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EFFECT OF SUSPENDED PARTICLES ON THERMAL INSTABILITY OF WALTERS' (MODELB') FLUID IN A BRINKMAN POROUS MEDIUM

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ABSTRACT

In this paper, the effect of suspended particles on thermal instability of incompressible Walters' (ModelB') elastico-viscous fluid in a porous medium is considered. For the porous medium, the Brinkman model is employed. By applying normal mode analysis method, the dispersion relation has been derived and solved analytically. It is observed that the medium permeability, suspended particles, gravity field and viscoelasticity introduce oscillatory modes. For stationary convection, it is observed that the Darcy number has stabilizing effect whereas the suspended particles and medium permeability has destabilizing effects on the system. The effects of suspended particles, Darcy number and medium permeability has also been shown graphically.

Keywords: *Brinkman porous medium, Suspended particles, Thermal instability, Viscoelasticity, Walters' (Model B') fluid.*

MSC 2010: *76A05; 76A10; 76E06; 76E25; 76S05.*

NOMENCLATURE

P_l	Dimensionless medium permeability
g	Gravitational acceleration
$\mathbf{g}$	Gravitational acceleration vector
m	Mass of suspended particle
D_A	Modified Darcy number
p	Pressure
K'	Stokes drag coefficient
N	Suspended particle number density
p_1	Thermal Prandtl number
v	Velocity of fluid
v_d	Velocity of suspended particles
k	Wave number of disturbance

Greek Symbols

β	Adverse temperature gradient
$\tilde{\mu}$	Effective viscosity of the porous medium
$\boldsymbol{\rho}$	Fluid density
μ	Fluid viscosity
μ'	Fluid viscoelasticity
υ	Kinematic viscosity
υ'	Kinematic viscoelasticity
ε	Medium porosity
δ	Perturbation in respective physical quantity
θ	Perturbation in temperature
η	Radius of suspended particles
κ	Thermal diffusivity
α	Thermal coefficient of expansion

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1. INTRODUCTION:

In recent years, considerable interest has been evinced in the study of thermal instability in a porous medium, because it has various applications in geophysics, food processing and nuclear reactors. A detailed account of the thermal instability of a Newtonian fluid, under varying assumptions of hydrodynamics and hydromagnetics has been given by Chandrasekhar [1]. Lapwood [2] has studied the convective flow in a porous medium using linearized stability theory. The Rayleigh instability of a thermal boundary layer in flow through a porous medium has been considered by Wooding [3]. Scanlon and Segel [4] have considered the effect of suspended particles on the onset of Be′nard convection and found that the critical Rayleigh number was reduced solely because the heat capacity of the pure gas was supplemented by the particles. The suspended particles were thus found to destabilize the layer.

Sharma and Sunil [5] have studied the thermal instability of an Oldroydian viscoelastic fluid with suspended particles in hydromagnetics in a porous medium. There are many elastico-viscous fluids that cannot be characterized by Maxwell's constitutive relations or Oldroyd's constitutive relations. One such class of fluids is Walters' (Model B') elastico-viscous fluid having relevance in chemical technology and industry. Walters' [6] reported that the mixture of polymethyl methacrylate and pyridine at 25^oC containg 30.5g of polymer per litre with density 0.98g per litre behaves very nearly as the Walters' (Model B') elastico-viscous fluid. Walters' (Model B') elastico-viscous fluid form the basis for the manufacture of many important polymers and useful products.

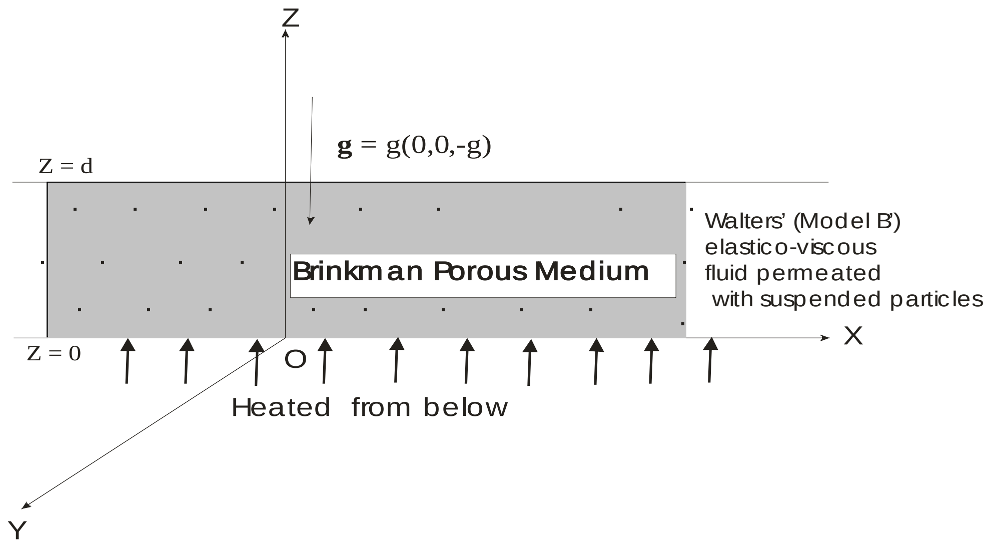
The investigation in porous media has been started with the simple Darcy model and gradually was extended to Darcy-Brinkman model. A good account of convection problems in a porous medium is given by Vafai and Hadim [7], Ingham and Pop [8] and Nield and Bejan [9]. Kuznetsov and Nield [10] have studied thermal instability in a porous medium layer saturated by a nanofluid: Brinkman model. Sharma and Rana [11] have studied thermal instability of a incompressible Walters' (Model B') elastico-viscous in the presence of variable gravity field and rotation in porous medium. Recently, Rana and Kango [12] have been studied the effect of rotation on thermal instability of Compressible Walters' (Model B') elastico-viscous fluid in porous medium.

The interest for investigations of non-Newtonian fluids is also motivated by a wide range of engineering applications which include ground pollutions by chemicals which are non-Newtonian like lubricants and polymers and in the treatment of sewage sludge in drying beds. Recently, polymers are used in agriculture, communications appliances and in bio medical applications. Examples of these applications are filtration processes, packed bed reactors, insulation system, ceramic processing, enhanced oil recovery, chromatography etc.

Keeping in mind the importance in various applications mentioned above, our interest, in the present paper is to study the effect of suspended particles on thermal instability of incompressible Walters' (Model B') elastico-viscous fluid in a Brinkman porous medium. This necessitates of an additional parameter, namely, a Darcy number.

2. MATHEMATICAL MODEL AND PERTURBATION EQUATIONS:

Here we consider an infinite, horizontal, incompressible Walters' (Model B') elastico-viscous fluid of depth d, bounded by the planes z = 0 and z = d in an isotropic and homogeneous medium of porosity ε and permeability k_1, which is acted upon by gravity **g**(0, 0, -g). This layer is heated from below such that a steady adverse temperature gradient $\beta\left(=\left|\frac{dT}{dz}\right|\right)$ is maintained. The character of equilibrium of this initial static state is determined by supposing that the system is slightly disturbed and then following its further evolution.



Schematic Sketch of Physical Situation

Let $\rho, \upsilon, \upsilon', p, \varepsilon$, T, α and $\boldsymbol{v}$(0, 0, 0), denote respectively, the density, kinematic viscosity, kinematic viscoelasticity, pressure, medium porosity, temperature, thermal coefficient of expansion and velocity of the fluid.

The equations expressing the conservation of momentum, mass, temperature and equation of state for Walters' (Model B') elastico-viscous fluid in a Brinkman porous medium are

$$\frac{1}{\varepsilon}\left[\frac{\partial \boldsymbol{v}}{\partial t}+\frac{1}{\varepsilon}(\boldsymbol{v}.\nabla)\boldsymbol{v}\right]=-\frac{1}{\rho_0}\nabla p+\boldsymbol{g}\left(1+\frac{\delta\rho}{\rho_0}\right)-\frac{1}{k_1}\left(\upsilon-\upsilon'\frac{\partial}{\partial t}\right)\boldsymbol{v}+\frac{\tilde{\mu}}{\rho_0}\nabla^2\boldsymbol{v}+\frac{K'N}{\rho_0\in}(\boldsymbol{v_d}-\boldsymbol{v}), \tag{1}$$

$$\nabla.\boldsymbol{v}=0, \tag{2}$$

$$E\frac{\partial T}{\partial t}+(\boldsymbol{v}.\nabla)T+\frac{mNC_{pt}}{\rho_0 C_f}\left[\in\frac{\partial}{\partial t}+v_d.\nabla\right]T=\kappa\nabla^2\mathrm{T}, \tag{3}$$

$$\rho=\rho_0[1-\alpha(T-T_0)], \tag{4}$$

where the suffix zero refers to values at the reference level z = 0.

Here Here $v_d(\bar{x},t)$ and $N(\bar{x},t)$ denote the velocity and number density of the particles respectively,$K'=6\pi\eta\rho v$, where η is particle radius, is the Stokes drag coefficient, $v_d=(l,r,s)$ and $\bar{x}=(x,y,z)$.

$$E=\varepsilon+(1-\varepsilon)\left(\frac{\rho_s c_s}{\rho_0 c_f}\right)$$

which is constant, κ is the thermal diffusivity, $\rho_{s,}\ c_s;\ \rho_{0,}\ c_f$ denote the density and heat capacity of solid (porous) matrix and fluid, respectively.

If mN is the mass of particles per unit volume, then the equations of motion and continuity for the particles are

$$mN\left[\frac{\partial v_d}{\partial t}+\frac{1}{\varepsilon}(v_d.\nabla)v_d\right]=K'N(v-v_d), \tag{5}$$

$$\varepsilon\frac{\partial N}{\partial t}+\nabla.(Nv_d)=0, \tag{6}$$

The presence of particles adds an extra force term proportional to the velocity difference between particles and fluid and appears in the equation of motion (1). Since the force exerted by the fluid on the particles is equal and opposite to that exerted by the particles on the fluid, there must be an extra force term, equal in magnitude but opposite in sign, in the equations of motion for the particles (6). The buoyancy force on the particles is neglected. Interparticles reactions are not considered either since we assume that the distance between the particles are quite large compared with their diameters. These assumptions have been used in writing the equations of motion (5) for the particles.
The initial state of the system is taken to be quiescent layer (no settling) with a uniform particle distribution number. The initial state is

$$\boldsymbol{v}=(0,0,0), T=-\beta z+T_0, \rho=\rho_0(1+\alpha\beta z), \tag{7}$$

is an exact solution to the governing equations.

Let $\boldsymbol{v}$(u, v, w), θ, δp and $\delta\rho$ denote, respectively, the perturbations in fluid velocity $\boldsymbol{v}$(0,0,0), temperature T, pressure p and density ρ.

The change in density $\delta\rho$ caused by perturbation θ in temperature is given by

$$\delta\rho=-\alpha\rho_0\theta. \tag{8}$$

The linearized perturbation equations governing the motion of fluid are

$$\frac{1}{\varepsilon}\frac{\partial \boldsymbol{v}}{\partial t}=-\frac{1}{\rho_0}\nabla\delta p-g\frac{\delta\rho}{\rho_0}-\frac{1}{k_1}\left(\upsilon-\upsilon'\frac{\partial}{\partial t}\right)\boldsymbol{v}+\frac{\tilde{\mu}}{\rho_0}\nabla^2\boldsymbol{v}+\frac{K'N}{\rho_0\in}(\boldsymbol{v_d}-\boldsymbol{v}), \tag{9}$$

$$\nabla.\boldsymbol{v}=0\ , \tag{10}$$

$$(E+b\varepsilon)\frac{\partial\theta}{\partial t}=\beta(w+bs)+\kappa\nabla^2\theta, \tag{11}$$

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where $b = \frac{mN C_{pt}}{\rho_0 C_f}$ and w, s are the vertical fluid and particles velocity.

In the Cartesian form, equations (9)-(11) with the help of equation (8) can be expressed as

$$\frac{1}{\varepsilon}\frac{\partial u}{\partial t} = -\frac{1}{\rho_0}\frac{\partial}{\partial x}(\delta p) - \frac{1}{k_1}\left(\upsilon - \upsilon'\frac{\partial}{\partial t}\right)u + \frac{\tilde{\mu}}{\rho_0}\nabla^2 u - \frac{mN}{\in\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\rho_0}\frac{\partial u}{\partial t}, \tag{12}$$

$$\frac{1}{\varepsilon}\frac{\partial v}{\partial t} = -\frac{1}{\rho_0}\frac{\partial}{\partial y}(\delta p) - \frac{1}{k_1}\left(\upsilon - \upsilon'\frac{\partial}{\partial t}\right)v + \frac{\tilde{\mu}}{\rho_0}\nabla^2 v - \frac{mN}{\in\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\rho_0}\frac{\partial v}{\partial t}, \tag{13}$$

$$\frac{1}{\varepsilon}\frac{\partial w}{\partial t} = -\frac{1}{\rho_0}\frac{\partial}{\partial z}(\delta p) + g\alpha\theta - \frac{1}{k_1}\left(\upsilon - \upsilon'\frac{\partial}{\partial t}\right)w + \frac{\tilde{\mu}}{\rho_0}\nabla^2 w - \frac{mN}{\in\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\rho_0}\frac{\partial w}{\partial t}, \tag{14}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{15}$$

$$(E + b\varepsilon)\frac{\partial \theta}{\partial t} = \beta(w + bs) + \kappa\nabla^2\theta. \tag{16}$$

Operating equation (12) and (13) by $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial y}$ respectively, adding and using equation (8), we get

$$\frac{1}{\varepsilon}\frac{\partial}{\partial t}\left(\frac{\partial w}{\partial z}\right) = \frac{1}{\rho_0}\left(\nabla^2 - \frac{\partial^2}{\partial z^2}\right)\delta p - \frac{1}{k_1}\left(\upsilon - \upsilon'\frac{\partial}{\partial t}\right)\left(\frac{\partial w}{\partial z}\right) + \frac{\tilde{\mu}}{\rho_0}\nabla^2\left(\frac{\partial w}{\partial z}\right) - \frac{mN}{\in\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\rho_0}\frac{\partial}{\partial t}\left(\frac{\partial w}{\partial z}\right). \tag{17}$$

Operating equation (14) and (17) by $\left(\nabla^2 - \frac{\partial^2}{\partial z^2}\right)$ and $\frac{\partial}{\partial z}$ respectively and adding to eliminate δp between equations (14) and (17), we get

$$\frac{1}{\varepsilon}\frac{\partial}{\partial t}(\nabla^2 \mathrm{w}) = -\frac{1}{k_1}\left(\upsilon - \upsilon'\frac{\partial}{\partial t}\right)\nabla^2\mathrm{w} + \frac{\tilde{\mu}}{\rho_0}\nabla^4\mathrm{w} + \mathrm{g}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\alpha\theta - \frac{mN}{\in\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\rho_0}\frac{\partial}{\partial t}(\nabla^2\mathrm{w}), \tag{18}$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$.

3. THE DISPERSION RELATION:

Following the normal mode analyses, we assume that the perturbation quantities have x, y and t dependence of the form

$$[w, \theta] = [W(z), \Theta(z)]exp(ilx + imy + nt), \tag{19}$$

where l and m are the wave numbers in the x and y directions, $k = (l^2 + m^2)^{1/2}$ is the resultant wave number and n is the frequency of the harmonic disturbance, which is, in general, a complex constant.

Using expression (19) in equations (18) and (16) become

$$\frac{n}{\varepsilon}\left[\frac{d^2}{dz^2} - k^2\right]W = -gk^2\alpha\Theta - \frac{1}{k_1}(\upsilon - \upsilon' n)\left(\frac{d^2}{dz^2} - k^2\right)W + \frac{\tilde{\mu}}{\rho_0}\left(\frac{d^2}{dz^2} - k^2\right)^2 W - \frac{mN}{\in\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\rho_0}\left(\frac{d^2}{dz^2} - k^2\right)W, \tag{20}$$

$$(E + b\varepsilon)\frac{\partial \theta}{\partial t} = \beta(w + bs) + \kappa\left(\frac{d^2}{dz^2} - k^2\right)\Theta. \tag{21}$$

Equation (20) and (21) in non-dimensional form, become

$$\left[1 + \left(\frac{P_l}{\varepsilon} + \frac{MP_l}{\varepsilon(1+\tau_1\sigma)} - F\right)\sigma - D_A(D^2 - a^2)\right](D^2 - a^2)W + \frac{ga^2d^2P_l\alpha\Theta}{\upsilon} = 0, \tag{22}$$

$$[D^2 - a^2 - E_1 p_1\sigma]\Theta = -\frac{\beta d^2}{\kappa}\left(\frac{B+\tau_1\sigma}{1+\tau_1\sigma}\right)W, \tag{23}$$

where we have put

$a = kd$, $\sigma = \frac{nd^2}{\upsilon}$, $F = \frac{\upsilon'}{d^2}$ and $P_l = \frac{k_1}{d^2}$, is the dimensionless medium permeability, $p_1 = \frac{\upsilon}{\kappa}$, is the thermal Prandtl number and $D_A = \frac{\tilde{\mu}k_1}{\mu d^2}$, is the Darcy number modified by the viscosity ratio.

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Eliminating Θ between equations (22) and (23), we obtain

$$\left[1+\left(\frac{P_l}{\varepsilon}+\frac{MP_l}{\varepsilon(1+\tau_1\sigma)}-F\right)\sigma-D_A(D^2-a^2)\right](D^2-a^2)(D^2-a^2-Ep_1\sigma)W-Ra^2P_l\left(\frac{B+\tau_1\sigma}{1+\tau_1\sigma}\right)W=0, \quad (24)$$

where $R=\frac{g\alpha\beta\, d^4}{\upsilon\kappa}$, is the thermal Rayleigh number.

Here we assume that the temperature at the boundaries is kept fixed, the fluid layer is confined between two boundaries and adjoining medium is electrically non-conducting. The boundary conditions appropriate to the problem are (Chandrasekhar [1])

$$W = D^2W = D^4 W = \Theta = 0 \text{ at } z = 0 \text{ and } 1. \quad (25)$$

The case of two free boundaries, though a little artificial is the most appropriate for stellar atmospheres. Using the boundary conditions (25), we can show that all the even order derivatives of W must vanish for z = 0 and z = 1 and hence the proper solution of W characterizing the lowest mode is

$$W=W_0\sin\pi z\,; \quad W_0 \text{ is a constant.} \quad (26)$$

Substituting equation (27) in (25), we get

$$R_1xP=\left[1+\left(\frac{P}{\varepsilon}+\frac{MP}{\varepsilon(1+\tau_1 i\sigma_1)}-\pi^2F\right)i\sigma_1+D_{A_1}(1+x)\right](1+x)(1+x+E_1p_1i\sigma_1)\left(\frac{1+\tau_1\pi^2 i\sigma_1}{B+\tau_1\pi^2 i\sigma_1}\right). \quad (27)$$

where we have put $R_1=\frac{R}{\pi^4}$, $T_{A_1}=\frac{T_A}{\pi^4}$, $D_{A_1}=\frac{D_A}{\pi^2}$ $x=\frac{a^2}{\pi^2}$, $i\sigma_1=\frac{\sigma}{\pi^2}$, $P=\pi^2P_l$, $\tau=\frac{m}{K'}$, $\tau_1=\frac{\tau\upsilon}{d^2}$, $M=\frac{mN}{\rho_0}$,
$E_1=E+b\in$, B = b+1.

Equation (27) is required dispersion relation accounting for the onset of thermal instability in Walters' (Model B') elastico-viscous fluid permeated with suspended particles in a Brinkman porous medium.

4. STABILITY OF THE SYSTEM AND OSCILLATORY MODES:

Here, we examine the possibility of oscillatory modes, if any, in Walters' (Model B') elastico-viscous fluid due to the presence of suspended particles, viscoelasticity, medium perpeability and gravity field. Multiply equation (22) by W^* the complex conjugate of W, integrating over the range of z and making use of equations (23) with the help of boundary conditions (25), we obtain

$$\left[1+\left(\frac{P_l}{\varepsilon}+\frac{MP_l}{\varepsilon(1+\tau_1\sigma)}-F\right)\sigma\right]I_1-D_AI_2-\frac{ga^2\alpha\kappa P_l}{\upsilon\beta}\left(\frac{1+\tau_1\sigma^*}{B+\tau_1\sigma^*}\right)(I_3+Ep_1\sigma^*I_4)=0, \quad (28)$$

where

$$I_1=\int_0^1(|DW|^2+a^2|W|^2)\,dz,$$

$$I_2=\int_0^1(|DW|^4+2a^2|DW|^2+a^4|W|^2)\,dz,$$

$$I_3=\int_0^1(|D\Theta|^2+a^2|\Theta|^2)\,dz,$$

$$I_4=\int_0^1|\Theta|^2dz,$$

The integral part I_1-I_4 are all positive definite. Putting $\sigma=i\sigma_i$ in equation (28), where σ_i is real and equating the imaginary parts, we obtain

$$\sigma_i\left[\left(\frac{P_l}{\varepsilon}+\frac{MP_l}{\varepsilon(1+\tau_1{}^2\sigma_i{}^2)}-F\right)I_1+\frac{ga^2\alpha\kappa P_l}{\upsilon\beta}\left\{\left(\frac{\tau_1(B-1)}{B^2+\tau_1{}^2\sigma_i{}^2}\right)I_3+\left(\frac{\tau_1(B-1)}{B^2+\tau_1{}^2\sigma_i{}^2}\right)E_1p_1I_4\right\}\right]=0, \quad (29)$$

Equation (29) implies that $\sigma_i=0$ or $\sigma_i\neq0$ which mean that modes may be non-oscillatory or oscillatory. The oscillatory modes introduced due to presence of viscosity, viscoelasticity, suspended particles and medium permeability which were non-existent in their absence.

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5. THE STATIONARY CONVECTION:

For stationary convection, putting $\sigma = 0$ in equations (27), we obtain

$$R_1 = \frac{(1+x)^2}{xPB}\left[1 + (1+x)D_{A_1}\right] \tag{30}$$

Equation (30) expresses the modified Rayleigh number R_1 as a function of the dimensionless wave number x and the parameters B, D_{A_1}, P and Walters' (Model B') elastico-viscous fluid behave like an ordinary Newtonian fluid since elastico-viscous parameter F vanishes with σ .

To study the effects of suspended particles, Darcy number and medium permeability, we examine the behavior of $\frac{dR_1}{dB}$, $\frac{dR_1}{dD_{A_1}}$ and $\frac{dR_1}{dP}$ analytically.

From equation (30), we get

$$\frac{dR_1}{dB} = -\left(\frac{1+x}{xPB^2}\right)\left[1 + (1+x)D_{A_1}\right], \tag{31}$$

which is negative. Hence, suspended particles have destabilizing effect on the thermal instability of Walters' (Model B') elastico-viscous fluid in a Brinkman porous medium. This destabilizing effect is an agreement of the earlier work of Scanlon and Segel [7], Sharma and Sunil [8], Sharma and Rana [9] and Rana and Kango [6].
From equation (30), we get

$$\frac{dR_1}{dD_{A_1}} = \frac{(1+x)^3}{xPB}, \tag{32}$$

which is positive implying thereby the stabilizing effect of Darcy number on the thermal instability of Walters' (Model B') elastico-viscous fluid permeated with suspended particles in a Brinkman porous medium.
It is evident from equation (31) that

$$\frac{dR_1}{dP} = -\frac{(1+x)^2}{xBP^2}\left[1 + (1+x)D_{A_1}\right], \tag{33}$$

From equation (33), we observe that medium permeability has destabilizing effect on the thermal instability of Walters' (Model B') elastico-viscous fluid permeated with suspended particles in a Brinkman porous medium. This destabilizing effect is an agreement of the earlier work of Scanlon and Segel [7], Sharma and Sunil [8], Sharma and Rana [9] and Rana and Kango [6].

The dispersion relation (30) is analyzed numerically. Graphs have been plotted by giving some numerical values to the parameters, to depict the stability characteristics.

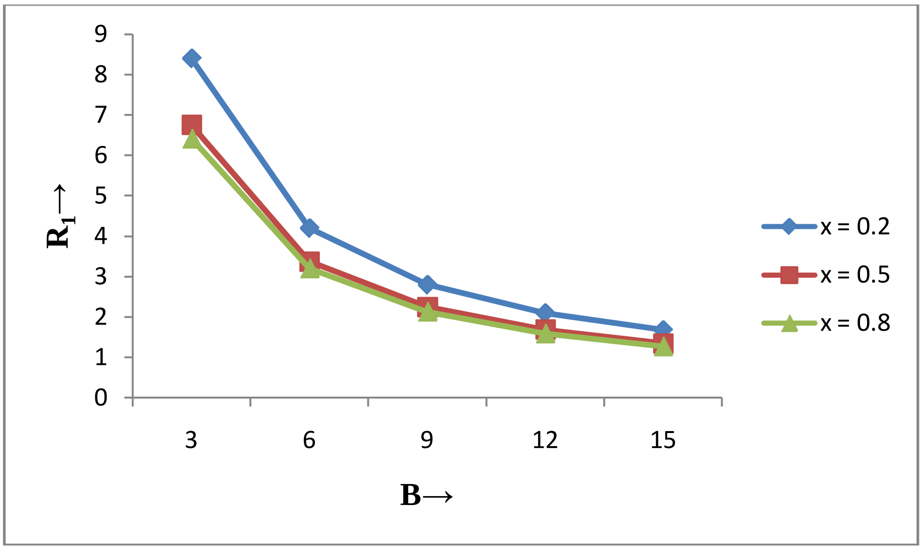

Fig.1. Variation of Rayleigh number R_1 with suspended particles B for $P = 2$ and $D_{A_1} = 10$ for fixed wave numbers $x = 0.2$, $x = 0.5$ and $x = 0.8$.

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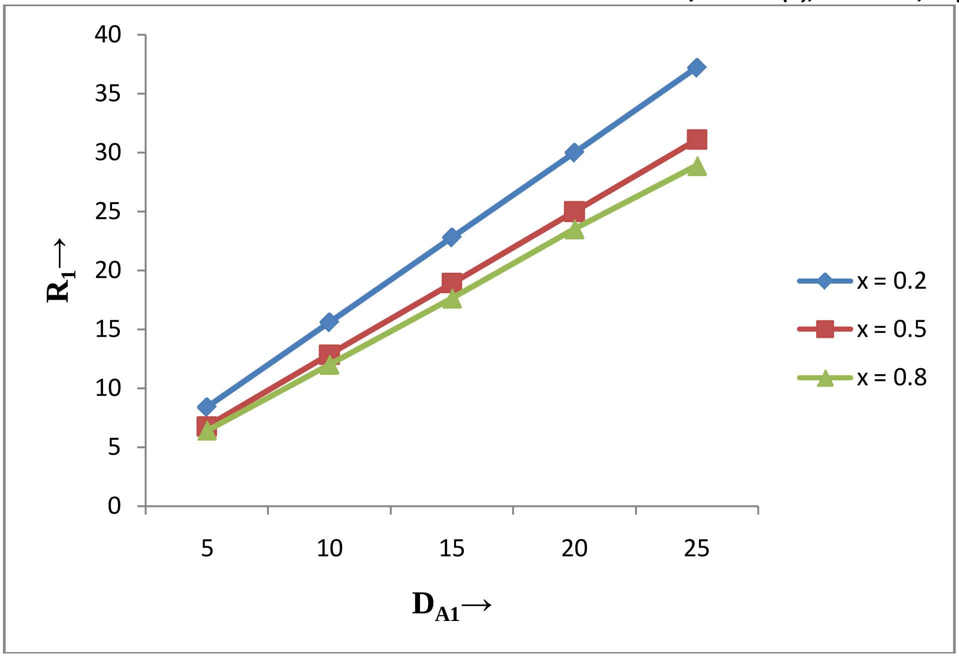

Fig.2. Variation of Rayleigh number R_1 with Darcy number D_{A_1} for *P = 2* and *B = 3* for fixed wave numbers *x = 0.2*, *x = 0.5* and *x = 0.8*.

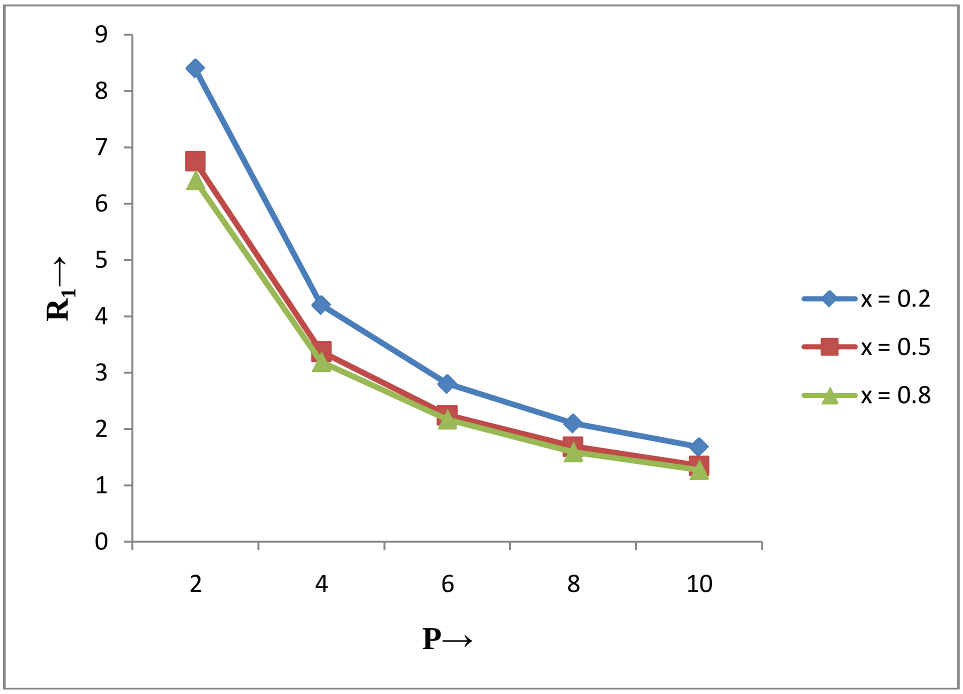

Fig.3. Variation of Rayleigh number R_1 with medium permeability *P* for *B = 3* and $D_{A_1} = 10$ for fixed wave numbers *x = 0.2*, *x = 0.5* and *x = 0.8*.

In fig.1, Rayleigh number R_1 is plotted against suspended particles *B* for *P = 2* and $D_{A_1} = 10$ for fixed wave numbers *x = 0.2*, *x = 0.5* and *x = 0.8*. This shows that suspended particles has a destabilizing effect on the thermal instability of Walters' (Model B') elastico-viscous fluid in a Brinkman porous medium for fixed wave numbers *x = 0.2*, *x = 0.5* and *x = 0.8*. In Fig.2, Rayleigh number R_1 is plotted against with Darcy number D_{A_1} for *P = 2*, *B = 3* for fixed wave numbers x = 0.2, *x = 0.5* and x = 0.8. This shows that Darcy number has a stabilizing effect on the thermal instability of Walters' (Model B') elastico-viscous fluid permeated with suspended particles in a Brinkman porous medium.

In fig.3, Rayleigh number R_1 is plotted against medium permeability *P* for $D_{A_1} = 10$ and *B = 3* for fixed wave numbers x = 0.2, *x = 0.5* and x = 0.8. This shows that medium permeability has a destabilizing effect on the thermal instability of Walters' (Model B') elastico-viscous fluid permeated with suspended particles in a Brinkman porous medium.

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6. CONCLUSION:

The effect of susprnded particles on thermal instability of Walters' (Model B') elastico-viscous fluid heated from below in a Brinkman porous medium has been investigated. The dispersion relation, including the effects of suspended particles, Darcy number, medium permeability and viscoelasticity on the thermal instability of Walters' (Model B') fluid in porous medium is derived. From the analysis, the main conclusions are as follows:

(i) For the case of stationary convection, Walters' (Model B') elastico-viscous fluid behaves like an ordinary Newtonian fluid as elastico-viscous parameter F vanishes with σ.

(ii) The expressions for $\frac{dR_1}{dB}, \frac{dR_1}{dD_{A_1}}$ and $\frac{dR_1}{dP}$ are examined analytically and it has been found that the Darcy number has stabilizing effect whereas the suspended particles and medium permeability has a destabilizing effect on the system.

(iii) The effects of suspended particles, Darcy number and medium permeability on thermal instability of Walters' (Model B') elastico-viscous fluid permeated with suspended particles in a Brinkman porous medium have also been shown graphically in figures 1, 2 and 3 respectively.

(iv) The oscillatory modes introduced due to presence of viscoelasticity, suspended particles, gravity field and medium permeability, which were non-existent in their absence.

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