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RELATIVISTIC COSMOLOGICAL MASS MODEL

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ABSTRACT

In this paper, the problem of spherically symmetric homogeneous space time with perfect fluid and viscous fluid distribution are considered separately in Einstein theory alongwith the cosmological term. In order to obtain the exact solution, we have assumed the barotropic equation of state $p = \gamma\rho$, $-1 \le \gamma \le 1$. It is observed that fluid distribution generates isotropic false vacuum model when $\gamma = -1$ and the solution becomes singular at $r = -b$. The cream of this paper is the viscous fluid distribution for the space time yields the case of perfect fluid distribution without imposing any condition and restriction.

Key Words: *De-generate vacuum model, perfect fluid, viscousfluid cosmological constant, isotropy, geodesic.*

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1. INTRODUCTION:

We know that Einstein theory of general relativity is a co-ordinate invariant theory which serves as basis for constructing models of the universe. Many authors have taken attempts to modify and generalise the Einstein's theory by incorporating Mach's principle and other desired features which are lacking in original theory. Out of the authors, *Buchdahl* (1959), *Bramhachary* (1960), *Jains* at.al (1968) and *Mohanty et. al.* (2000, 2002a) have studied in various angles.

The cosmological constant Λ has been introduced in 1917 by Einstein to modify his own equations of general relativity. Now this cosmological term plays an important role in modern theories. A number of observations suggest that the universe possess a non-zero cosmological constant (Krauss and Turner, 1995). The cosmological term corresponds to the energy density or vacuum in context of quantum field theory, which provides a repulsive force opposing the gravitational pull between the galaxies. If the cosmological term is a large value than its energy plus the matter in the universe which can sum of to numbers that inflation predicts. Thus it is highly necessary to study about the cosmological constant.

Hence we have devoted to investigate the role of cosmological constant (Λ) and solutions in spherically symmetric homogenous space time in presence of perfect and viscous fluid separately. The field equations with Λ have been derived and also solutions. Moreover, the space time has been designed as de-generate vacuum universe stating $p + \rho = 0$. Also some physical and geometrical properties have been discussed in this paper.

2. FIELD EQUATION:

We have taken the static spherically symmetric metric of the form

$$ds^2 = e^{\nu} dt^2 - e^{\lambda} dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right) \tag{1}$$

where υ and λ are functions of 'r' only.

The Einstein's field equations for the metric (1) gravitating perfect fluid with cosmological term Λg_{ij} can be written as

$$R_{ij} - \frac{1}{2} g_{ij} R + \Lambda g_{ij} = -\frac{k}{4\pi} T_{ij} \tag{2}$$

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where T_{ij} is the energy momentum tensor of gravitating macro matter field represented by perfect fluid and it is given by

$$T_{ij} = (\rho + p)u_i u_j - pg_{ij} \tag{3}$$

which yields the components $T_{11} = pe^{\lambda}, T_{22} = pr^2, T_{33} = pr^2 \sin^2\theta \text{ and } T_{44} = \rho e^{\nu}$ (3a) together with

$$g^{ij}u_i u_j = 1 \tag{4}$$

which gives $u_1 = u_2 = u_3 = 0 \text{ and } u_4^2 = e^{\nu}$. (4a)

Here u_i is the four velocity vector of the fluid, ρ and p are energy density and proper pressure of the distribution respectively.

Using co-moving co-ordinate system the Einstein's field equation(2) for the metric (1) yields

$$e^{-\lambda}\left(\frac{\nu_1}{r} + \frac{1}{r^2}\right) - \frac{1}{r^2} + \Lambda = \frac{kp}{4\pi}, \tag{5}$$

$$e^{-\lambda}\left(\frac{\nu_1}{2} + \frac{\nu_{11}^2}{4} - \frac{\nu_1\lambda_1}{4} + \frac{\nu_1 - \lambda_1}{2r}\right) + \Lambda = \frac{kp}{4\pi} \tag{6}$$

and $e^{-\lambda}\left(\frac{\lambda_1}{r} - \frac{1}{r^2}\right) + \frac{1}{r^2} - \Lambda = \frac{k\rho}{4\pi}$, (7)

where the index'1' indicates the ordinary differentation with respect to 'r'.

3. SOLUTIONS AND MODEL:

Since the system of equations of section-2 is under determined, we can take the barotropic equation of state.

$$p = \gamma\rho, \ -1 \le \gamma \le 1 \tag{8}$$

as an additional condition.

If $\gamma = -1$, the equation (8) makes the form

$$\rho + p = 0, \tag{9}$$

for which, the metric(1) represent de-generate vacuum universe (false vacuum universe)

Now adding equation (5) and(7), we get

$$e^{-\lambda}\left(\frac{\lambda_1 + \nu_1}{r} - \frac{1}{r^2}\right) = \frac{k}{4\pi}(p + \rho) \tag{10}$$

by Using equation (9) and (10) we obtain

$$\lambda_1 + \nu_1 = 0 \tag{11}$$

On integration, equation (11) yields

$$\lambda + \nu = a, \tag{12}$$

where 'a' is the constant of integration.

But we can take a = 0 as λ = ν = 0 at r → ∞.Thus the equation (12) reduces to

$$\lambda+\nu=0. \tag{13}$$

using equation (13) in equation (6), we obtain

$$e^{-\lambda}\left(\frac{\lambda_1}{r}-\frac{\lambda_1^2}{2}+\frac{\lambda_{11}}{2}\right)-\Lambda+\frac{kp}{4\pi}=0 \tag{14}$$

If we consider $\Lambda=\dfrac{kp}{4\pi}$ then equation (14) reduces to

$$\frac{\lambda_1}{r}-\frac{\lambda_1^2}{2}+\frac{\lambda_{11}}{2}=0 \tag{15}$$

on integration, equation (15) yield

$$\lambda_1 e^{-\lambda}=\frac{b}{r^2} \tag{16}$$

where b is the constant of integration. Again integrating equation (16), we get

$$e^{-\lambda}=\frac{b}{r}+c \tag{17}$$

where c is the constant of integration. Putting c = 1 in equation (17), it reduces to

$$\lambda=\ell_n\left(\frac{r}{r+b}\right) \tag{18}$$

Using equation (18) in (13), we get

$$\nu=\ell_n\left(\frac{r}{r+b}\right) \tag{19}$$

Hence the degenerate vacuum model of the metric (1) is designed as

$$ds^2=\left(\frac{r}{r+b}\right)^{-1}dt^2-\left(\frac{r}{r+b}\right)dr^2-r^2\left(d\theta^2+\sin^2 d\theta^2\right) \tag{20}$$

4. The Einstein field equation for the metric (1) gravitating viscous fluid with cosmological term Λg_{ij} is taken as in equation (2) and T_{ij} is the energy momentum tensor for viscous fluid is given by

$$T_{ij}=\rho u_i u_j-\left(\overline{p}-\xi\overline{\theta}\right)H_{ij}-2\eta\,\sigma_{ij} \tag{21}$$

where ρ is the matter density, $\overline{\rho}$ is the isotropic pressure, u_i is the four velocity, η coefficient of shear, ξ is the bulk viscosity,

$$\overline{\theta}=u^{\alpha}{}_{;\alpha}=\frac{\partial u^{\alpha}}{\partial x^{\alpha}}+\Gamma^{\alpha}_{\alpha\beta}\,u^{\beta} \tag{22}$$

is the expansion factor.

$H_{ij}=g_{ij}-u_i u_j$ is the projection tensor (23)

and

$$\sigma_{ij}=\frac{1}{2}\left(u_{i;\alpha}\,H_j^{\alpha}+u_{j;\alpha}\,H_i^{\alpha}\right)-\frac{1}{3}\overline{\theta}H_{ij} \tag{24}$$

the shear tensor,

where $H_j^{\alpha} = g^{\alpha\beta} H_{j\beta}$. (25)

Now, the expansion factor $\overline{\theta}$ in co-moving co-ordinate system is

$$\overline{\theta} = u_{j1}^{1} + u_{j2}^{2} + u_{j3}^{3} + u_{j4}^{4} = 0 \tag{26}$$

The non-venishing components of the projection tensor H_{ij} associated with the metric (1) are

$$\left.\begin{aligned} H_{11} &= -e^{\lambda} \\ H_{22} &= -r^2 \\ \text{and} \quad H_{33} &= -r^2 \sin^2\theta \end{aligned}\right\} \tag{27}$$

Using equation (27) in equation (25), the non vanishing components are

$$H_1^1 = H_2^2 = H_3^3 = 1 \tag{28}$$

The non vanishing components of the co-variant derivative of four velocity vector is

$$u_{1;4} = -\frac{\nu_1}{2} e^{\frac{\nu}{2}} \tag{29}$$

Equation (24) for the metric (1) yield

$$\sigma_{ij} = 0 \tag{30}$$

Hence, with the help of equation (26), (27) & (30), the non-vanishing components of energy momentum tensor T_{ij} are

$$\left.\begin{aligned} T_{11} &= \overline{p} e^{\lambda}, \\ T_{22} &= \overline{p} r^2 \\ T_{33} &= \overline{p} r^2 \sin^2\theta \\ \text{and} \quad T_{44} &= \rho e^{\nu} \end{aligned}\right\} \tag{31}$$

where $\overline{p} = p + \xi u_{;\alpha}^{\alpha}$ (32)

in which p is the internal pressure.

Using equation (26) in equation (32) we get

$$\overline{p} = p \tag{33}$$

Now equation (31) can be written as

$$\left.\begin{aligned} T_{11} &= p e^{\lambda}, \\ T_{22} &= p r^2 \\ T_{33} &= p r^2 \sin^2\theta \\ \text{and} \quad T_{44} &= \rho e^{\nu} \end{aligned}\right\} \tag{34}$$

Since the energy momentum tensor T_{ij} obtained in equation (34) for viscous fluid distribution are identical with the energy momentum tensor T_{ij} for perfect fluid distribution obtained in equation (4a), the field equations and solutions are also same for the matric (1).

5. GEOMETRICAL AND PHYSICAL PROPERTIES:

In this geometrical and physical properties of the Universe (20)

(i) Einstein Space:

It is shown that the space time (1) is not an Einstein space as

$R_{ij} \neq \frac{R}{4} g_{ij}$ for i, j = 1,2,3,4 .

(ii) Motion of test Particle:

The motion of test particle is given by geodesic equation

$$\frac{d^2x^i}{ds^2} + \Gamma^i_{jk} \frac{dx^j}{ds}\frac{dx^k}{ds} = 0 \tag{35}$$

Where s is the parameter along the path of the particle. The r, θ and φ components of the geodesic equations are

$$\frac{d^2r}{ds^2} + \frac{\lambda_1}{2}\left(\frac{dr}{ds}\right)^2 = 0 \tag{36a}$$

$$\frac{d^2r}{ds^2} - re^{-\lambda}\left(\frac{d\theta}{ds}\right)^2 = 0 \tag{36b}$$

$$\frac{d^2r}{ds^2} - r\sin^2\theta\, e^{-\lambda}\left(\frac{d\varphi}{ds}\right)^2 = 0 \tag{36c}$$

$$\frac{d^2\theta}{ds^2} + \frac{1}{r}\frac{dr}{ds}\cdot\frac{d\theta}{ds} = 0 \tag{36d}$$

$$\frac{d^2\theta}{ds^2} - \sin\theta\cdot\cos\theta\left(\frac{d\varphi}{ds}\right)^2 = 0 \tag{36e}$$

$$\frac{d^2\varphi}{ds^2} + \frac{1}{r}\frac{dr}{ds}\cdot\frac{d\varphi}{ds} = 0 \tag{36f}$$

and $$\frac{d^2\varphi}{ds^2} + \cot\theta\,\frac{d\theta}{ds}\cdot\frac{d\varphi}{ds} = 0 \tag{36g}$$

But we have already obtained in equation (4a) is $\frac{dr}{ds} = 0$, $\frac{d\theta}{ds} = 0$ *and* $\frac{d\varphi}{ds} = 0$. Thus particle has zero velocity in r, θ and φ directions as well. Subsequently it is obtained from equation (36a) – (36g) that

$$\frac{d^2r}{ds^2} = \frac{d^2\theta}{ds^2} = \frac{d^2\varphi}{ds^2} = 0. \tag{37}$$

It concludes from equation (37) that the particle remains at rest for ever in the same direction.

(iii) The expansion scalar $\bar{\theta}$ is zero and it implies that the universe has no expansion or contraction.

(iv) the verticity tensor ω = 0 as $\omega_{ij} = \frac{1}{2}\left(u_{i;j} - u_{j;i}\right) - \frac{1}{2}\left(\dot{u}_i u_j - \dot{u}_j u_i\right) = 0$. Thus the universe is non-rotating in nature.

(v) The shear tensor σ = 0 as $\sigma_{ij} = 0$ which indicates that the universe (20) is non shearing in nature and hence isotropic.

(vi) The acceleration of the universe (20) is also obtained as

$$\dot{u}_i = -\frac{\nu_1}{2} = \frac{b}{2r\left(r+b\right)} \neq 0.$$

Hence both the fluids are non-geodesic in nature.

6. CONCLUSION:

We have obtained a cream that the viscous fluid distribution yields the same solution as that of the perfect fluid distribution for the same metric without imposing any condition or restriction. The universe obtained in equation (20) is the achievement of general relativity theory in the field of celestical mechanics which corresponds to Newton's treatment of classical gravitational theory.

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