International Journal of Mathematical Archive-3(2), 2012, Page: 373-379
IJMA Available online through www.ijma.info ISSN 2229 – 5046

INTUITIONISTIC FUZZY ALMOST OPEN MAPPINGS IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

A. MANIMARAN*& K. ARUN PRAKASH

Department of Mathematics, Kongu Engineering College, Perundurai-638 052, Erode, Tamilnadu, India

E-mail: manimaranthangaraj@gmail.com

P. THANGARAJ

Department of Computer Science and Engineering, Bannariamman Institute of Technology, Sathyamangalam, Erode, Tamilnadu, India

E-mail: ctptr@yahoo.co.in

(Received on: 16-01-12; Accepted on: 15-02-12)

ABSTRACT

This paper is devoted to the study of intuitionistic fuzzy almost open and Intuitionistic fuzzy almost closed mappings in intuitionistic fuzzy topological spaces. Some of its properties are studied and relationships with other existing intuitionistic fuzzy closed mappings were discussed.

AMS Classification: *54A40.*

Keywords and Phrases: *Intuitionistic fuzzy topology, Intuitionistic fuzzy open set, Intuitionistic fuzzy closed set, Intuitionistic fuzzy regular open set, Intuitionistic fuzzy almost open mapping, Intuitionistic fuzzy almost closed mapping.*

---

1. INTRODUCTION:

After the introduction of fuzzy sets by Zadeh [8], there have been a number of generalizations of this fundamental concept. The notion of intuitionistic fuzzy sets introduced by Atanassov [1] is one among them. Using the notion of intuitionistic fuzzy sets, Coker [2, 3] introduced the notion of intuitionistic fuzzy topological spaces. In this paper, we introduce the concept of intuitionistic fuzzy almost open mapping. We have also studied some of the properties of intuitionistic fuzzy almost open mapping and their relationship between other existing intuitionistic fuzzy open mappings.

2. PRELIMINARIES:

Before entering to our work, we recall the following notations, definitions and intuitionistic fuzzy sets as given by Atanassov [1], Coker [3]. Throughout this paper, (X,τ), (Y,σ) and (Z,η) always means an intuitionistic fuzzy topological spaces in which no separation axioms are assumed unless otherwise mentioned.

Definition 2.1: [1] Let *X* be a non-empty fixed set. An intuitionistic fuzzy set (*IFS*, for short), A is an object having the form

$$A=\{< x,\mu_A(x),\gamma_A(x)>: x\in X\}$$

where the mapping $\mu_A : X\to I$ and $\gamma_A : X\to I$ denotes respectively the degree of membership (namely $\mu_A(x)$) and the non-membership (namely $\gamma_A(x)$) of each element $x\in X$ to a set *A*, and $0\le\mu_A(x)+\gamma_A(x)\le 1$ for each $x\in X$. Obviously, every set *A* on a non-empty set *X* is an *IFS* having the form

$$A=\{< x,\mu_A(x),\gamma_A(x)>: x\in X\}$$

Corresponding author: *A. MANIMARAN*, ***E-mail:*** *manimaranthangaraj@gmail.com*

Definition 2.2: [1] Let *X* be a non-empty set and let the *IFS*'s *A* and *B* in the form

$A = \{< x, \mu_A(x), \gamma_A(x) >: x \in X\}$; $B = \{< x, \mu_B(x), \gamma_B(x) >: x \in X\}$

Let $\{A_j : j \in J\}$ be an arbitrary family of *IFSs* in (X, τ). Then,

(i) $A \le B$ if and only if $\forall x \in X$, $\mu_A(x) \le \mu_B(x)$ and $\gamma_A(x) \ge \gamma_B(x)$
(ii) $\overline{A} = \{< x, \gamma_A(x), \mu_A(x) >: x \in X\}$
(iii) $\cap A_j = \{< x, \wedge \mu_{A_j}(x), \vee \gamma_{A_j}(x) >: x \in X\}$
(iv) $\cup A_j = \{< x, \vee \mu_{A_j}(x), \wedge \gamma_{A_j}(x) >: x \in X\}$
(v) $\tilde{1} = \{< x, 1, 0 >: x \in X\}$ and $\tilde{0} = \{< x, 0, 1 >: x \in X\}$
(vi) $\overline{\overline{A}} = A$, $\overline{\underset{\sim}{1}} = \underset{\sim}{0}$ and $\overline{\underset{\sim}{0}} = \underset{\sim}{1}$.

Definition 2.3: [3] An intuitionistic fuzzy topology (*IFT*, for short) on a non-empty set *X* is a family τ of *IFSs* in *X* satisfying the following axioms:

(i) $\underset{\sim}{1}, \underset{\sim}{0} \in \tau$
(ii) $A_i \cap A_2 \in \tau$ for some $A_i, A_2 \in \tau$
(iii) $\bigcup A_j \in \tau$ for any $\{A_j : j \in J\} \in \tau$

In this case, the ordered pair (X, τ) is called intuitionistic fuzzy topological space (*IFTS*, for short) and each *IFS* in τ is known as an intuitionistic fuzzy open set (*IFOS*, for short) in *X*. The complement of an intuitionistic fuzzy open set is called intuitionistic fuzzy closed set (*IFCS*, for short).

Definition 2.4: [3] Let (X, τ) be an *IFTS* and let $A = \{< x, \mu_A(x), \gamma_A(x) >: x \in X\}$ be an *IFS* in *X*. Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure of *A* are defined by

$$\text{int}(A) = \bigcup\{G / G \text{ is an IFOS in X and } G \subseteq A\}$$

$$\text{cl}(A) = \bigcap\{K / K \text{ is an IFCS in X and } A \subseteq K\}$$

Remark 2.5: [3] For any *IFS A* in (X, τ), we have $\text{cl}(\overline{A}) = \overline{\text{int}(A)}$ and $\text{int}(\overline{A}) = \overline{\text{cl}(A)}$.

Definition 2.6: [4] An *IFS A* in an *IFTS* (X, τ) is called an intuitionistic fuzzy regular open set (*IFROS*) if $\text{int}(\text{cl}(A)) = A$. The complement of intuitionistic fuzzy regular open set is called intuitionistic fuzzy regular closed (*IFRCS*, for short). The family of all *IFROS* (*IFRCS*) of (X, τ) is denoted by *IFROS*(X) (*IFRCS*(X)).

Definition 2.7: [4] An *IFS* $A = \{< x, \mu_A(x), \gamma_A(x) >: x \in X\}$ in an *IFT* (X, τ) is called an intuitionistic fuzzy semi-open set (*IFSOS*, for short), if $A \subseteq \text{cl}(\text{int}(A))$. The complement of an *IFSOS* is called an *IFSCS.*

Definition 2.8: [5] Let *A* be a fuzzy set in an *IFTS*(X, τ). Then, semiclosure (briefly scl) and semi-interior (briefly sint) are given as

$$\text{scl}(A) = \bigcap\{B / A \subseteq B, B \text{ is fuzzy semiclosed}\};$$
$$\text{sint}(A) = \bigcup\{B / B \subseteq A, B \text{ is fuzzy semiopen}\}.$$

Definition 2.9: [4] An *IFS* $A = \{< x, \mu_A(x), \gamma_A(x) >: x \in X\}$ in an *IFTS* (X, τ) is called an *intuitionistic fuzzy α-open set* (IFαOS) if $A \subseteq \text{int}(\text{cl}(\text{int}(A)))$.

Definition 2.10: [6] An *IFS* $A = \{< x, \mu_A(x), \gamma_A(x) >: x \in X\}$ in an *IFTS* (X, τ) is called an *intuitionistic fuzzy semi-pre open set* (*IFSPOS*) if there exists *B*∈*IFPO*(*X*), such that $B \subseteq A \subseteq \text{cl}(B)$.

Definition 2.11: [4] Let (X,τ) be an *IFTS* and $A=\{<x,\mu_A(x),\gamma_A(x)>: x\in X\}$ be an *IFS* in *X*. The *intuitionistic fuzzy α-interior* and *intuitionistic fuzzy α-closure* of A are defined by

$$\alpha\operatorname{int}(A)=\bigcup\{G / G \text{ is an IF}\alpha\text{OS in X and } G\subseteq A\}$$

$$\alpha\operatorname{cl}(A)=\bigcap\{K / K \text{ is an IF}\alpha\text{CS in X and } A\subseteq K\}$$

Definition 2.12: [7] An intuitionistic fuzzy point (*IFP* for short), written $p_{(\alpha,\beta)}$ is defined to be an *IFS* of X given

by $p_{(\alpha,\beta)}(x)=\begin{cases}(\alpha,\beta) & \text{if } \quad x=p\\ (0,1) & \text{otherwise}\end{cases}$

Definition 2.13: [7] Let $p_{(\alpha,\beta)}$ be an *IFP* of an *IFTS* X. An *IFS A* of *X* is called an intuitionistic fuzzy neighborhood (*IFN*) of $p_{(\alpha,\beta)}$, if there exists an *IFRO* set *B* in *X* such that $p_{(\alpha,\beta)}\in B\le A$.

Definition 2.14: [3] Let *X* and *Y* be two non-empty sets and $f:(X,\tau)\to(Y,\sigma)$ be a mapping. If $B=\{<y,\mu_B(y),\gamma_B(y)>: y\in Y\}$ is an *IFS* in *Y*, then the pre image of *B* under f is denoted and defined by $f^{-1}(B)=\{<x,f^{-1}(\mu_B)(x),f^{-1}(\gamma_B)(x)>: x\in X\}$, since, μ_B,γ_B are fuzzy sets, we explain that

$$f^{-1}(\mu_B)(x)=\mu_B(f(x))$$

Definition 2.15: [7] A mapping $f:(X,\tau)\to(Y,\delta)$ is said to be intuitionistic fuzzy closed mapping if $f(B)$ is an *IFCS* of *Y* for each *IFCS B* of *X*.

Definition 2.16: [4] A mapping $f:(X,\tau)\to(Y,\delta)$ is said to be intuitionistic fuzzy almost continuous mapping if $f^{-1}(B)$ is an *IFOS* of *X* for each *IFROS B* of *Y*.

3. INTUITIONISTIC FUZZY ALMOST OPEN MAPPINGS:

Definition 3.1: [5] A mapping $f:(X,\tau)\to(Y,\sigma)$ is said to be an intuitionistic fuzzy almost open (*IFAO*, for short) mapping, if for each *IFROS U* of *X*, f(*U*) is an *IFOS* in *Y*.

Definition 3.2: [5] A mapping $f:(X,\tau)\to(Y,\sigma)$ is said to be an intuitionistic fuzzy almost closed (*IFAC*, for short) mapping, if for each *IFRCS* set *A* of *X*, f(*A*) is an *IFCS* in *Y*.

Example 3.3: Let $X=\{a,b,c\}=Y$, $\tau=\{\underset{\sim}{0},\underset{\sim}{1},A\}$ and $\sigma=\{\underset{\sim}{0},\underset{\sim}{1},B\}$ where $A=\left\{<x,\left(\frac{a}{0.1},\frac{b}{0.2},\frac{c}{0.4}\right),\left(\frac{a}{0.3},\frac{b}{0.5},\frac{c}{0.6}\right)>,x\in X\right\}$ and $B=\left\{<y,\left(\frac{a}{0.2},\frac{b}{0.1},\frac{c}{0.4}\right),\left(\frac{a}{0.5},\frac{b}{0.3},\frac{c}{0.6}\right)>,y\in Y\right\}$, $\operatorname{cl}(A)=A^c$ and $\operatorname{int}(\operatorname{cl}(A))=A$. Therefore, *A* is an *IFROS* in X.

Define an intuitionistic fuzzy mapping $f:(X,\tau)\to(Y,\sigma)$ by $f(a)=b, f(b)=a,\ f(c)=c$, the image of an *IFS A* is $f(A)=\left\{<y,\left(\frac{a}{0.2},\frac{b}{0.1},\frac{c}{0.4}\right),\left(\frac{a}{0.5},\frac{b}{0.3},\frac{c}{0.6}\right)>,y\in Y\right\}$. Clearly, f(A) is an *IFOS* and hence f is an *IFAO* mapping.

Example 3.4: Let $X=\{a,b,c\}=Y$, $\tau=\{\underset{\sim}{0},\underset{\sim}{1},A\}$ and $\sigma=\{\underset{\sim}{0},\underset{\sim}{1},B\}$, where

$$A=\left\{<x,\left(\frac{a}{0.2},\frac{b}{0.3},\frac{c}{0.4}\right),\left(\frac{a}{0.3},\frac{b}{0.4},\frac{c}{0.5}\right)>,x\in X\right\} \text{and}\ B=\left\{<y,\left(\frac{a}{0.7},\frac{b}{0.1},\frac{c}{0.2}\right),\left(\frac{a}{0.1},\frac{b}{0.6},\frac{c}{0.4}\right)>,y\in Y\right\}.$$

Then (X,τ) and (Y,σ) are *IFTS* on *X* and *Y* respectively. $\operatorname{cl}(A)=A^c$ and $\operatorname{int}(\operatorname{cl}(A))=A$. Therefore, *A* is an *IFROS* in *X*. Define an intuitionistic fuzzy mapping $f:(X,\tau)\to(Y,\sigma)$ by $f(a)=c,\ f(b)=a,\ f(c)=b$, then

$$f(A)=\left\{<y,\left(\frac{a}{0.3},\frac{b}{0.4},\frac{c}{0.2}\right),\left(\frac{a}{0.4},\frac{b}{0.5},\frac{c}{0.3}\right)>,y\in Y\right\}.$$

Thus, f (A) is not an *IFOS* in *Y*. Therefore f is not an *IFAO* mapping.

Example 3.5: Let $X = \{a,b,c\} = Y$, $\tau = \{\underset{\sim}{0}, \underset{\sim}{1}, A\}$ and $\sigma = \{\underset{\sim}{0}, \underset{\sim}{1}, C\}$, where

$A = \left\{< x, \left(\frac{a}{0.1}, \frac{b}{0.2}, \frac{c}{0.3}\right), \left(\frac{a}{0.4}, \frac{b}{0.5}, \frac{c}{0.6}\right) >, x \in X\right\}$ and $B = \left\{< x, \left(\frac{a}{0.4}, \frac{b}{0.5}, \frac{c}{0.6}\right), \left(\frac{a}{0.1}, \frac{b}{0.2}, \frac{c}{0.3}\right) >, x \in X\right\}$,

$C = \left\{< y, \left(\frac{a}{0.3}, \frac{b}{0.1}, \frac{c}{0.2}\right), \left(\frac{a}{0.6}, \frac{b}{0.4}, \frac{c}{0.5}\right) >, y \in Y\right\}$. Then (X,τ) and (Y,σ) are *IFTS* on *X* and *Y* respectively. $\mathrm{int}(B) = A$ and $\mathrm{cl}(\mathrm{int}(B)) = B$. Therefore, *B* is an *IFRCS* in *X*.

Define an intuitionistic fuzzy mapping $f : (X,\tau) \to (Y,\sigma)$ by $f(a) = b$, $f(b) = c$, $f(c) = a$ then

$f(B) = \left\{< y, \left(\frac{a}{0.6}, \frac{b}{0.4}, \frac{c}{0.5}\right), \left(\frac{a}{0.3}, \frac{b}{0.1}, \frac{c}{0.2}\right) >, y \in Y\right\}$.

Clearly, f is an *IFAC* mapping.

Example 3.6: Let $X = \{a,b,c\} = Y$, $\tau = \{\underset{\sim}{0}, \underset{\sim}{1}, A\}$ and $\sigma = \{\underset{\sim}{0}, \underset{\sim}{1}, C\}$, where

$A = \left\{< x, \left(\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.1}\right), \left(\frac{a}{0.6}, \frac{b}{0.5}, \frac{c}{0.2}\right) >, x \in X\right\}$ and $B = \left\{< x, \left(\frac{a}{0.6}, \frac{b}{0.5}, \frac{c}{0.2}\right), \left(\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.1}\right) >, x \in X\right\}$,

$C = \left\{< y, \left(\frac{a}{0.4}, \frac{b}{0.3}, \frac{c}{0.6}\right), \left(\frac{a}{0.1}, \frac{b}{0.4}, \frac{c}{0.2}\right) >, y \in Y\right\}$, $\mathrm{int}(B) = A$ and $\mathrm{cl}(\mathrm{int}(B)) = B$.

Therefore, *B* is an *IFRCS* in *X*. Define an intuitionistic fuzzy mapping $f : (X,\tau) \to (Y,\sigma)$ by $f(a) = b$, $f(b) = c$, $f(c) = a$ then

$f(B) = \left\{< y, \left(\frac{a}{0.2}, \frac{b}{0.6}, \frac{c}{0.5}\right), \left(\frac{a}{0.1}, \frac{b}{0.3}, \frac{c}{0.4}\right) >, y \in Y\right\}$.

Clearly, f is not an *IFAC* mapping.

Theorem 3.7: Every intuitionistic fuzzy closed mapping is an *IFAC* mapping.

Proof: Let $f : (X,\tau) \to (Y,\sigma)$ be an intuitionistic fuzzy closed mapping and let *B* be an *IFRCS* in *X*. Since every *IFRCS* is an *IFCS*, *B* is an *IFCS* in *X*. By the hypothesis, f (*B*) is an *IFCS* in *Y*. Hence f is an *IFAC* mapping. The converse of the above theorem is not true in general as shown in the following example.

Example 3.8: Let $X = \{a,b\} = Y$, $\tau = \{\underset{\sim}{0}, \underset{\sim}{1}, A\}$ and $\sigma = \{\underset{\sim}{0}, \underset{\sim}{1}, B\}$, where $A = \left\{< x, \left(\frac{a}{0.7}, \frac{b}{0.5}\right), \left(\frac{a}{0.2}, \frac{b}{0.4}\right) >, x \in X\right\}$ and $B = \left\{< y, \left(\frac{a}{0.2}, \frac{b}{0.3}\right), \left(\frac{a}{0.3}, \frac{b}{0.2}\right) >, y \in Y\right\}$ then $\tau = \{\underset{\sim}{0}, \underset{\sim}{1}, A\}$ and $\sigma = \{\underset{\sim}{0}, \underset{\sim}{1}, B\}$ are *IFT*s on *X* and *Y* respectively. Define *a mapping* $f : (X,\tau) \to (Y,\sigma)$ by *f*(*a*) = *u*, *f*(*b*) = *v*. Clearly, then, $\underset{\sim}{0}, \underset{\sim}{1}$ are the only *IFRCS* in *X*. Now, $f\left(\underset{\sim}{0}\right) = \underset{\sim}{0}$ and $f\left(\underset{\sim}{1}\right) = \underset{\sim}{1}$ are *IFCS* in *Y*. Hence *f* is an *IFAC* mapping. But $f(\overline{A})$ is not an *IFCS* in *Y*, where $\overline{A}$ is an *IFCS* in *X*.

Therefore, *f* is not an intuitionistic fuzzy closed mapping.

Theorem 3.9: Let $f : (X,\tau) \to (Y,\sigma)$ be a mapping, then the following are equivalent:

(i) f is an *IFAC* mapping;
(ii) f is an *IFAO* mapping

Proof:
(i) $\Rightarrow$(ii): Let *B* be an *IFROS* in *X*. then $\overline{B}$ be an *IFRCS* in *X*. By the hypothesis $f(\overline{B})=\overline{f(B)}$ is an *IFCS* in *Y*. Therefore f(B) is an *IFOS* in *Y*. Hence f is an *IFAO* mapping.

(ii) $\Rightarrow$(i): Obvious.

Theorem 3.10: Let $f:(X,\tau)\to(Y,\sigma)$ be a mapping, then f is an *IFAC* mapping if for each *IFP* $p_{(\alpha,\beta)}\in Y$ and for each *IFOS* *B* in X such that $f^{-1}(p_{(\alpha,\beta)})\in B$, $cl(B)$ is an intuitionistic fuzzy neighborhood of $p_{(\alpha,\beta)}\in Y$.

Proof: Let $p_{(\alpha,\beta)}\in Y$ and *A* be an *IFROS* in *X*. Then *A* is an *IFOS* in *X*. By hypothesis $f^{-1}(p_{(\alpha,\beta)})\in A$ and $p_{(\alpha,\beta)}\in f(A)$ in *Y*. Since $cl(A)$ is an intuitionistic fuzzy neighborhood of $p_{(\alpha,\beta)}$, then there exists an *IFOS* *B* in *Y* such that $p_{(\alpha,\beta)}\in B\subseteq cl(A)$.We have $p_{(\alpha,\beta)}\in f(A)\subseteq cl(f(A))$.

Now $B=\{p_{(\alpha,\beta)}/p_{(\alpha,\beta)}\in B\}=f(A)$. Therefore f (A) is an *IFOS* in Y. Hence f is an *IFAO* mapping. By the above theorem f is an *IFAC* mapping.

Theorem 3.11: Let $f:(X,\tau)\to(Y,\sigma)$ be a mapping, if f is an *IFAC* mapping, then $cl(A)\subseteq f(cl(A))$ for every *IFSPOS* A in *Y*.

Proof: Let *A* be an *IFSPOS* in *X*. Then $cl(A)$ is an *IFRCS* in *X*. By the hypothesis $f(cl(A))$ is an *IFCS* in *Y*. this implies $cl(f(cl(A)))=f(cl(A))$. Then $cl(f(A))\subseteq cl(f(cl(A)))=f(cl(A))$. Hence $cl(f(A))\subseteq f(cl(A))$.

Theorem 3.12: Let $f:(X,\tau)\to(Y,\sigma)$ be a mapping, if $cl(f(B))\subseteq f(cl(B))$ for every *IFS* *B* in *X*, then f is an *IFAC* mapping.

Proof: Let *B* be an *IFRCS* in *X*. By the hypothesis $cl(f(B))\subseteq f(cl(B))$. Since every *IFRCS* is an *IFCS*, *B* is an *IFCS* in *X*. Therefore $cl(B)=B$. Hence $cl(f(B))\subseteq f(B)$. This implies that $f(B)$ is an *IFCS* in *Y*. thus f is an *IFAC* mapping.

Theorem 3.13: If $f:(X,\tau)\to(Y,\sigma)$ is a mapping, then the following statements are equivalent:
(i) f is an *IFAC* mapping;
(ii) $cl(f(A))\subseteq f(\alpha cl(A))$ for every *IFSPOS A* in *X*;
(iii) $cl(f(A))\subseteq f(\alpha cl(A))$ for every *IFSOS A* in *X*.

Proof: (i)$\Rightarrow$(ii): Let *A* be an *IFSPOS* in *X*. then $cl(A)$ is an *IFRCS* in *X*. By the hypothesis $f(cl(A))$ is an *IFCS* in *X*.

Therefore $cl(f(cl(A)))=f(cl(A))$. Now $cl(f(A))\subseteq cl(f(cl(A)))=f(cl(A))$. Since $cl(A)$ is an *IFRCS* in X,

$cl(int(cl(A)))=cl(A)$. Therefore $cl(f(A))\subseteq f(cl(A))=f(cl((int(cl(A)))))\subseteq f(A\cup cl((int(cl(A)))))\subseteq f(\alpha cl(A))$.

(ii)$\Rightarrow$(iii): Since every *IFSOS* is an *IFSPOS*, the proof follows immediately.

(iii)$\Rightarrow$(i): Let A be an *IFRCS* in *X*. By the hypothesis $cl(f(A))\subseteq f(\alpha cl(A))\subseteq f(cl(A))=f(A)\subseteq cl(f(A))$.

Hence $cl(f(A))=f(A)$, which implies $f(A)$ is an *IFCS* in *Y*. Thus f is an *IFAC* mapping.

Theorem 3.14: Let $f:(X,\tau)\to(Y,\sigma)$ be an *IFAC* mapping, then $int(cl(f(B)))\subseteq f(cl(B))$ for every *IFRCS* B in X.

Proof: Let *B* be an *IFRCS* in X. By the hypothesis f(*B*) is an *IFCS* in *Y*. Then $cl(f(B))=f(B)$. Now $int(cl(f(B)))\subseteq int(f(B))\subseteq f(B)\subseteq f(cl(B))$. Hence $int(cl(f(B)))\subseteq f(cl(B))$.

Theorem 3.15: Let $f:(X,\tau)\to(Y,\sigma)$ be an *IFAC* mapping, then $f(int(B))\subseteq cl(int(f(B)))$ for every *IFROS B* in *X*.

Proof: Proof is similar to the above theorem.

Theorem 3.16: Let $f:(X,\tau)\to(Y,\sigma)$ be a bijective mapping, then the following statements are equivalent:

(i) f is an *IFAO* mapping;
(ii) f is an *IFAC* mapping;
(iii) f^{-1} is an intuitionistic fuzzy almost continuous mapping.

Proof: (i) $\Rightarrow$ (ii): Obvious.

(ii) $\Rightarrow$ (iii): Let *A* be an *IFRCS* in *X*. By assumption $f(A)$ is an *IFCS* in *Y*. Then $(f^{-1})^{-1}(A)=f(A)$ is an *IFCS* in *Y*.

Hence f is an intuitionistic fuzzy almost continuous mapping.

(iii) $\Rightarrow$ (i): Let *A* be an *IFROS* in *X*. By hypothesis $(f^{-1})^{-1}(A)=f(A)$ is an *IFOS* in *Y*. Hence f is an *IFAO* mapping.

Theorem 3.17: If $f:(X,\tau)\to(Y,\sigma)$ is an intuitionistic fuzzy almost open mapping, then $scl(f(A))\subseteq f(cl(A))$ for every *IFSPOS A* in *X*.

Proof: Let *A* be an *IFSPOS* in *X*. Then cl(A) is an *IFRCS* in *X*. By the hypothesis f(cl(A)) is an *IFCS* in *Y*. Then f(cl(A)) is an *IFSCS* in *Y* and thus $scl(f(cl(A)))=f(cl(A))$. Now $scl(f(A))\subseteq scl(f(cl(A)))=f(cl(A))$. Therefore $scl(f(A))\subseteq f(cl(A))$.

Theorem 3.18: A mapping $f:(X,\tau)\to(Y,\sigma)$ is an *IFAO* if and only if for each *IFSCS* F of X, $f(int(F))\subset int(f(F))$.

Proof: Suppose that f is an *IFAO* mapping and let F be an *IFSCS* of *X*. Then $int(cl(f(F)))\subset F$ and $f(int(cl(f(F))))$ is an *IFOS* in *Y*. Therefore, we have. $f(int(F))\subset int(f(F))$. Conversely, let *U* be an *IFROS* of *X*. Then *U* is an *IFSCS*. By the hypothesis, we have $f(U)=f(int(U))\subset int(f(U))$. Thus, $f(U)$ is an *IFOS* in *Y* and hence f is an *IFAO* mapping.

Theorem 3.19: A mapping $f:(X,\tau)\to(Y,\sigma)$ is an *IFAO* if and only if for any intuitionistic fuzzy subset *S* of *Y* and any *IFRCS F* of *X* containing $f^{-1}(S)$, there exists a closed set *G* of *Y* containing *S* such that $f^{-1}(G)\subset F$.

Proof: Suppose that f is an *IFAO*. Let $S\subset Y$ and *F* be an *IFRCS* of *X* containing $f^{-1}(S)$. Put $G=Y-f(X-F)$. Since $f^{-1}(S)\subset F$, we have $S\subset G$. Since f is an *IFAO* mapping and *F* is *IFRCS* in *X*, *G* is an *IFCS* in *Y*. It follows from a straight forward calculation that $f^{-1}(G)\subset F$.

Conversely, Let *U* be an *IFROS* in *X* and $S=Y=f(U)$, then $X-U$ is an *IFRCS* containing $f^{-1}(S)$. By hypothesis, there exists an *IFCS G* of *Y* containing *S* such that $f^{-1}(G)\subset X-U$. Thus, we have $f(U)\subset Y-G$. On the other hand we have $f(U)=Y-S\supset Y-G$ and hence $f(U)=Y-G$. Consequently, $f(U)$ is an *IFROS* in *Y* and f is an *IFAO* mapping.

Theorem 3.20: A mapping $f:(X,\tau)\to(Y,\sigma)$ is an *IFAO* and *A* is an *IFROS* of *X*, then the restriction $f_A:(A,\tau/_A)\to(Y,\sigma)$ is an *IFAO* mapping.

Proof: Let *U* be an *IFROS* in the subspace *A*. Since *A* is an *IFROS* in *X*, so is *U* and hence $f(U)$ is an *IFROS* in *Y*. Therefore, f/A is an *IFAO* mapping.

Theorem 3.21: A mapping $f:(X,\tau)\to(Y,\sigma)$ is an *IFAO* mapping. If *A* is an *IFROS* in *X* such that $A=f^{-1}(B)$ for some intuitionistic fuzzy subset *B* of *Y*, then a mapping $f_A:A\to B$ is defined by $f_A(x)=f(x)$ for all intuitionistic fuzzy points $x\in A$ is an *IFAO* mapping.

Proof: Let *U* be an *IFROS* in the subspace *A*. Since *A* is an *IFROS* in *X*, we ave $U-int_A(cl_A(U))=A\cap int_X(cl_X(U))$. Since, f is an *IFAO* mapping, $f(int_X(cl_X(U)))$ is an *IFO* in *Y*. Therefore, $f_A(U)=B\cap f(int_X(cl_X(U)))$ is an *IFO* in the subspace *B* and hence f_A is an *IFAO* mapping.

REFERENCES:

[1] Atanassov K, *Intuitionistic fuzzy sets*, Fuzzy Sets and Systems 20 (1986), 87-96.

[2] Coker. D, *On fuzzy compactness in intuitionistic fuzzy topological spaces*, J. Fuzzy Math. 3 (1995), 899 - 909.

[3] Coker. D, *An introduction to intuitionistic fuzzy topological spaces*, Fuzzy Sets and Systems 88 (1997), 81- 89,

[4] Gurcay. H, Coker. D, *On fuzzy continuity in intuitionistic fuzzy topological spaces*, J. Fuzzy Math. 5 (1997), 365 - 378.

[5] Jun. Y.B Kang. J.O and Song S.Z, *Intuitionistic fuzzy irresolute and continuous mappings*, Far East J. Math. Sci. 17(2) (2005), 201-216.

[6] Jun. Y.B and Song S.Z, *Intuitionistic fuzzy semi-preopen sets and intuitionistic fuzzy semi-precontinuous mappings*, J. Appl. Math & Computing, Vol.19 (2005). No.1-2, pp 467-474.

[7] Lee S. J, and Lee E.P, The *category of intuitionistic fuzzy topological spaces*, Bull. Korean Math. Soc. 37 (2000). No.63-76.

[8] Zadeh LA. *Fuzzy sets*, Information and Control, 8 (1965), 338 - 353.

****

