International Journal of Mathematical Archive-3(2), 2012, Page: 354-361
IJMA Available online through www.ijma.info ISSN 2229 – 5046

ORDERED INTUITIONISTIC FUZZY PRE SEMI BASICALLY DISCONNECTED SPACES

D. Amsaveni*, M. K. Uma and E. Roja

Sri Sarada College for Women, Salem-636016, Tamilnadu, India
E-mail: d_amsaveni@rediffmail.com

(Received on: 13-01-12; Accepted on: 17-02-12)

ABSTRACT

In this paper, a new class of intuitionistic fuzzy topological spaces called ordered intuitionistic fuzzy pre semi basically disconnected spaces is introduced. Tietze extension theorem for ordered intuitionistic fuzzy pre semi basically disconnected spaces has been discussed as in [15] and several other properties are also discussed.

Key words: *Ordered intuitionistic fuzzy pre semi basically disconnected spaces, ordered intuitionistic fuzzy pre semi continuous functions, and lower F_σ (resp.upper F_σ) intuitionistic fuzzy pre semi continuous functions.*

---

1. INTRODUCTION:

After the introduction of the concept of fuzzy sets by Zadeh [18], several researches were conducted on the generalizations of the notion of fuzzy set. The concept of "Intuitionistic fuzzy sets" was first published by Atanassov [2] and many works by the same author and his colleagues appeared in the literature [3-5]. Later this concept was generalized to "Intuitionistic L-fuzzy sets" by Atanassov and stoeva [6]. An introduction to intuitionistic fuzzy topological space was introduced by Dogan Coker [9]. Several types of fuzzy connectedness in intuitionistic fuzzy topological spaces were defined by Coker (1997). The construction is based on the idea of intuitionistic fuzzy set developed by Atanassov (1983, 1986; Atanassov and Stoeva, 1983). In this paper a new class of intuitionistic fuzzy topological spaces namely, ordered intuitionistic fuzzy pre semi basically disconnected spaces is introduced by using the concepts of [11,13,15]. 'Intuitionistic fuzzy pre semi closed sets' was introduced by [1]. Tietze extension theorem for ordered intuitionistic fuzzy pre semi basically disconnected spaces has been discussed as in [15].

2. PRELIMINARIES:

Throughout this paper let X be a non empty set and $I=[0,1]$.

Definition: 2.1[4] Let X be a non empty fixed set. An intuitionistic fuzzy set (*IFS* for short) A is an object having the form $A=\{\langle x,\mu_A(x),\gamma_A(x)\rangle : x\in X\}$ where the functions $\mu_A : X\to I$ and $\gamma_A : X\to I$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non membership (namely $\gamma_A(x)$) of each element $x\in X$ to the set A, respectively, and $0\le \mu_A(x)+\gamma_A(x)\le 1$ for each $x\in X$.

Remark: 2.1[4] For the sake of simplicity, we shall use the symbol $A=\langle x,\mu_A,\gamma_A\rangle$.

Definition: 2.2[9] Let X be a non empty fixed set. Then, $0_\sim=\{\langle x,0,1\rangle : x\in X\}$ and $1_\sim=\{\langle x,1,0\rangle : x\in X\}$.

Definition: 2.3[9] Let X be a non empty set. An intuitionistic fuzzy topology (*IFT* for short) on a non empty set X is a family τ of intuitionistic fuzzy sets (*IFSs* for short) in X satisfying the following axioms:
(T_1) $0_\sim, 1_\sim \in\tau$,
(T_2) $G_1\cap G_2\in\tau$ for any $G_1,G_2\in\tau$,

***Corresponding author: D. Amsaveni*, *E-mail: d_amsaveni@rediffmail.com**

(T$_3$) $\bigcup_{\subseteq\tau} G_i \in \tau$ for any arbitrary family $\{G_i : i \in J\}$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (*IFTS* for short) and any *IFS* in τ is known as an intuitionistic fuzzy open set (*IFOS* for short) in X.

Definition: 2.4[9] Let (X, τ) be an *IFTS* and $A = \langle x, \mu_A, \gamma_A \rangle$ be an *IFS* in X. Then the fuzzy interior and fuzzy closure of A are defined by

$$cl(A) = \bigcap\{K : K \text{ is an } IFCS \text{ in } X \text{ and } A \subseteq K\},\ \operatorname{int}(A) = \bigcup\{G : G \text{ is an } IFOS \text{ in } X \text{ and } G \subseteq A\}.$$

NOTATION: 2.1 An *IFTS* (X, T) represent intuitionistic fuzzy topological spaces and for a subset *A* of a space (X, T), *IFcl(A), IFint(A), IFPScl(A), IFPSint(A),* $IF\ sp\operatorname{int}(A)$, $IF\ spcl(A)$ and $\overline{A}$ denote an intuitionistic fuzzy closure of *A*, an intuitionistic fuzzy interior of *A*, intuitionistic fuzzy pre semi closure of *A*, an intuitionistic fuzzy semi pre interior of *A*, an intuitionistic fuzzy pre semi interior of *A*, an intuitionistic fuzzy semi pre closure of *A* and the complement of *A* in X respectively.

Definition: 2.5 17] A subset *A* of an *IFTS* (X, T) is called an *IF* semi pre open set if $A \subseteq IFcl(IF\operatorname{int}(IFcl(A)))$ and an *IF* semi pre closed set if $IF\operatorname{int}(IFcl(IF\operatorname{int}(A))) \subseteq A$;

Definition: 2.6[14] A subset *A* of an *IFTS* (X, T) is called an *IF* generalized closed (briefly *IF g*-closed) set if $IF\ cl(A) \subseteq U$ whenever $A \subseteq U$ and *U* is *IF* open in (X, T). The complement of an *IF g*-closed set is called an IF *g*-open set;

Definition: 2.7[1] A subset *A* of an *IFTS* (X, T) is called intuitionistic fuzzy pre semi closed (*IF* pre semi closed for short) if $IF\ spcl(A) \subseteq U$ whenever $A \subseteq U$ and *U* is *IF g*-open in (X, T).

Definition: 2.8[1] A subset *A* of an *IFTS* (X, T) is called intuitionistic fuzzy pre semi open(*IF* pre semi open for short) if $\overline{A}$ is *IF* pre semi closed.

Definition: 2.9[1] A function $f : (X, T) \to (Y, S)$ is called intuitionistic fuzzy pre semi continuous (*IF* pre semi continuous for short) if $f^{-1}(V)$ is an *IF* pre semi closed set of (X, T) for every *IF* closed set *V* of (Y, S).

Definition: 2.10 Let (X, T) be an *IFTS*. Let *A* be any *IF* pre semi open F_σ in (X, T). If *IF* pre semi closure of *A* is *IF* pre semi open , then (X, T) is said to be intuitionistic fuzzy pre semi basically disconnected(for short, *IF* pre semi basically disconnected).

Definition: 2.11[11] An ordered set on which there is given a fuzzy topology is called an ordered fuzzy topological space.

3. ORDERED INTUITIONISTIC FUZZY PRE SEMI BASICALLY DISCONNECTED SPACES:

In this section, the concept of ordered intuitionistic fuzzy pre semi basically disconnected spaces is introduced. Some of its characterizations and properties are studied.

Definition: 3.1 An ordered set on which there is given an intuitionistic fuzzy topology is called an ordered intuitionistic fuzzy topological space (for short ordered *IFTS*).

Definition: 3.2 An *IFS A* in a partially ordered set $(X, T, \leq)$ is said to be an

(a) Increasing *IFS* if $x \leq y$ implies $A(x) \subseteq A(y)$.

That is, $\mu_A(x) \leq \mu_A(y)$ and $\gamma_A(x) \geq \gamma_A(y)$.

(b) Decreasing *IFS* if $x \le y$ implies $A(x) \supseteq A(y)$.

That is, $\mu_A(x) \ge \mu_A(y)$ and $\gamma_A(x) \le \gamma_A(y)$.

Definition: 3.3 Let $(X, T, \le)$ be an intuitionistic fuzzy topological space (for short *IFTS*) and *A* be an intuitionistic fuzzy set (for short *IFS*) in (X, T). *A* is called an intuitionistic fuzzy G_δ (for short *IF* G_δ) if $A = \bigcap_{i=1}^{\infty} A_i$ where each A_i is *IF* open.

Definition: 3.4 Let $(X, T, \le)$ be an intuitionistic fuzzy topological space (for short *IFTS*) and *A* be an intuitionistic fuzzy set (for short *IFS*) in $(X, T, \le)$. *A* is called an intuitionistic fuzzy F_σ (*IF* F_σ) if $A = \bigcup_{i=1}^{\infty} A_i$ where each $\overline{A_i}$ is *IF* open.

Definition: 3.5 Let $(X, T, \le)$ be an ordered *IFTS* and let *A* be any *IFS* in $(X, T, \le)$, *A* is called increasing *IF* pre semi open if $IF\,sp\,\mathrm{int}(A) \supseteq U$ whenever $A \supseteq U$ and *U* is *IF g*-closed in $(X, T, \le)$. The complement of an increasing *IF* pre semi open set is called decreasing *IF* pre semi closed.

Note: 3.1 (a) Let $(X, T, \le)$ be an ordered *IFTS*. An *IFS A* in $(X, T, \le)$ which is both intuitionistic fuzzy pre semi open and *IF* F_σ is denoted by *IF* pre semi open F_σ.

(b) Let $(X, T, \le)$ be an *IFTS*. An *IFS A* in $(X, T, \le)$ which is both intuitionistic fuzzy pre semi closed (for short *IF* pre semi closed) and *IF* G_δ is denoted by *IF* pre semi closed G_δ.

(c) An *IFS A* which is both *IF* pre semi open F_σ and *IF* pre semi closed G_δ is denoted by intuitionistic fuzzy pre semi *COGF* (*IF* pre semi *COGF*).

Definition: 3.6 Let $(X, T, \le)$ be an *IFTS*. For any *IFS A* in $(X, T, \le)$,

$I^{IFPSG_\delta}(A)$ = increasing intuitionistic fuzzy pre semi closure G_δ of *A*

= $\bigcap\{B/B$ is an increasing intuitionistic fuzzy pre semi closed G_δ set and $B \supseteq A\}$,

$D^{IFPSG_\delta}(A)$ = decreasing intuitionistic fuzzy pre semi G_δ closure of *A*

= $\bigcap\{B/B$ is a decreasing intuitionistic fuzzy pre semi closed G_δ set and $B \supseteq A\}$,

$I^{0IFPSF_\sigma}(A)$ = increasing intuitionistic fuzzy pre semi F_σ interior of *A*

= $\bigcup\{B/B$ is an increasing intuitionistic fuzzy pre semi open F_σ set and $B \subseteq A\}$,

$D^{0IFPSF_\sigma}(A)$ = decreasing intuitionistic fuzzy pre semi F_σ interior of *A*

= $\bigcup\{B/B$ is a decreasing intuitionistic fuzzy pre semi open F_σ set and $B \subseteq A\}$.

Clearly, $I^{IFPSG_\delta}(A)$ (resp. $D^{IFPSG_\delta}(A)$ is the smallest increasing (resp.decreasing) intuitionistic fuzzy pre semi closed G_δ set containing *A* and $I^{0IFPSF_\sigma}(A)$ (resp. $D^{0IFPSF_\sigma}(A)$ is the largest increasing (resp.decreasing) intuitionistic fuzzy pre semi open F_σ set contained in *A*.

Definition: 3.7 Let $(X,T,\leq)$ be an *IFTS*. Let *A* be any *IF* pre semi open F_σ in $(X,T,\leq)$. If *IF* pre semi closure of *A* is *IF* pre semi open F_σ, then $(X,T,\leq)$ is said to be intuitionistic fuzzy pre semi basically disconnected (for short, *IF* pre semi basically disconnected).

Proposition: 3.1 For any *IFS A* of an ordered intuitionistic fuzzy topological space $(X,T,\leq)$, the following statements are hold.

(a) $\overline{I^{IFPSG_\delta}(A)} = D^{0IFPSF_\sigma}(\overline{A})$.

(b) $\overline{D^{IFPSG_\delta}(A)} = I^{0IFPSF_\sigma}(\overline{A})$.

(c) $\overline{I^{0IFPSF_\sigma}(A)} = D^{IFPSG_\delta}(\overline{A})$.

(d) $\overline{D^{0IFPSF_\sigma}(A)} = I^{IFPSG_\delta}(\overline{A})$.

Proof: (a): Since $I^{IFPSG_\delta}(A)$ is an increasing *IF* pre semi closed G_δ set containing *A*, $\overline{I^{IFPSG_\delta}(A)}$ is a decreasing *IF* pre semi open F_σ set such that $\overline{I^{IFPSG_\delta}(A)} \subseteq \overline{A}$. Let *B* be another decreasing *IF* pre semi open F_σ set such that $B \subseteq \overline{A}$. Then $\overline{B}$ is an increasing *IF* pre semi closed G_δ set such that $\overline{B} \supseteq A$. It follows that $I^{IFPSG_\delta}(A) \subseteq \overline{B}$. That is, $B \subseteq \overline{I^{IFPSG_\delta}(A)}$. Thus, $\overline{I^{IFPSG_\delta}(A)}$ is the largest decreasing *IF* pre semi open F_σ set such that $\overline{I^{IFPSG_\delta}(A)} \subseteq \overline{A}$. That is, $\overline{I^{IFPSG_\delta}(A)} = D^{0IFPSF_\sigma}(\overline{A})$.

The proofs of (b), (c) and (d) can be proved in a similar manner.

Definition: 3.8 Let $(X,T,\leq)$ be an ordered *IFTS* . Let *A* be any increasing (resp. decreasing) *IF* pre semi open F_σ (resp. *IF* pre semi closed G_δ) set in $(X,T,\leq)$. If $I^{IFPSG_\delta}(A)$ (resp. $D^{IFPSG_\delta}(A)$) is increasing (resp. decreasing) *IF* pre semi open F_σ in $(X,T,\leq)$, then $(X,T,\leq)$ is said to be upper F_σ (resp. lower F_σ) intuitionistic fuzzy pre semi basically disconnected (for short *IF* pre semi basically disconnected). An *IFTS* $(X,T,\leq)$ is said to be ordered *IF* pre semi basically disconnected if it is both upper F_σ and lower F_σ intuitionistic fuzzy pre semi basically disconnected.

Proposition: 3.2 For an *IFTS* $(X,T,\leq)$, the followings are equivalent:

(a) $(X,T,\leq)$ is upper F_σ *IF* pre semi basically disconnected.

(b) For each decreasing *IF* pre semi closed G_δ set *A*, $D^{0IFPSF_\sigma}(A)$ is decreasing *IF* pre semi closed G_δ.

(c) For each increasing *IF* pre semi open F_σ set *A*, $D^{IFPSG_\delta}(\overline{I^{IFPSG_\delta}(A)}) = \overline{I^{IFPSG_\delta}(A)}$.

(d) For each pair of increasing *IF* pre semi open F_σ set *A* and decreasing *IF* pre semi open F_σ set *B* in $(X,T,\leq)$ with $\overline{I^{IFPSG_\delta}A} = B$, $D^{IFPSG_\delta}B = \overline{I^{IFPSG_\delta}A}$.

Proof: (a)$\Rightarrow$(b): Let *A* be any decreasing *IF* pre semi closed G_δ set. We claim that $D^{0IFPSF_\sigma}(A)$ is decreasing *IF* pre semi closed G_δ. Now, $\overline{A}$ is increasing *IF* pre semi open F_σ. So by assumption (a), $I^{IFPSG_\delta}(\overline{A})$ is increasing *IF* pre semi open F_σ. That is, $D^{0IFPSF_\sigma}(A)$ is decreasing *IF* pre semi closed G_δ.

(b)$\Rightarrow$(c): Let *A* be an increasing *IF* pre semi open F_σ set. Then, $\overline{I^{IFPSG_\delta}(A)} = D^{0IFPSF_\sigma}(\overline{A})$. Consider $I^{IFPSG_\delta}(A) + D^{IFPSG_\delta}(\overline{I^{IFPSG_\delta}(A)}) = I^{IFPSG_\delta}(A) + D^{IFPSG_\delta}(D^{0IFPSF_\sigma}(\overline{A}))$. Since *A* is increasing *IF* pre semi open F_σ, $\overline{A}$ is a decreasing *IF* pre semi closed G_δ set and by (b), $D^{0IFPSF_\sigma}(\overline{A})$ is a decreasing *IF* pre semi closed G_δ set. Therefore, $D^{IFPSG_\delta}(D^{0IFPSF_\sigma}(\overline{A})) = D^{0IFPSF_\sigma}(\overline{A})$.Now, $I^{IFPSG_\delta}(A) + D^{IFPSG_\delta}(D^{0IFPSF_\sigma}(\overline{A})) = I^{IFPSG_\delta}(A) + (D^{0IFPSF_\sigma}(\overline{A})) = I^{IFPSG_\delta}(A) + \overline{I^{IFPSG_\delta}(A)}$.

Hence, $D^{IFPSG_\delta}(D^{0IFPSF_\sigma}(\overline{A})) = \overline{I^{IFPSG_\delta}(A)}$. By Proposition 3.1, (c) holds.

(c)$\Rightarrow$(d): Let *A* be any increasing *IF* pre semi open F_σ set and *B* be any decreasing *IF* pre semi open F_σ set such that $\overline{I^{IFPSG_\delta}(A)} = B$. By Proposition 3.1, $\overline{B} = D^{0IFPSF_\sigma}(\overline{A})$.

By(c), $D^{IFPSG_\delta}(\overline{I^{IFPSG_\delta}(A)}) = D^{IFPSG_\delta}(I^{0IFPSF_\sigma}(\overline{A})) = B$ (3.1)

Therefore, $D^{IFPSG_\delta}(\overline{I^{IFPSG_\delta}(A)}) = D^{IFPSG_\delta}(B)$ (3.2)

From (3.1) and (3.2), we have, $\overline{I^{IFPSG_\delta}(A)} = D^{IFPSG_\delta}(B)$.

(d)$\Rightarrow$(a): Let *A* be any increasing *IF* pre semi open F_σ set. Let $\overline{I^{IFPSG_\delta}(A)} = B$. From (d), it follows that $\overline{D^{IFPSG_\delta}(B)} = I^{IFPSG_\delta}(A)$. Hence, $(X, T, \le)$ is upper F_σ *IF* pre semi basically disconnected space.

Proposition: 3.3 Let $(X, T, \le)$ be an ordered *IFTS*. Then $(X, T, \le)$ is an upper F_σ *IF* pre semi basically disconnected space if and only if for any decreasing *IF* pre semi open F_σ set *A* and decreasing *IF* pre semi closed G_δ set *B* such that $A \subseteq B$, $D^{IFPSG_\delta}(A) \subseteq D^{0IFPSF_\sigma}(B)$.

Notation: 3.1 An ordered *IFS* which is both decreasing (resp. increasing) *IF* pre semi open F_σ and *IF* pre semi closed G_δ is called a decreasing (resp. increasing) *IF* pre semi clopen set (for short *IF* pre semi *COGF*).

Remark: 3.1 Let $(X, T, \le)$ be an upper F_σ *IF* pre semi basically disconnected space. Let $\left\{A_i, \overline{B_i} / i \in N\right\}$ be a collection such that A_i's are decreasing *IF* pre semi open F_σ sets, B_i's are decreasing *IF* pre semi closed G_δ sets and let $A, \overline{B}$ be decreasing *IF* pre semi open F_σ and increasing *IF* pre semi open F_σ sets respectively. If $A_i \subseteq A \subseteq B_j$ and $A_i \subseteq B \subseteq B_j$ for all $i, j \in N$, then there exists a decreasing *IF* pre semi *COGF* set *C* such that $D^{IFPSG_\delta}(A_i) \subseteq C \subseteq D^{0IFPSF_\sigma}(B_j)$ for all $i, j \in N$.

Proposition: 3.4 Let $(X, T, \le)$ be an upper F_σ *IF* pre semi basically disconnected space. Let $\left(A_q\right)_{q \in Q}$ and $\left(B_q\right)_{q \in Q}$ be the monotone increasing collections of decreasing *IF* pre semi open F_σ sets and decreasing *IF* pre semi closed G_δ sets of $(X, T, \le)$ respectively and suppose that $A_{q_1} \subseteq B_{q_2}$ whenever $q_1 < q_2$ (Q is the set of rational numbers). Then there exists a monotone increasing collection $\left\{C_q\right\}_{q \in Q}$ of decreasing *IF* pre semi *COGF* sets of $(X, T, \le)$ such that $D^{IFPSG_\delta}(A_{q_1}) \subseteq C_{q_2}$ and $C_{q_1} \subseteq D^{0IFPSF_\sigma}(B_{q_2})$ whenever $q_1 < q_2$.

4. PROPERTIES AND CHARACTERIZATIONS OF INTUITIONISTIC FUZZY PRE SEMI BASICALLY DISCONNECTED SPACES:

In this section various properties and characterizations of intuitionistic fuzzy pre semi basically disconnected spaces are discussed.

Definition: 4.1 Let $(X,T,\leq)$ be an ordered *IFTS*. A function $f : X \to R(I)$ is called lower F_σ (resp.upper F_σ) *IF* pre semi continuous, if $f^{-1}(R_t)$ (resp. $f^{-1}(L_t)$) is an increasing or decreasing *IF* pre semi open F_σ set (resp. *IF* pre semi open F_σ / *IF* pre semi closed G_δ) for each $t \in R$.

Proposition: 4.1 Let $(X,T,\leq)$ be an ordered *IFTS*. Let A be an *IFS* in *X*, and let $f : X \to R(I)$ be such that

$$f(x)(t) = \begin{cases} 1 & if \;\; t < 0, \\ A(x) & if \;\; 0 \leq t \leq 1, \\ 0 & if \;\; t > 1, \end{cases}$$

for all $x \in X$. Then f is lower F_σ (resp.upper F_σ) *IF* pre semi continuous iff *A* is an increasing or decreasing *IF* pre semi open F_σ (resp. *IF* pre semi open F_σ / *IF* pre semi closed G_δ) set.

Definition: 4.2 The characteristic function of any *IFS A* in *X* is the function $\chi_A : X \to I(L)$ defined by $\chi_A(x) = (A(x)), x \in X$.

Proposition: 4.2 Let $(X,T,\leq)$ be an ordered *IFTS*, and let A be an *IFS* in X. Then χ_A is lower F_σ (resp.upper F_σ) *IF* pre semi continuous iff *A* is an increasing or decreasing *IF* pre semi open F_σ (resp. *IF* pre semi open F_σ / *IF* pre semi closed G_δ) set.

Proof: The proof follows from Proposition 4.1.

Definition: 4.3 Let $(X,T,\leq)$ and $(Y,S,\leq)$ be *IFTSs*. A function $f : (X,T,\leq) \to (Y,S,\leq)$ is called increasing F_σ (resp. decreasing G_δ) intuitionistic fuzzy strongly pre semi continuous (for short, increasing F_σ (resp. decreasing G_δ) *IF* strongly pre semi continuous) if $f^{-1}(A)$ is increasing F_σ (resp. decreasing G_δ) *IF* pre semi clopen in $(X,T,\leq)$ for every *IF* pre semi open F_σ set in $(Y,S,\leq)$. If *f* is both increasing F_σ and decreasing G_δ *IF* strongly pre semi continuous, then it is called ordered *IF* strongly pre semi continuous.

Proposition: 4.3 Let $(X,T,\leq)$ be an ordered *IFTS*. Then the following statements are equivalent:

(a) $(X,T,\leq)$ is upper F_σ *IF* pre semi basically disconnected,

(b) If $g,h : X \to R(I)$, *g* is lower F_σ *IF* pre semi continuous, *h* is upper F_σ *IF* pre semi continuous and $g \subseteq h$ then there exists an increasing F_σ *IF* strongly pre semi continuous function, $f : (X,T,\leq) \to R(I)$ such that $g \subseteq f \subseteq h$.

(c) If $\overline{A}$ is increasing *IF* pre semi open F_σ set and *B* is decreasing *IF* pre semi open F_σ set such that $B \subseteq A$, then there exists an increasing F_σ *IF* strongly pre semi continuous function $f : (X,T,\leq) \to [0,1](I)$ such that $B \subseteq \overline{L_1} f \subseteq R_0 f \subseteq A$.

Proof: (a) $\Rightarrow$ **(b)** can be established by the concepts of increasing F_σ (resp. decreasing G_δ) *IF* pre semi clopen set in $(X,T,\leq)$ and the theorem 3.7 of Kubiak [15] with some slight suitable modifications.

(b) $\Rightarrow$ **(c):** Suppose $\overline{A}$ is an increasing *IF* pre semi open F_σ set and *B* is an decreasing *IF* pre semi closed G_δ set, such that $B \subseteq A$. Then $\chi_B \subseteq \chi_A$ and χ_B, χ_A are lower F_σ and upper F_σ *IF* pre semi continuous functions respectively. Hence by (b), there exists an increasing F_σ *IF* strongly pre semi continuous function $f:(X,T,\leq)\to R(I)$ such that $\chi_B \subseteq f \subseteq \chi_A$. Clearly, $f(x)\in[0,1](I)$ for all $x\in X$ and $B=(\overline{L_1})\chi_B \subseteq (\overline{L_1})f \subseteq R_0 f \subseteq R_0\chi_A = A$.

(c) $\Rightarrow$ **(a):** This follows from Proposition 3.2 and the fact that $(\overline{L_1})f$ and $R_0 f$ are decreasing *IF* pre semi closed G_δ and decreasing *IF* pre semi open F_σ sets respectively.

5. TIETZE EXTENSION THEOREM FOR ORDERED *IF* PRE SEMI BASICALLY DISCONNECTED SPACES:

In this section Tietze extension theorem for ordered *IF* pre semi basically disconnected spaces is studied.

Proposition: 5.1 Let $(X,T,\leq)$ be an upper F_σ *IF* pre semi basically disconnected space and let $A\subset X$ be such that χ_A is increasing *IF* pre semi open F_σ set in $(X,T,\leq)$. Let $f:(A,\ T/A)\to[0,1](I)$ be an increasing F_σ *IF* strongly pre semi continuous function. Then *f* has an increasing F_σ *IF* strongly pre semi continuous extension over $(X,T,\leq)$.

Proof: Let $g,h: X\to[0,1](I)$ be such that $g=f=h$ on A, and $g(x)=\langle 0\rangle$, $h(x)=\langle 1\rangle$ if $x\notin A$.
We now have,

$$R_t g=\begin{cases} B_t\cap\chi_A & if\ \ t\geq 0,\\ 1 & if\ \ t<0,\end{cases}$$

Where B_t is increasing *IF* pre semi open F_σ set such that $B_t/A=R_t f$ and

$$L_t h=\begin{cases} A_t\cap\chi_A & if\ \ t\leq 1,\\ 1 & if\ \ t>1,\end{cases}$$

where A_t is increasing *IF* pre semi open F_σ such that $A_t/A=L_t f$. Thus, *g* is lower F_σ *IF* pre semi continuous, *h* is upper F_σ *IF* pre semi continuous and $g\subseteq h$. By Proposition 4.2, there is an increasing F_σ *IF* strongly pre semi continuous function $F:(X,T,\leq)\to[0,1](I)$ such that $g\subseteq F\subseteq h$; hence $F\equiv f$ on A.

ACKNOWLEDGEMENT:

The authors express their sincere thanks to the referee for his valuable comments regarding the improvement of the paper.

REFERENCES:

[1] D. Amsaveni, M. K. Uma and E. Roja, Intutionistic Fuzzy Pre Semi Closed Sets, *International Journal of Mathematical Sciences and applications.* (Accepted)

[2] K. Atanassov, Intuitionstic fuzzy sets,in: V. Sgurev, Ed., VII ITKR's Session, Sofia (une 1983 Central Sci. and Techn. Library, Bulg. Academy of sciences, 1984).

[3] K. Atanssov, Intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, 20 (1986), 87-96.

[4] K Atanassov, Review and new results on intuitionistic fuzzy sets, Preprint IM-MFAIS 1-88, Sofia, 1988.

[5] K. Atanassov and S. Stoeva, Intuitionistic fuzzy sets, in: *Polish symp. On Internal & Fuzzy Mathematics, Poznan* (August 1983), 23-26.

[6] K. Atanassov and S. Stoeva, Intuitionistic L-fuzzy sets, in: R. Trappl, Ed., *Cybernetics and system Research,* Vol. 2 (Elsevier, Amsterdam, 1984), 539-540.

[7] A.S. Bin Shahna, On fuzzy strong semi continuity and fuzzy pre - continuity, *Fuzzy Sets and Systems,* 44 (1991), 303-308.

[8] D. Coker, An introduction to intuitionistic fuzzy topological spaces, *Fuzzy Sets and Systems,* 88 (1997),81-89.

[9] Gurcay, H., Coker, D., and Haydar, Es. A., On fuzzy continuity in intuitionistic fuzzy topological spaces, *The J. fuzzy mathematics,* 1997, 365-378.

[10] Joung Kon Jeon, Young Bae Jun, and Jin Han Park, Intuitionistic fuzzy alpha-continuity and intuitionistic fuzzy pre continuity, *International Journal of Mathematics and Mathematical Sciences,* 2005, 3091-3101.

[11] Katsaras A.K., Ordered fuzzy topological spaces, J. Math. Anal. Appl. 84(1981), 44-58.

[12] Necla Turanli, Dogan Coker, Fuzzy connectedness in intuitionistic fuzzy topological spaces, *Fuzzy sets and systems* 116(2000), 369-375.

[13] G. Thangaraj and G. Balasubramanian, *On fuzzy basically disconnected spaces,* The Journal of Fuzzy Mathematics, 9(2001), 103-110.

[14] Thakur, S. S. and Singh, S., On fuzzy semi-pre open sets and fuzzy semi-precontinuity, *Fuzzy sets and systems*, 1998, 383-391.

[15] Tomasz Kubiak, L-fuzzy normal space and Tietze extension theorem, J. Math. Anal. Appl., 25(1987), 141-153.

[16] M.K. Uma, E. Roja and G. Balasubramanian, Tiezte extension theorem for ordered fuzzy pre extremally disconnected spaces, *East Asian Math J.,* 24 (2008), 213-221.

[17] Young Bae Jun and Seok- Zun Song, Intuitionistic fuzzy semi-pre open sets and Intuitionistic semi-pre continuous mappings, *jour. of Appl. Math & computing*, 2005,467-474.

[18] L. A. Zadeh, Fuzzy sets, *Inform and control,* 8 (1965), 338-353.

****