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PERISTALTIC PUMPING OF A FOURTH GRADE FLUID THROUGH A POROUS MEDIUM IN AN ASYMMETRIC CHANNEL

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ABSTRACT

In this paper, the peristaltic flow of a fourth grade fluid through a porous medium in an asymmetric channel under the assumption of long wavelength is investigated. Series solutions of axial velocity and pressure gradient are given by using regular perturbation technique when Deborah number is small. Numerical computations have been performed for the pressure gradient and pressure rise. The effects of various pertinent parameters on the pressure gradient and pumping characteristics are discussed in detail.

Key words: *Asymmetric channel, Darcy number, Fourth grade fluid, peristaltic pumping.*

1. INTRODUCTION:

Most studies on the peristaltic motion assume the physiological fluids to behaving like Newtonian fluids with constant viscosity. However, this approach fails to give an ample understanding of the peristaltic mechanism involved in small blood vessels, lymphatic vessels, intestine, and ductus efferentus of the male reproductive tracts. In these body organs, the viscosity of the fluid varies across the thickness of the duct. Also, the assumption that the chyme in small intestine is a Newtonian material of variable viscosity is not adequate in reality. Chyme is undoubtedly a non-Newtonian fluid. Some authors feel that the main factor responsible for moving the chyme along the intestine is a gradient in the frequency of segmentation (a process of oscillating contraction and relaxation of smooth muscles in the intestine wall) along the length of intestine. Moreover, peristaltic waves die out after traveling a very short distance; peristaltic waves which travel the entire length of small intestine do not occur in humans except under abnormal conditions. Also, in transport of spermatozoa in the cervical canal, there are some other important factors, responsible for the transport of semen in ductus efferentes. The phenomenon of peristalsis has also been proposed as a mechanism for the transport of spermatic fluid (semen) in vasdeferens. Movement through vasdeferens is accomplished by means of peristaltic action of contractile cells in the duct wall (Semans and Longworthy [10]). However, there is no doubt that peristalsis aids in moving semen in ductus efferentus, the chyme in the intestine, and flow of semen in vas deferens. The above review of physiological flows indicates that non-Newtonian viscoelastic rheology is the correct way of properly describing the peristaltic flow through channels and tubes. The effects of third order fluid on peristaltic transport in a planar channel were studied by Siddiqui et al. [11] and the corresponding axisymmetric tube results are obtained by Hayat et al. [4]. Peristaltic flow of a MHD third grade fluid in a tube has studied by Hayat and Ali [5]. Also, Hayat et al. [6] investigated the peristaltic transport of a third order fluid under the effect of a magnetic field in a planar channel. Hayat et al. [7] have discussed peristaltic transport of a third order fluid in a channel. Peristaltic transport of a fourth grade fluid in an inclined asymmetric channel has studied by Haroun [3].

Flow through a porous medium has been of considerable interest in recent years particularly among geophysical fluid dynamicists. Examples of natural porous media are beach sand, sand stone, limestone, rye bread, wood, the human lung, bile duct, gall bladder with stones and in small blood vessels. Flow through a porous medium has analyzed by a number of workers by employing Darcy's law (Scheidegger [9]). Some studies about this point have been made by Varshney [13] and Raptis and Perdikis [8]. Elshehawey et al. [1] investigated the peristaltic flow of a generalized Newtonian fluid through a porous medium. Peristaltic motion of a Newtonian fluid through a porous medium in asymmetric channel was discussed by Elshehaway et al. [2]. Recently, Subba Reddy et al. [12] have studied the long wavelength approximation to MHD peristaltic flow of a Bingham fluid through a porous medium in an inclined channel.

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In view of these, we studied the peristaltic flow of a fourth grade fluid through a porous medium in an asymmetric channel under the assumption of long wavelength. Series solutions of axial velocity and pressure gradient are given by using regular perturbation technique when Deborah number is small. Numerical computations have been performed for the pressure gradient and pressure rise. The effects of various pertinent parameters on the pressure gradient and pressure rise are studied in detail through graphs.

2. MATHEMATICAL FORMULATION:

We consider the peristaltic flow of an incompressible fourth grade fluid through a porous medium in a two-dimensional asymmetric channel of width $a_1 + a_2$.The flow is induced by sinusoidal wave trains propagating with constant speed c along the channel walls. Fig. 1 shows the physical model of the problem.

The geometry of the wall surfaces is defined

$$Y = H_1(X,t) = a_1 + b_1 cos\left[\frac{2\pi}{\lambda}(X - ct)\right] \qquad \text{(Upper wall)} \tag{2.1}$$

$$Y = H_2(X,t) = -a_2 - b_2 cos\left[\frac{2\pi}{\lambda}(X - ct) + \theta\right] \qquad \text{(Lower wall)} \tag{2.2}$$

where b_1, b_2 are the amplitudes of the upper and lower waves, λ is the wavelength, θ is the phase difference which varies in the range $0 \leq \theta \leq \pi$ and t is the time. Further, a_1, a_2, b_1, b_2 and θ satisfies the condition $b_1^2 + b_2^2 + 2b_1b_2cos\theta \leq (a_1 + a_2)^2$ so that walls will not intersect with each other.

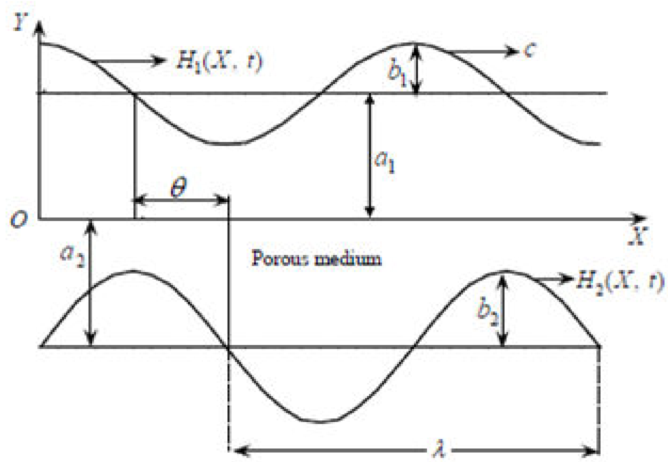

Fig. 1 Schematic diagram of the asymmetric channel

$$\rho\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial X} + V\frac{\partial}{\partial Y}\right)U = -\frac{\partial P}{\partial X} + \frac{\partial S_{XX}}{\partial X} + \frac{\partial S_{XY}}{\partial Y} - \frac{\mu}{k}U \tag{2.3}$$

$$\rho\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial X} + V\frac{\partial}{\partial Y}\right)V = -\frac{\partial P}{\partial Y} + \frac{\partial S_{XY}}{\partial X} + \frac{\partial S_{YY}}{\partial Y} - \frac{\mu}{k}V \tag{2.4}$$

and the equation of continuity is

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{2.5}$$

where ρ is the density, U and V are the velocity components in X and Y directions, P is the pressure, k is the permeability of the porous medium and S_{XX} , S_{XY} , S_{YY} are the components of extra stress tensor.

The constitutive equations for an incompressible fourth grade fluid are

$$T = -pI + S \tag{2.6}$$

$$\begin{aligned} S = {} & \mu A_1 + \alpha_1 A_2 + \alpha_2 A_1^2 + \beta_1 A_3 + \beta_2(A_1A_2 + A_2A_1) + \beta_3(trA_1^2)A_1 + \gamma_1 A_4 \\ & + \gamma_2(A_1A_3 + A_3A_1) + \gamma_3 A_2^2 + \gamma_4(A_1^2A_2 + A_2A_1^2) + \gamma_2(trA_2)A_2 + \\ & + \gamma_6(trA_2)A_1^2 + [\gamma_7 trA_3 + \gamma_8 tr(A_2A_1)]A_1 \end{aligned} \tag{2.7}$$

where μ is constant viscosity and $\alpha_1, \alpha_2, \beta_1, \beta_2, \beta_3, \gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_2, \gamma_6, \gamma_7, \gamma_8$ being material constants and A_n representing the Rivlin-Ericksen tensors defined by

$$A_n = \frac{dA_{n-1}}{dt} + A_{n-1}(grad\ V) + (grad\ V)^T A_{n-1}, n > 1 \tag{2.8}$$

$$A_1 = (grad\ V) + (grad\ V)^T \tag{2.9}$$

$$x = X - ct, y = Y, u(x,y) = U - c, v(x,y) = V, p(x,y) = P(X,Y,t) \tag{2.10}$$

Introducing the non-dimensional parameters and variables

$$\bar{x} = \frac{x}{\lambda}, \bar{y} = \frac{y}{a_1}, \bar{u} = \frac{u}{c}, \bar{v} = \frac{v}{c\delta}, \bar{S} = \frac{a_1}{\mu c} S, \bar{p} = \frac{p a_1^2}{\mu c \lambda}, \bar{t} = \frac{ct}{\lambda}, h_1 = \frac{H_1}{a_1}, h_2 = \frac{H_2}{a_1}$$

$$\delta = \frac{a}{\lambda}, \phi_1 = \frac{b_1}{a_1}, \phi_2 = \frac{b_2}{a_1}, d = \frac{a_2}{a_1}, \tag{2.11}$$

Using Eqs. (2.10), we obtain Eqs. (2.3) - (2.2) as

$$\rho\left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right)u = -\frac{\partial p}{\partial x} + \frac{\partial S_{xx}}{\partial x} + \frac{\partial S_{xy}}{\partial y} - \frac{\mu}{k}(u + c) \tag{2.12}$$

$$\rho\left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right)v = -\frac{\partial p}{\partial y} + \frac{\partial S_{xy}}{\partial x} + \frac{\partial S_{yy}}{\partial y} - \frac{\mu}{k}v \tag{2.13}$$

and the equation of continuity is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.14}$$

where (u, v) are velocity components in the wave frame.

Using Eqs. (2.9), we obtain Eqs. (2.12) - (2.14) as

$$Re\delta\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \delta\frac{\partial S_{xx}}{\partial x} + \frac{\partial S_{xy}}{\partial y} - \frac{1}{Da}(u + 1) \tag{2.15}$$

$$Re\delta^3\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \delta^2\frac{\partial S_{xy}}{\partial x} + \delta\frac{\partial S_{yy}}{\partial y} - \frac{\delta^2}{Da}v \tag{2.16}$$

and the equation of continuity is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.17}$$

where

$$\begin{aligned} S = {} & A_1 + \lambda_1 A_2 + \lambda_2 A_1^2 + \xi_1 A_3 + \xi_2(A_1A_2 + A_2A_1) + \xi_3(trA_1^2)A_1 + \eta_1 A_4 \\ & + \eta_2(A_1A_3 + A_3A_1) + \eta_3 A_2^2 + \eta_4(A_1^2A_2 + A_2A_1^2) + \eta_2(trA_2)A_2 + \\ & + \eta_6(\mathrm{trA_2})A_1^2 + [\eta_7 \mathrm{trA_3} + \eta_8 \mathrm{tr}(A_2A_1)]A_1 \end{aligned} \tag{2.18}$$

$$A_n = \left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right)A_{n-1} + A_{n-1}(grad\ V) + (grad\ V)^T A_{n-1}$$

Here the wave number δ, Reynolds number Re, Darcy number Da, the material coefficients $\lambda_1, \lambda_1, \xi_1, \xi_2, \xi_3, \eta_1, \eta_2, \eta_3, \eta_4, \eta_2, \eta_6, \eta_7, \eta_8$ are

$$\delta = \frac{a}{\lambda}, Re = \frac{\rho a_1 c}{\mu}, \ Da = \frac{k}{a_1^2},$$

$$\lambda_1 = \frac{\alpha_1 c}{\mu a_1}, \lambda_2 = \frac{\alpha_2 c}{\mu a_1}, \xi_1 = \frac{\beta_1 c^2}{\mu a_1^2}, \ \ \xi_2 = \frac{\beta_2 c^2}{\mu a_1^2},$$

$$\xi_3 = \frac{\beta_3 c^2}{\mu a_1^2}, \eta_1 = \frac{\gamma_1 c^3}{\mu a_1^3}, \eta_2 = \frac{\gamma_2 c^3}{\mu a_1^3}, \eta_3 = \frac{\gamma_3 c^3}{\mu a_1^3}, \tag{2.19}$$

$$\eta_4 = \frac{\gamma_4 c^3}{\mu a_1^3}, \eta_2 = \frac{\gamma_2 c^3}{\mu a_1^3}, \eta_6 = \frac{\gamma_6 c^3}{\mu a_1^3}, \eta_7 = \frac{\gamma_7 c^3}{\mu a_1^3}, \eta_8 = \frac{\gamma_8 c^3}{\mu a_1^3},$$

The corresponding dimensionless boundary conditions in wave frame of reference are given by

$$u = -1 \quad \text{at} \quad y = h_1 = 1 + \phi_1 cos2\pi x \tag{2.20}$$

$$u = -1 \quad \text{at} \quad y = h_2 = -d - \phi_2 cos(2\pi x + \theta) \tag{2.21}$$

where $\phi_1 = \frac{b_1}{a_1}, \phi_2 = \frac{b_2}{a_1}, d = \frac{a_2}{a_1}$.

Under the assumption of long wavelength, the Eqs. (2.15) and (2.16), become

$$0 = -\frac{\partial p}{\partial x} + \frac{\partial S_{xy}}{\partial y} - \frac{1}{Da}(u+1) \tag{2.22}$$

$$0 = -\frac{\partial p}{\partial y} \tag{2.23}$$

here $S_{xy} = \frac{\partial u}{\partial y} + 2\Gamma\left(\frac{\partial u}{\partial y}\right)^3$, in which the Deborah number Γ is $\Gamma = \xi_2 + \xi_3$.

Eqs. (2.22) and (2.23) can be rewritten as

$$\frac{dp}{dx} = \frac{\partial}{\partial y}\left[\frac{\partial u}{\partial y} + 2\Gamma\left(\frac{\partial u}{\partial y}\right)^3\right] - \sigma^2(u+1) \tag{2.24}$$

here $\sigma^2 = \frac{1}{Da}$.

The volume flow rate in wave frame of reference is given by

$$q = \int_{h_2}^{h_1} u dy \tag{2.25}$$

The instantaneous flux $Q(X,t)$in the laboratory frame is

$$Q(X,t) = \int_{h_2}^{h_1}(u+1)dy = \int_{h_2}^{h_1} u dy + \int_{h_2}^{h_1} dy = q + h_1 - h_2 \tag{2.26}$$

The average flux over one period $(T = \lambda/c)$ of the peristaltic wave is

$$\bar{Q} = \frac{1}{T}\int_0^T Q dt = \frac{1}{T}\int_0^T (q + h_1 - h_2)dt = q + 1 + d \tag{2.27}$$

3. SOLUTION:

Eq. (2.24) is non-liner differential equation, so that it is not possible to obtain a closed form solution, so we seek a perturbation solution. We expand the flow quantities in a power series of the small parameter Deborah number Γ as follows:

$$u = u_0 + \Gamma u_1 + O(\Gamma^2), \tag{3.1}$$

$$p = p_0 + \Gamma p_1 + O(\Gamma^2), \tag{3.2}$$

$$q = q_0 + \Gamma q_1 + O(\Gamma^2), \tag{3.3}$$

Substituting these equations into the Eqs. (2.24), (2.20) and (2.21) we obtain

3.1 System of order zero:

$$\frac{\partial^2 u_0}{\partial y^2} - \sigma^2(u_0 + 1) = \frac{dp_0}{dx}, \tag{3.4}$$

Together with the boundary conditions

$$u_0 = -1 \;\; at \;\; y = h_1 \tag{3.5}$$

$$u_0 = -1 \;\; at \;\; y = h_2 \tag{3.6}$$

3.2 System of order one:

$$\frac{\partial^2 u_1}{\partial y^2} - \sigma^2 u_1 = \frac{dp_1}{dx} - 2\frac{\partial}{\partial y}\left(\frac{\partial u_0}{\partial y}\right)^3 \tag{3.7}$$

Together with the boundary conditions

$$u_1 = 0 \quad at \quad y = h_1 \tag{3.8}$$

$$u_1 = 0 \quad at \quad y = h_2 \tag{3.9}$$

3.3 Zeroth-order solution:

Solving Eq. (3.4) using boundary conditions (3.5) and (3.6), we get

$$u_0 = \frac{1}{\sigma^2}\left(\frac{dp_0}{dx}\right)[A_1 \cosh \sigma y + A_2 \sinh \sigma y - 1] - 1. \tag{3.10}$$

where $A_1 = \frac{\sinh \sigma h_2 - \sinh \sigma h_1}{\sinh \sigma(h_2 - h_1)}$ and $A_2 = \frac{\cosh \sigma h_1 - \cosh \sigma h_2}{\sinh \sigma(h_2 - h_1)}$. and the volume flow rate q_0 is given by

$$q_0 = \int_{h_2}^{h_1} u_0 dy$$

$$= \frac{1}{\sigma^3}\left(\frac{dp_0}{dx}\right)\frac{[2 - 2\cosh \sigma(h_1 - h_2) - \sigma(h_1 - h_2)\sinh \sigma(h_2 - h_1)]}{\sinh[\sigma(h_2 - h_1)]} - (h_1 - h_2) \tag{3.11}$$

From Eq. (3.11)

$$\frac{dp_0}{dx} = \frac{[q_0 + (h_1 - h_2)]\sigma^3 \sinh[\sigma(h_2 - h_1)]}{2 - 2\cosh \sigma(h_1 - h_2) - \sigma(h_1 - h_2)\sinh \sigma(h_2 - h_1)} \tag{3.12}$$

3.4 First order solution:

Substituting Eq. (3.10) into Eq. (3.7) and solving Eq. (3.7) using boundary conditions (3.8) and (3.9), we obtain

$$u_1 = \frac{1}{\sigma^2}\left(\frac{dp_1}{dx}\right)[A_1 \cosh \sigma y + A_2 \sinh \sigma y - 1] + \frac{1}{4M^4}\left(\frac{dp_0}{dx}\right)^3 [A_9 \cosh \sigma y + A_{10} \sinh \sigma y - 3A_3 \cosh 3\sigma y - \\ - 3A4 \sinh 3\sigma y - 4A2\sigma y \sinh \sigma y - 4A6\sigma y \cosh \sigma y \tag{3.13}$$

where $A_3 = \frac{1}{4}[A_1^3 + 3A_1A_2^2]$, $A_4 = \frac{1}{4}[A_2^3 + 3A_1^2A_2]$,

$A_2 = \frac{3}{4}[A_1A_2^2 - A_1^3]$, $A_6 = \frac{3}{4}[A_2^3 - A_1^2A_2]$,

$A_7 = 3A_3 \cosh 3\sigma h_1 + 3A_4 \sinh 3\sigma h_1 + 4A_2\sigma h_1 \sinh \sigma h_1 + 4A_6\sigma h_1 \cosh \sigma h_1$,

$A_8 = 3A_{31} \cosh 3\sigma h_2 + 3A_4 \sinh 3\sigma h_2 + 4A_2\sigma h_2 \sinh \sigma h_2 + 4A_6\sigma h_2 \cosh \sigma h_2$,

$A_9 = \frac{A_7 \sinh \sigma h_2 - A_8 \sinh \sigma h_1}{\sinh[\sigma(h_2 - h_1)]}$, $A_{10} = \frac{A_8 \cosh \sigma h_1 - A_7 \cosh \sigma h_2}{\sinh[\sigma(h_2 - h_1)]}$, and the volume flow rate q_1 is given by

$$q_1 = \int_{h_2}^{h_1} u_1 dy$$

$$= \frac{1}{\sigma^3}\frac{dp_1}{dx}\left[\frac{2 - 2\cosh \sigma(h_1 - h_2) - \sigma(h_1 - h_2)\sinh \sigma(h_2 - h_1)}{\sinh[\sigma(h_2 - h_1)]}\right] + \frac{1}{4\sigma^4}\left(\frac{dp_0}{dx}\right)^3 A_{11} \tag{3.14}$$

From Eq. (3.14), we have

$$\frac{dp_1}{dx} = \frac{\left[q_1 - \frac{1}{4\sigma^4}\left(\frac{dp_0}{dx}\right)^3 A_{11}\right]\sigma N^3 \sinh[\sigma(h_2 - h_1)]}{2 - 2\cosh \sigma(h_1 - h_2) - \sigma(h_1 - h_2)\sinh \sigma(h_2 - h_1)} \tag{3.15}$$

where $A_{11} = \frac{(A_9+4A_2)}{\sigma}[\sinh\sigma h_1 - \sinh\sigma h_2] + \frac{(A_{10}+4A_6)}{N}[\cosh\sigma h_1 - \cosh\sigma h_2] - \frac{A_3}{\sigma}[\sinh 3\sigma h_1 - \sinh 3\sigma h_2] - \frac{A_4}{\sigma}[\cosh 3\sigma h_1 - \cosh 3\sigma h_2] - 4A_2[h_1\cosh\sigma h_1 - h_2\cosh\sigma h_2] - 4A_6[h_1\sinh\sigma h_1 - h_2\sinh\sigma h_2]$

Substituting from equations (3.12) and (3.15) into the Eq. (3.2) and using the relation $\frac{dp}{dx} = \frac{dp_0}{dx} + \Gamma\frac{dp_1}{dx}$ and neglecting terms greater than $O(\Gamma^2)$, we get

$$\frac{dp}{dx} = \left(\frac{[q+(h_1-h_2)]\sigma^3\sinh[\sigma(h_2-h_1)]}{2-2\cosh\sigma(h_1-h_2)-\sigma(h_1-h_2)\sinh\sigma(h_2-h_1)}\right) + \frac{\Gamma}{4}\left(\frac{\left[(q+(h_1-h_2))^3\right]\sigma^8 A_{11}\sinh[\sigma(h_2-h_1)]}{[2-2\cosh\sigma(h_1-h_2)-\sigma(h_1-h_2)\sinh\sigma(h_2-h_1)]^4}\right) \tag{3.16}$$

The pressure rise per wave length (Δp) and is defined as

$$\Delta p = \int_0^1 \frac{dp}{dx}\,dx\,, \tag{3.17}$$

The above integral numerically evaluated using the MATLAB software.

4. DISCUSSION OF THE RESULTS:

Fig. 2 depicts the variation of axial pressure gradient $\frac{dp}{dx}$ with Debaroh number Γ for $\phi_1 = 0.5,\ \phi_2 = 0.7,\ d = 1.2$, $\bar{Q} = -1\,, Da = 0.1$ and $\theta = \frac{\pi}{6}$. It is observed that, the axial pressure gradient dp/dx increases with an increase in Deborah number Γ. Further, it is observed that, the axial pressure gradient is more for fourth grade fluid $(0 < \Gamma < 1)$ than that of Newtonian fluid $(\Gamma = 0)$.

The variation of axial pressure gradient $\frac{dp}{dx}$ with Darcy number Da for $\phi_1 = 0.5,\ \phi_2 = 0.7, \bar{Q} = -1\,, d = 1.2$, $\Gamma = 0.01$and $\theta = \frac{\pi}{6}$ is depicted in Fig. 3. It is noted that, the axial pressure gradient dp/dx decreases on increasing Darcy numberDa.

Fig. 4 shows the variation of the axial pressure gradient dp/dx with phase shift θ for $\phi_1 = 0.5,\ \phi_2 = 0.7, \bar{Q} = -1\,, d = 1.2, Da = 0.1$ and $\Gamma = 0.01$. It is observed that, the axial pressure gradient dp/dx decreases with an increase in phase shift θ.

The variation of axial pressure gradient $\frac{dp}{dx}$ with upper wave ϕ_1 for $Da = 0.1,\ \phi_2 = 0.7, \bar{Q} = -1\,, d = 1.2$, $\Gamma = 0.01$ and $\theta = \frac{\pi}{6}$ is shown in Fig. 5. It is found that, the axial pressure gradient dp/dx increases with an increase in amplitude of the upper wave ϕ_1. The same trend is observed for lower wave amplitude ϕ_2 as shown in Fig. 6.

Fig. 7 illustrates the variation of the axial pressure gradient dp/dx with width of the channel d for $\Gamma = 0.01,\ \phi_1 = 0.5$, $\phi_2 = 0.7, \bar{Q} = -1\,, Da = 0.1$ and $\theta = \frac{\pi}{6}$. It is noted that, the axial pressure gradient dp/dx decreases with an increase in d.

The variation of pressure rise Δp with time averaged flux $\bar{Q}$ for different values of Deborah number Γ with $\phi_1 = 0.5$, $\phi_2 = 0.7,\ d = 1.2,\ Da = 0.1$ and $\theta = \frac{\pi}{6}$ is shown in Fig. 8. It is found that, the time-averaged flux $\bar{Q}$ increases with increasing Deborah number Γ in both the pumping region $(\Delta p > 0)$and free-pumping $(\Delta p = 0)$, while it decreases with increasing Deborah number Γ in the co-pumping $(\Delta p < 0)$ region.

Fig. 9 depicts the variation of Δp with time-averaged flux $\bar{Q}$ for different values of Darcy number Da with $\phi_1 = 0.5$, $\phi_2 = 0.7,\ d = 1.2,\ \Gamma = 0.01$ and $\theta = \frac{\pi}{6}$. It is observed that, any two pumping curves intersecting in the first quadrant, to the left of this point of intersection, the time-averaged flux $\bar{Q}$ decreases with increasing Darcy number Da and to the right of this point of intersection, the $\bar{Q}$ increases with increasing Da.

Fig. 10 shows the variation of Δp with time-averaged flux $\bar{Q}$ for different values of phase shift θ with $\phi_1 = 0.5$, $\phi_2 = 0.7,\ d = 1.2,\ \Gamma = 0.01$ and $Da = 0.1$. It is noted that, the time-averaged flux $\bar{Q}$ decreases with an increase in θ in both the pumping region and free pumping region, while it increases with increasing θ in the co-pumping regions for appropriately chosen $\Delta p(< 0)$.

The variation of Δp with time-averaged flux $\bar{Q}$ for different values of amplitude of the upper wave ϕ_1 with $M = 1, \phi_2 = 0.7, d = 1.2, \Gamma = 0.01$ and $\theta = \frac{\pi}{6}$ is presented in Fig. 11. It is found that, the time-averaged flux $\bar{Q}$ increases on

increasing ϕ_1 in both the pumping region and free-pumping region, while it decreases with increasing ϕ_1 in the co-pumping region for appropriately chosen $\Delta p (< 0)$. The same phenomenon is observed for the lower wave amplitude ϕ_2 as presented in Fig. 12.

Fig. 13 shows the variation of Δp with time-averaged flux $\bar{Q}$ for different values of width of the channel d with $M = 1, \phi_1 = 0.5, \phi_2 = 0.7, \Gamma = 0.01$ and $\theta = \frac{\pi}{6}$. It is observed that, the time-averaged flux $\bar{Q}$ decreases with an increase d in both the pumping region and free pumping region, while it increases with increasing d in the co-pumping region.

5. CONCLUSIONS:

In this paper, we studied the effect of magnetic field on the peristaltic pumping of an incompressible fourth grade fluid through a porous medium in an asymmetric channel under assumption of long wavelength. A perturbation technique is obtained for the case in which Deborah number Γ is small. It is found that, the axial pressure gradient and the pumping increases with increasing Γ, ϕ_1 and ϕ_2, while they decreases with increasing Da, θ and d.

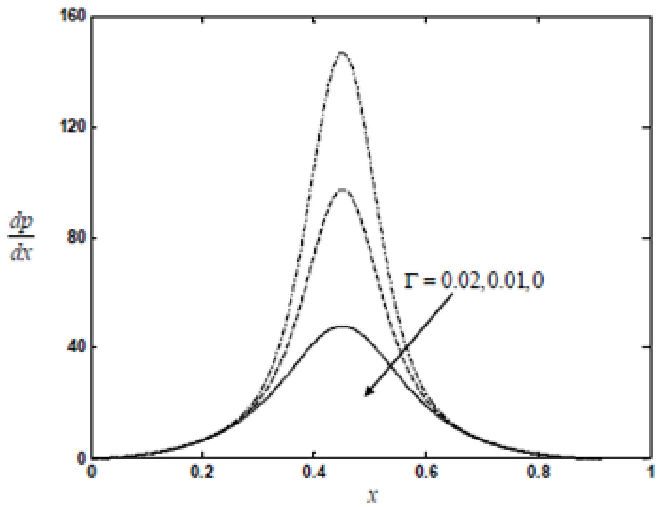

Fig. 2 Profiles of axial pressure gradient $\frac{dp}{dx}$ for different values of Deborah number Γ for $\phi_1 = 0.5$, $\phi_2 = 0.7$, $d = 1.2$, $\bar{Q} = -1$, $Da = 0.1$ and $\theta = \frac{\pi}{6}$.

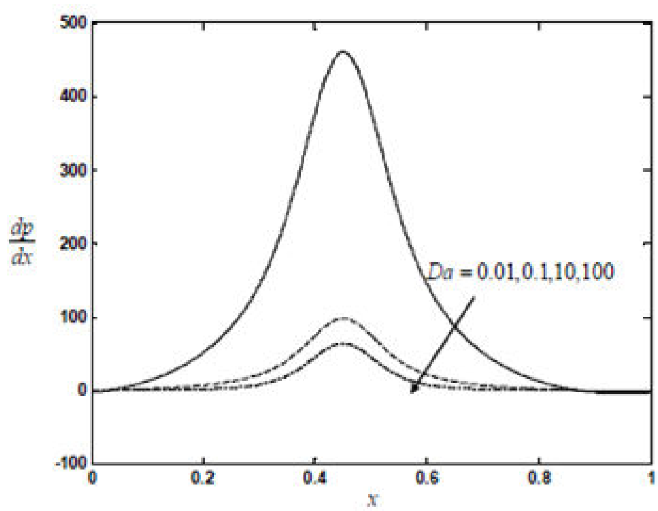

Fig. 3 Profiles of axial pressure gradient $\frac{dp}{dx}$ for different values of Darcy number Da for $\phi_1 = 0.5$, $\phi_2 = 0.7$, $d = 1.2$, $\bar{Q} = -1$, $\Gamma = 0.01$ and $\theta = \frac{\pi}{6}$.

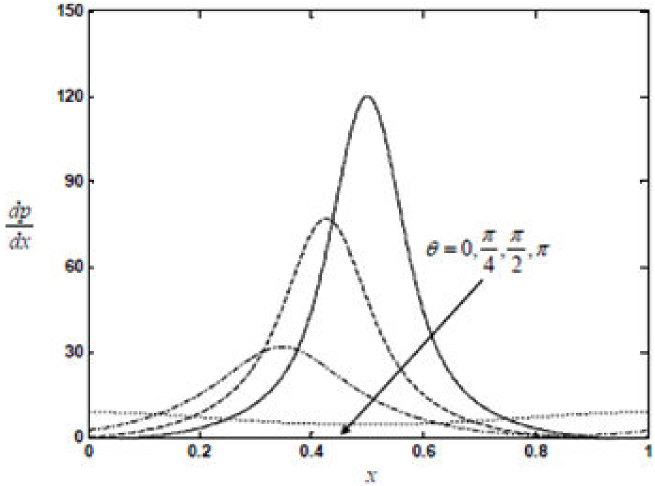

Fig. 4 Profiles of axial pressure gradient $\frac{dp}{dx}$ for different values of phase shift θ for $\phi_1 = 0.5$, $\phi_2 = 0.7$, $d = 1.2, \bar{Q} = -1$, $\Gamma = 0.01$ and $Da = 0.1$.

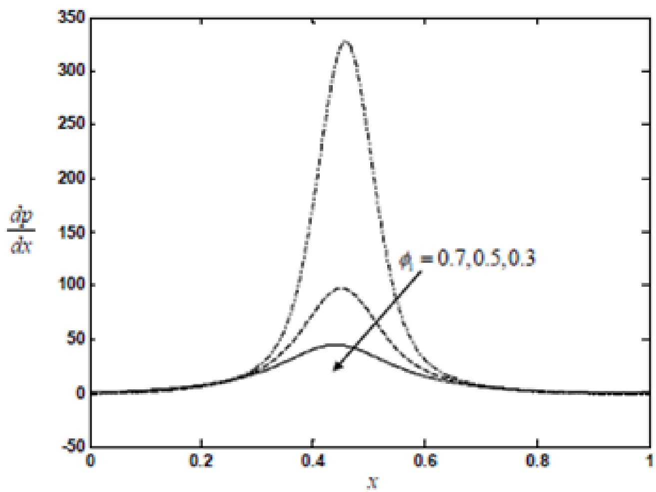

Fig. 5 Profiles of axial pressure gradient $\frac{dp}{dx}$ for different values of ϕ_1 for $\Gamma = 0.01$, $\phi_2 = 0.7$, $d = 1.2, Da = 0.1$ and $\theta = \frac{\pi}{6}$.

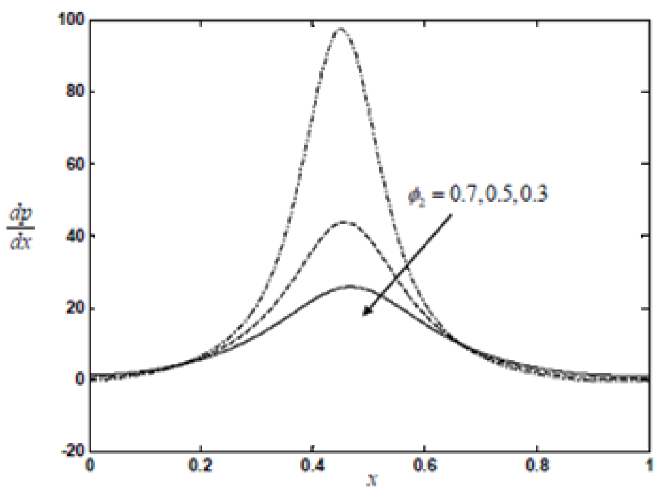

Fig. 6 Profiles of axial pressure gradient $\frac{dp}{dx}$ for different values of ϕ_2 for $\Gamma = 0.01$, $\phi_1 = 0.5$, $d = 1.2$, $Da = 0.1, \bar{Q} = -1$ and $\theta = \frac{\pi}{6}$.

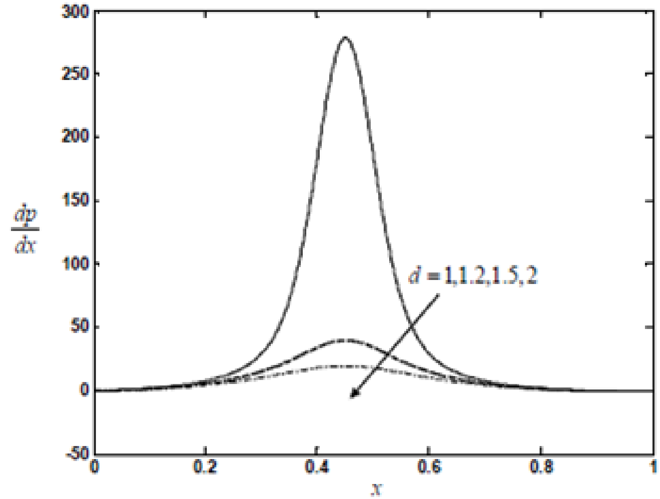

Fig. 7 Profiles of axial pressure gradient $\frac{dp}{dx}$ for different values of d for $\Gamma = 0.01$, $\phi_1 = 0.5$, $\phi_2 = 0.7$, $Da = 0.1, \bar{Q} = -1$ and $\theta = \frac{\pi}{6}$.

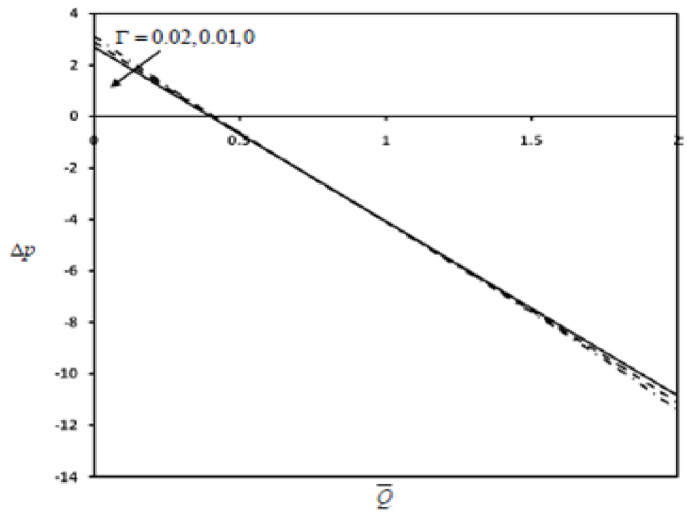

Fig. 8 The variation of pressure rise Δp with time averaged flux $\bar{Q}$ for different values of Deborah number Γ with $\phi_1 = 0.5$, $\phi_2 = 0.7$, $d = 1.2$, $Da = 0.1$ and $\theta = \frac{\pi}{6}$.

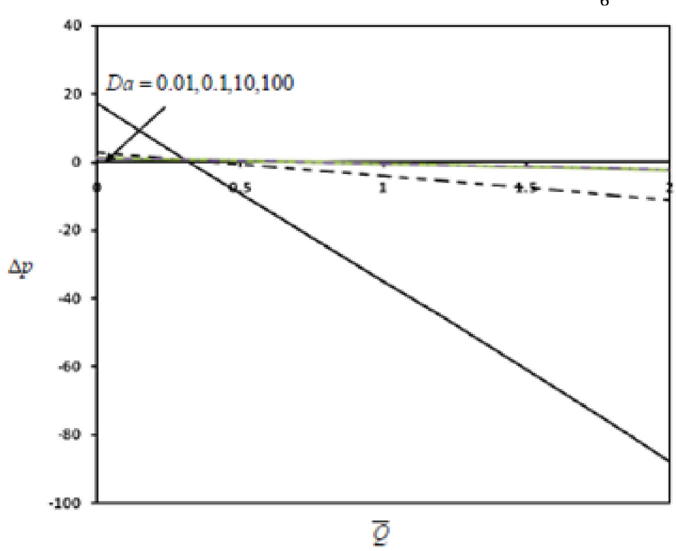

Fig. 9 The variation of pressure rise Δp with time averaged flux $\bar{Q}$ for different values of Darcy number Da with $\phi_1 = 0.5$, $\phi_2 = 0.7$, $d = 1.2$, $\Gamma = 0.01$ and $\theta = \frac{\pi}{6}$.

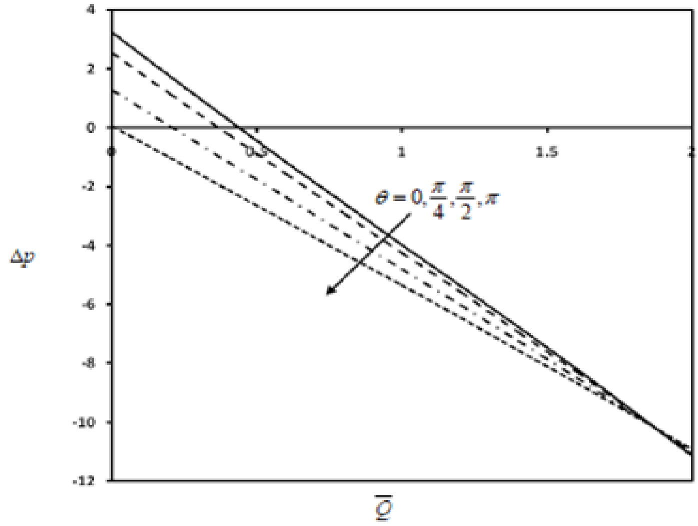

Fig. 10 The variation of pressure rise Δp with time averaged flux $\bar{Q}$ for different values of phase shift θ with $\phi_1 = 0.5$, $\phi_2 = 0.7$, $d = 1.2$, $\Gamma = 0.01$ and $Da = 0.1$.

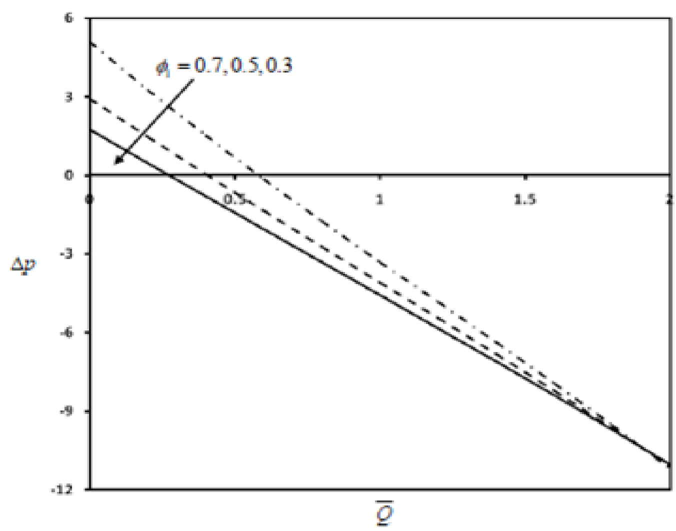

Fig. 11 The variation of pressure rise Δp with time averaged flux $\bar{Q}$ for different values of ϕ_1 with $Da = 0.1$, $\phi_2 = 0.7, d = 1.2$, $\Gamma = 0.01$ and $\theta = \frac{\pi}{6}$.

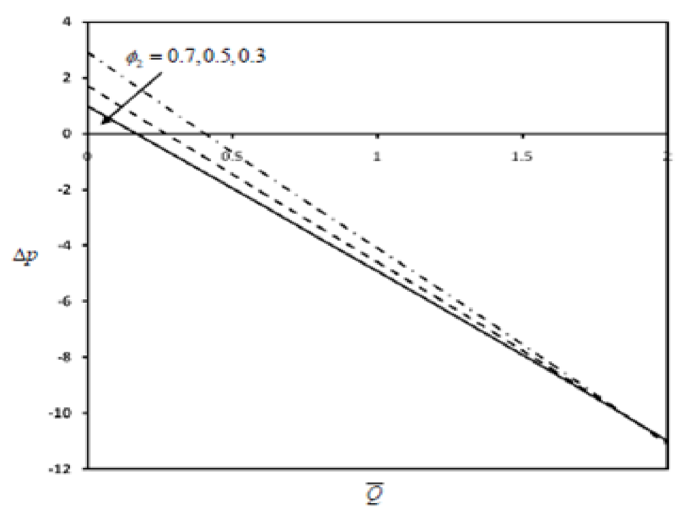

Fig. 12 The variation of pressure rise Δp with time averaged flux $\bar{Q}$ for different values of ϕ_2 with $Da = 0.1, \phi_1 = 0.5$, $d = 1.2$, $\Gamma = 0.01$ and $\theta = \frac{\pi}{6}$.

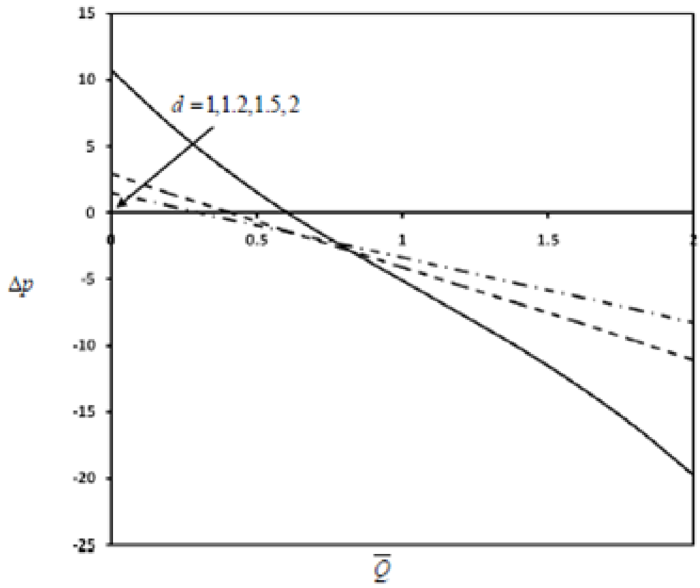

Fig. 13 The variation of pressure rise Δp with time averaged flux $\bar{Q}$ for different values of d with $Da = 0.1, \phi_1 = 0.5$, $\phi_2 = 0.7$, $\Gamma = 0.01$ and $\theta = \frac{\pi}{6}$.

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