Mathematical Archive-3(1), 2012, Page: 253-259
Available online through www.ijma.info ISSN 2229 – 5046

COMMON FIXED POINT THEOREMS IN INTUITIONISTIC FUZZY METRIC SPACES VIA ABSORBING MAPS

[1]Saurabh Manro*, [1]S. S. Bhatia, [2]Sanjay Kumar and [3]Jatinderjit Singh

[1]School of Mathematics and Computer Applications, Thapar University, Patiala (Punjab), India
E-mail: sauravmanro@hotmail.com

[2]Deenbandhu Chhotu Ram University of Science and Technology, Murthal (Sonepat), India

[3]Govt. High School, Sirthala (Ludhiana, Punjab), India

(Received on: 22-10-11; Accepted on: 07-11-11)

ABSTRACT

In this paper, we obtain common fixed point theorems in intuitionistic fuzzy metric spaces by using notion of absorbing maps. Our result generalizes many known results in intuitionistic fuzzy metric spaces.

***Key words**: Intuitionistic fuzzy metric space, Reciprocal continuous mappings, Absorbing maps, Common fixed point.*

***AMS (2010) Subject Classification**: 47H10, 54H25.*

1. INTRODUCTION:

Atanassove [4] introduced and studied the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets. In 2004, Park [9] defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norms and continuous t-conorms. Recently, in 2006, Alaca et al.[1] using the idea of Intuitionistic fuzzy sets, defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norm and continuous t- conorms as a generalization of fuzzy metric space due to Kramosil and Michalek [5].In the literature, many results have been proved for contraction maps in different settings such as fuzzy metric spaces [3, 7], intuitionistic fuzzy metric spaces [1, 4, 6].

In this paper, we obtain common fixed point theorems in intuitionistic fuzzy metric spaces by using notion of absorbing maps. Our result generalizes many known results in intuitionistic fuzzy metric spaces.

2. PRELIMINARIES:

The concepts of triangular norms (*t*-norms) and triangular conorms (*t*-conorms) are known as the axiomatic skelton that we use are characterization fuzzy intersections and union respectively. These concepts were originally introduced by Menger [8] in study of statistical metric spaces.

Definition: 2.1[11] A binary operation $*$: $[0,1]\times[0,1] \rightarrow [0,1]$ is continuous *t*-norm if $*$ satisfies the following conditions:
(i) $*$ is commutative and associative;
(ii) $*$ is continuous;
(iii) $a * 1 = a$ for all $a \in [0,1]$;
(iv) $a * b \le c * d$ whenever $a \le c$ and $b \le d$ for all $a, b, c, d \in [0,1]$.

Definition: 2.2[11] A binary operation $\Diamond$: $[0, 1] \times [0, 1] \rightarrow [0, 1]$ is continuous t-conorm if $\Diamond$ satisfies the following conditions:
(i) $\Diamond$ is commutative and associative;
(ii) $\Diamond$ is continuous;
(iii) $a \Diamond 0 = a$ for all $a \in [0,1]$;
(iv) $a \Diamond b \le c \Diamond d$ whenever $a \le c$ and $b \le d$ for all $a, b, c, d \in [0,1]$.

***Corresponding author: [1]Saurabh Manro*,*E-mail: sauravmanro@hotmail.com**

Alaca et al. [1] using the idea of Intuitionistic fuzzy sets, defined the notion of intuitionistic fuzzy metric space with the help of continuous *t*-norm and continuous *t*-conorms as a generalization of fuzzy metric space due to Kramosil and Michalek [5] as :

Definition: 2.3[1] A 5-tuple $(X, M, N, *, \Diamond)$ is said to be an intuitionistic fuzzy metric space if *X* is an arbitrary set, *is a continuous *t*-norm, $\Diamond$ is a continuous *t*-conorm and *M, N* are fuzzy sets on $X^2 \times [0, \infty)$ satisfying the following conditions:

(i) $M(x, y, t) + N(x, y, t) \leq 1$ for all $x, y \in X$ and $t > 0$;
(ii) $M(x, y, 0) = 0$ for all $x, y \in X$;
(iii) $M(x, y, t) = 1$ for all $x, y \in X$ and $t > 0$ if and only if x = y;
(iv) $M(x, y, t) = M(y, x, t)$ for all $x, y \in X$ and $t > 0$;
(v) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$ for all $x, y, z \in X$ and $s, t > 0$;
(vi) $M(x, y, .) : [0, \infty) \rightarrow [0, 1]$ is left continuous for all $x, y \in X$;
(vii) $\lim_{t \to \infty} M(x, y, t) = 1$ for all $x, y \in X$ and $t > 0$;
(viii) $N(x, y, 0) = 1$ for all $x, y \in X$;
(ix) $N(x, y, t) = 0$ for all $x, y \in X$ and $t > 0$ if and only if x = y;
(x) $N(x, y, t) = N(y, x, t)$ for all $x, y \in X$ and $t > 0$;
(xi) $N(x, y, t) \Diamond N(y, z, s) \geq N(x, z, t + s)$ for all $x, y, z \in X$ and $s, t > 0$;
(xii) $N(x, y, .) : [0, \infty) \rightarrow [0, 1]$ is right continuous for all $x, y \in X$;
(xiii) $\lim_{t \to \infty} N(x, y, t) = 0$ for all $x, y \in X$.

Then *(M, N)* is called an intuitionistic fuzzy metric space on *X*. The functions *M(x, y, t)* and *N(x, y, t)* denote the degree of nearness and the degree of non-nearness between *x* and *y* w.r.t. t respectively.

Remark 2.1: [1] Every fuzzy metric space *(X, M, *)* is an intuitionistic fuzzy metric space of the form *(X, M, 1-M, *, ◊)* such that *t*-norm * and *t*-conorm ◊ are associated as

$$x \Diamond y = 1-((1-x) * (1-y)) \text{ for all } x, y \in X.$$

Remark: 2.2 [1] In intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$, *M(x, y, *)* is non-decreasing and *N(x, y, ◊)* is non-increasing for all $x, y \in X$.

Alaca, Turkoglu and Yildiz [1] introduced the following notions:

Definition: 2.4[1] Let $(X, M, N, *, \Diamond)$ be an intuitionistic fuzzy metric space. Then
(a) a sequence $\{x_n\}$ in *X* is said to be Cauchy sequence if, for all $t > 0$ and $p > 0$,

$$\lim_{n \to \infty} M(x_{n+p}, x_n, t) = 1 \text{ and } \lim_{n \to \infty} N(x_{n+p}, x_n, t) = 0.$$

(b) a sequence $\{x_n\}$ in *X* is said to be convergent to a point $x \in X$ if, for all $t > 0$,

$$\lim_{n \to \infty} M(x_n, x, t) = 1 \text{ and } \lim_{n \to \infty} N(x_n, x, t) = 0.$$

Definition: 2.5 [1] An intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$ is said to be complete if and only if every Cauchy sequence in *X* is convergent.

Example: 2.1[1] Let $X = \{1/n: n \in N\} \cup \{0\}$ and let * be the continuous *t*-norm and ◊ be the continuous *t*-conorm defined by $a * b = ab$ and $a \Diamond b = \min\{1, a+b\}$ respectively, for all $a, b \in [0,1]$. For each $t \in (0, \infty)$ and $x, y \in X$, define *(M, N)* by

$$M(x,y,t)=\begin{cases}\dfrac{t}{t+|x-y|}, & t>0,\\ 0 & t=0\end{cases} \quad \text{and} \quad N(x,y,t)=\begin{cases}\dfrac{|x-y|}{t+|x-y|}, & t>0,\\ 1 & t=0.\end{cases}$$

Clearly, $(X,M,N,*,\Diamond)$ is complete intuitionistic fuzzy metric space.

Definition: 2.6 Let *f* and *g* are two self maps on an intuitionistic fuzzy metric space $(X,M,N,*,\Diamond)$ then f is called *g*-absorbing if there exists a positive integer $R>0$ such that

$$M(gx,gfx,t)\geq M(gx,fx,t/R)=1, N(gx,gfx,t)\leq N(gx,fx,t/R)=0 \text{ for all } x\in X .$$

Definition: 2.7 Let *f* and *g* are two self maps on an intuitionistic fuzzy metric space $(X,M,N,*,\Diamond)$ then f is called pointwise *g*-absorbing if for given $x\in X$, there exists a positive integer $R>0$ such that

$$M(gx,gfx,t)\geq M(gx,fx,t/R)=1, N(gx,gfx,t)\leq N(gx,fx,t/R)=0 \text{ for all } x\in X .$$

Definition: 2.8[2] Let $(X,M,N,*,\Diamond)$ be an intuitionistic fuzzy metric space. *f* and *g* be self maps on *X*. A point $x\in X$ is called a coincidence point of *f* and *g* iff *fx* = *gx*. In this case, *w* = *fx* = *gx* is called a point of coincidence of f and g.

Definition 2.9[10]: A pair of self mappings *(f, g)* of intuitionistic fuzzy metric space $(X,M,N,*,\Diamond)$ is said to be reciprocal continuous if $\lim_{n\to\infty} fgx_n = fx$ and $\lim_{n\to\infty} gfx_n = gx$ whenever, there exists a sequence $\{x_n\}$ in *X* such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = x$ for some $x\in X$.

Alaca [1] proved the following results:

Lemma: 2.1 [1] Let (X, M, N, *, ◊) be intuitionistic fuzzy metric space and for all $x,y\in X$, $t>0$ and if for a number $k\in(0,1)$ such that

$$M(x,y,kt)\geq M(x,y,t) \text{ and } N(x,y,kt)\leq N(x,y,t)$$

Then, *x* = *y*.

3. MAIN RESULTS:

Theorem 3.1: Let *P* be pointwise *S*-absorbing and *Q* be the pointwise *T*-absorbing self maps on a complete intuitionistic fuzzy metric space $(X,M,N,*,\Diamond)$ with $a*b=\min\{a,b\}$ and $a\Diamond b=\max\{a,b\}$ for all $a,b\in[0,1]$ satisfying the conditions:

(3.1) $P(X)\subseteq T(X), Q(X)\subseteq S(X)$

(3.2) there exist $k\in(0,1)$ such that for all $x,y\in X$ and $t>0$,

$$M(Px,Qy,kt)\geq \min\{M(Sx,Ty,t), M(Px,Sx,t), M(Qy,Ty,t), M(Px,Ty,t)\},$$
$$N(Px,Qy,kt)\leq \max\{N(Sx,Ty,t), N(Px,Sx,t), N(Qy,Ty,t), N(Sx,Ty,t)\}.$$

If the pair *(P, S)* or *(Q, T)* is reciprocally continuous compatible maps then *P, Q, S* and *T* have a unique common fixed point in *X*.

Proof: Let x_0 be any point in *X*, construct a sequence $\{y_n\}$ in X such that by (3.1), we have $y_{2n-1} = Tx_{2n-1} = Px_{2n-2}$ and $y_{2n} = Sx_{2n} = Qx_{2n+1}$ for all *n* = 1,2,3,…

Now, we show that $\{y_n\}$ is a Cauchy sequence in *X*.

By (3.2), we have

$$
\begin{aligned}
M(y_{2n+1}, y_{2n+2}, kt) &= M(Px_{2n}, Qx_{2n+1}, kt) \\
&\geq \min\left\{M(Sx_{2n}, Tx_{2n+1}, t), M(Px_{2n}, Sx_{2n}, t), M(Qx_{2n+1}, Tx_{2n+1}, t), M(Px_{2n}, Tx_{2n+1}, t)\right\} \\
&= \min\left\{M(y_{2n}, y_{2n+1}, t), M(y_{2n+1}, y_{2n}, t), M(y_{2n}, y_{2n+1}, t), M(y_{2n+1}, y_{2n+1}, t)\right\} \\
& M(y_{2n+1}, y_{2n+2}, kt) \geq M(y_{2n}, y_{2n+1}, t)
\end{aligned}
$$

And

$$
\begin{aligned}
N(y_{2n+1}, y_{2n+2}, kt) &= N(Px_{2n}, Qx_{2n+1}, kt) \\
&\leq \max\left\{N(Sx_{2n}, Tx_{2n+1}, t), N(Px_{2n}, Sx_{2n}, t), N(Qx_{2n+1}, Tx_{2n+1}, t), N(Px_{2n}, Tx_{2n+1}, t)\right\} \\
&= \max\left\{N(y_{2n}, y_{2n+1}, t), N(y_{2n+1}, y_{2n}, t), N(y_{2n}, y_{2n+1}, t), N(y_{2n+1}, y_{2n+1}, t)\right\}
\end{aligned}
$$

$$N(y_{2n+1}, y_{2n+2}, kt) \leq N(y_{2n}, y_{2n+1}, t)$$

In general, $M(y_n, y_{n+1}, kt) \geq M(y_{n-1}, y_n, t), N(y_n, y_{n+1}, kt) \leq N(y_{n-1}, y_n, t)$.

$$M(y_n, y_{n+1}, kt) \geq M(y_{n-1}, y_n, t), N(y_n, y_{n+1}, kt) \leq N(y_{n-1}, y_n, t)$$

Therefore,

$$
\begin{aligned}
M(y_n, y_{n+1}, t) &\geq \min\left\{M(y_{n-1}, y_n, t/k), M(y_{n-2}, y_{n-1}, t/k^2), \ldots, M(y_0, y_1, t/k^n)\right\} \to 1 (n \to \infty) \\
N(y_n, y_{n+1}, t) &\leq \max\left\{N(y_{n-1}, y_n, t/k), N(y_{n-2}, y_{n-1}, t/k^2), \ldots, N(y_0, y_1, t/k^n)\right\} \to 0 (n \to \infty)
\end{aligned}
$$

Hence,

$$M(y_n, y_{n+p}, t) \geq \min\left\{M(y_n, y_{n+1}, (1-k)t), M(y_{n+1}, y_{n+p}, kt)\right\}$$

$$
\begin{aligned}
&\geq \min\left\{M(y_0, y_1, (1-k)t/k^n), M(y_{n+1}, y_{n+2}, t), M(y_{n+2}, y_{n+p}, (k-1)t)\right\} \\
&\geq \min\left\{M(y_0, y_1, (1-k)t/k^n), M(y_0, y_1, t/k^{n+1}), M(y_{n+2}, y_{n+3}, t), M(y_{n+3}, y_{n+p}, (k-2)t)\right\} \\
&\geq \min\left\{M(y_0, y_1, (1-k)t/k^n), M(y_0, y_1, t/k^{n+1}), M(y_0, y_1, (1-k)t/k^{n+2}), \ldots, M(y_0, y_1, (k-p)t/k^{n+p+1})\right\} \\
& n \to \infty \\
& M(y_n, y_{n+p}, t) \to 1
\end{aligned}
$$

and

$$
\begin{aligned}
& N(y_n, y_{n+p}, t) \leq \max\left\{N(y_n, y_{n+1}, (1-k)t), N(y_{n+1}, y_{n+p}, kt)\right\} \\
&\leq \max\left\{N(y_0, y_1, (1-k)t/k^n), N(y_{n+1}, y_{n+2}, t), N(y_{n+2}, y_{n+p}, (k-1)t)\right\} \\
&\leq \max\left\{N(y_0, y_1, (1-k)t/k^n), N(y_0, y_1, t/k^{n+1}), N(y_{n+2}, y_{n+3}, t), N(y_{n+3}, y_{n+p}, (k-2)t)\right\} \\
&\leq \max\left\{N(y_0, y_1, (1-k)t/k^n), N(y_0, y_1, t/k^{n+1}), N(y_0, y_1, (1-k)t/k^{n+2}), \ldots, N(y_0, y_1, (k-p)t/k^{n+p+1})\right\} \\
& n \to \infty \\
& N(y_n, y_{n+p}, t) \to 0.
\end{aligned}
$$

$\{y_n\}$ is a Cauchy sequence in *X*. But $(X, M, N, *, \Diamond)$ is complete intuitionistic fuzzy metric space so there exist a point $z \in X$ such that $\{y_n\} \to z$.

Therefore, $\{Px_{2n-2}\}, \{Tx_{2n-1}\}, \{Sx_{2n}\}, \{Qx_{2n+1}\} \to z$. As the pair *(P, S)* is reciprocally continuous. Therefore, $\lim_{n\to\infty} PSx_{2n} = Pz, \lim_{n\to\infty} SPx_{2n} = Sz$ and as *(P, S)* pair is compatible, therefore,

$$\lim_{n\to\infty} M(PSx_{2n}, SPx_{2n}, t) = 1, \lim_{n\to\infty} N(PSx_{2n}, SPx_{2n}, t) = 0$$

$$M(Pz, Sz, t) = 1, N(Pz, Sz, t) = 0.$$

Hence, $Pz = Sz$. Since, $P(X) \subseteq T(X)$, then there exist a point $u \in X$ such that $Pz = Tu$. By (3.2), we have

$$\begin{aligned} M(Pz, Qu, kt) &\geq \min\{M(Sz, Tu, t), M(Pz, Sz, t), M(Qu, Tu, t), M(Pz, Tu, t)\} \\ &= \min\{M(Pz, Pz, t), M(Pz, Pz, t), M(Qu, Pz, t), M(Pz, Pz, t)\} \end{aligned}$$

$$M(Pz, Qu, kt) \geq M(Pz, Qu, t),$$

$$N(Pz, Qu, kt) \geq \min\{M(Sz, Tu, t), M(Pz, Sz, t), M(Qu, Tu, t), M(Pz, Tu, t)\}$$

$$= \min\{M(Pz, Pz, t), M(Pz, Pz, t), M(Qu, Pz, t), M(Pz, Pz, t)\}$$

$$N(Pz, Qu, kt) \geq N(Pz, Qu, t),$$

Hence, by lemma 2.1, we have $Pz = Qu$. Thus, $Pz = Sz = Qu = Tu$. As *P* is *S*-absorbing then for $R > 0$, we have

$$M(Sz, SPz, t) \geq M(Sz, Pz, t/R) = 1$$

and $N(Sz, SPz, t) \leq N(Sz, Pz, t/R) = 0$

This gives, $Sz = SPz = Pz$.

Also, by (3.2), we have

$$\begin{aligned} M(Pz, PPz, kt) = M(PPz, Qu, kt) &\geq \min\{M(SPz, Tu, t), M(PPz, SPz, t), M(Qu, Tu, t), M(PPz, Tu, t)\} \\ &\geq \min\{M(Pz, Pz, t), M(PPz, Pz, t), 1, M(PPz, Pz, t)\} = M(Pz, Pz, t) = 1 \end{aligned}$$

and

$$\begin{aligned} N(Pz, PPz, kt) = M(PPz, Qu, kt) &\leq \max\{N(SPz, Tu, t), N(PPz, SPz, t), N(Qu, Tu, t), N(PPz, Tu, t)\} \\ &\leq \max\{N(Pz, Pz, t), N(PPz, Pz, t), 0, N(PPz, Pz, t)\} = N(Pz, Pz, t) = 0. \end{aligned}$$

therefore, we get, $Pz = PPz = SPz$. Therefore, Pz is a common fixed point of *P* and *S*.

Similarly, if *Q* is pointwise *T*-absorbing, therefore, we have

$$M(Tu, TQu, t) \geq M(Tu, Qu, t/R) = 1, N(Tu, TQu, t) \leq N(Tu, Qu, t/R) = 0.$$

This implies, $Tu = TQu = Qu$.

Now, by (3.2), we have

$$M(QQu,Qu,kt)=M(Pz,QQu,kt)\geq \min\{M(Sz,TQu,t),M(Pz,Sz,t),M(QQu,TQu,t),M(Pz,TQu,t)\}$$
$$=\min\{M(Sz,Qu,t),M(Pz,Pz,t),M(QQu,Qu,t),M(Pz,Qu,t)\}=M(QQu,Qu,t)$$

and

$$N(QQu,Qu,kt)=M(Pz,QQu,kt)\leq \max\{N(Sz,TQu,t),N(Pz,Sz,t),N(QQu,TQu,t),N(Pz,TQu,t)\}$$
$$=\max\{N(Sz,Qu,t),N(Pz,Pz,t),N(QQu,Qu,t),N(Pz,Qu,t)\}=N(QQu,Qu,t).$$

Therefore, by lemma 2.1, *QQu* = *Qu* = *TQu*. Hence, *Qu* = *Pz* is a common fixed point of *P, Q, S* and *T*.

UNIQUENESS:

Let w_1 and w_2 be two fixed points of *P, Q, S* and *T*. We show that $w_1 = w_2$. By (3.2), we have

$$M(w_1,w_2,kt)=M(Pw_1,Qw_2,kt)\geq \min\{M(Sw_1,Tw_2,t),M(Pw_1,Sw_1,t),M(Qw_2,Tw_2,t),M(Pw_1,Tw_2,t)\}$$
$$=\min\{M(w_1,w_2,t),M(w_1,w_1,t),M(w_2,w_2,t),M(w_1,w_2,t)\}=M(w_1,w_2,t)$$

and

$$N(w_1,w_2,kt)=N(Pw_1,Qw_2,kt)\leq \max\{N(Sw_1,Tw_2,t),N(Pw_1,Sw_1,t),N(Qw_2,Tw_2,t),N(Pw_1,Tw_2,t)\}$$
$$=\max\{N(w_1,w_2,t),N(w_1,w_1,t),N(w_2,w_2,t),N(w_1,w_2,t)\}=N(w_1,w_2,t).$$

By lemma 2.1, we get $w_1 = w_2$. Hence, *P, Q, S* and *T* have a unique common fixed point. Proof is similar when *Q* and *T* are assumed to be compatible and reciprocally continuous.

Theorem: 3.2 Let *P* be pointwise *S*-absorbing and *Q* be the pointwise *T*-absorbing self maps on intuitionistic fuzzy metric space $(X,M,N,*,\Diamond)$ with $a*b=\min\{a,b\}$ and $a\Diamond b=\max\{a,b\}$ for all $a,b\in[0,1]$ satisfying the conditions (3.1) and (3.2). If the range of one of the mappings *P, Q, S* or *T* be a complete subspace of *X* then *P, Q, S* and *T* have a unique common fixed point in *X*.

Proof: By using theorem (3.1), we show that $\{y_n\}$ is a Cauchy sequence in *X*. Let S(X) be a complete subspace of *X*. Therefore, there exist a point $u\in X$ such that $\{Px_{2n-2}\},\{Tx_{2n-1}\},\{Sx_{2n}\},\{Qx_{2n+1}\},\{y_n\}\to z=Su$. By using (3.2), we get

$$M(Pu,Qx_{2n+1},kt)\geq \min\{M(Su,Tx_{2n+1},t),M(Pu,Su,t),M(Qx_{2n+1},Tx_{2n+1},t),M(Pu,Tx_{2n+1},t)\}$$
$$n\to\infty$$
$$M(Pu,Su,kt)\geq \min\{M(Su,Su,t),M(Pu,Su,t),M(Su,Su,t),M(Pu,Su,t)\}=M(Pu,Su,t)$$
$$and$$
$$N(Pu,Qx_{2n+1},kt)\leq \max\{N(Su,Tx_{2n+1},t),N(Pu,Su,t),N(Qx_{2n+1},Tx_{2n+1},t),N(Pu,Tx_{2n+1},t)\}$$
$$n\to\infty$$
$$N(Pu,Su,kt)\leq \max\{N(Su,Su,t),N(Pu,Su,t),N(Su,Su,t),N(Pu,Su,t)\}=N(Pu,Su,t).$$

By lemma 2.1, we get *Pu* = *Su* = *Tw* for some $w\in X$.

Again, by (3.2), we have

$$M(Pu,Qw,kt)\geq \min\{M(Su,Tw,t),M(Pu,Su,t),M(Qw,Tw,t),M(Pu,Tw,t)\}$$
$$=\min\{M(Pu,Pu,t),M(Pu,Pu,t),M(Qw,Pu,t),M(Pu,Pu,t)\}=M(Qw,Pu,t)$$

and

$$N(Pu,Qw,kt)\leq \max\{N(Su,Tw,t),N(Pu,Su,t),N(Qw,Tw,t),N(Pu,Tw,t)\},$$

$$= \max\left\{N(Pu,Pu,t), N(Pu,Pu,t), N(Qw,Pu,t), N(Pu,Pu,t)\right\} = N(Qw,Pu,t)$$

By lemma 2.1, we get $Pu = Qw$. Therefore, $Pu = Qw = Su = Tw$. Since, P is pointwise S-absorbing, then we have

$$M(Su,SPu,t) \geq M(Su,Pu,t/R) = 1, N(Su,SPu,t) \leq N(Su,Pu,t/R) = 0.$$

Therefore, $Su = SPu = SSu$. Similarly, if Q is pointwise T-absorbing, therefore, we have

$$M(Tw,TQw,t) \geq M(Tw,Qw,t/R) = 1, N(Tw,TQw,t) \leq N(Tw,Qw,t/R) = 0.$$ Hence, $Tw = TQw = TTw$.

Therefore, $Su = Tw$ is a common fixed point of P, Q, S and T. Proof is similar if $T(X)$ or $Q(X)$ or $P(X)$ is complete subspace of X.

4. REFERENCES:

[1] C. Alaca, D. Turkoglu, and C. Yildiz, Fixed points in Intuitionistic fuzzy metric spaces, *Chaos, Solitons & Fractals*, 29(2006), 1073-1078.

[2] G, Jungck, Commuting mappings and fixed point, *Amer. Math. Monthly,* 83(1976), 261-263.

[3] G. Jungck and B. E. Rhoades, Fixed point Theorems for occasionally weakly compatible mappings, *Fixed point theory,* 7(2006), 286-296.

[4] K. Atanassov, Intuitionistic Fuzzy sets, *Fuzzy sets and system*, 20(1986) 87-96.

[5] I. Kramosil and J. Michalek, Fuzzy metric and Statistical metric spaces, *Kybernetica,* 11(1975), 326-334.

[6] S. Manro, Some fixed point theorems on expansion type maps in intuitionistic fuzzy metric spaces, *ARPN Journal of Systems and Software*, 1(8) (2011) 258-262.

[7] S. Manro, S. S. Bhatia, S. Kumar, Common fixed point theorems in fuzzy metric spaces, *Annals of Fuzzy Mathematics and Informatics*, 3(1)(2012), 151- 158.

[8] K. Menger, Statistical metrics, *Proc. Nat. Acad. Sci. (USA),* 28 (1942), 535- 537.

[9] J. H. Park, Intuitionistic fuzzy metric spaces, *Chaos, Solitons & Fractals*, 22(2004) 1039-1046.

[10] R.P. Pant, Common fixed point of four mappings, *Bull. Of Cal. Math. Soc*., 90 (1998) 281-286.

[11] B. Schweizer and A. Sklar, Probabilistic Metric Spaces, *North Holland Amsterdam,* 1983.

****

