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FORCED CONVECTION IN POROUS MEDIA

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ABSTRACT

An analytical approach for the effect of the transverse thermal dispersion on fully developed in a parallel plate channel filled with isotropic fluid saturated porous medium has been studied. The influence of various participating entities has been illustrated on the flow entities. It is observed that increase in the Darcy's number contributes to increase in the velocity field. Further, it is also noted that increase in γ contributes to decrease in the velocity field. Further, it is seen that as the pore size increases the velocity is found to be increasing. Also, as γ increases the velocity is found to be decreasing significantly and it is seen that as the Darcy's number increases the temperature is found to be increasing.

***Key words:** Thermal dispersion, porous media, Thermal conductivity and Forced convection.*

NOMENCLATURE:

k_f : Thermal conductivity of the fluid phase
k_s : Thermal conductivity of the solid Phase
$\Pr$: Prandtl number
μ_f : Dynamic viscosity of the fluid
μ_{eff} : Effective dynamic viscosity in Brinkman term
γ^2 : Ratio of μ_{eff} and μ_f
H : Half the distance between the plates of the channel
K : Permeability of the porous mediu
Da : Darcy number < 10^{-1}
C : Dimensionless experimental constant
d_p : Average diameter of the porous Particle
U : Mean velocity
G : Applied pressure gradient
c_f : Specific heat of the fluid
ε : Porosity
ρ_f : Density of the fluid
k_m : Thermal conductivity of the porous Matrix
Nu : Nusselt number

INTRODUCTION

Significant filtration in molecular diffusion must be taken into account for an additional heat transfer which appears due to hydrodynamic mixing of fluids at pore scale when forced convection in porous media is considered. Such an effect is generally termed as thermal diffusion, which results in apparent increase of thermal conductivity of the porous

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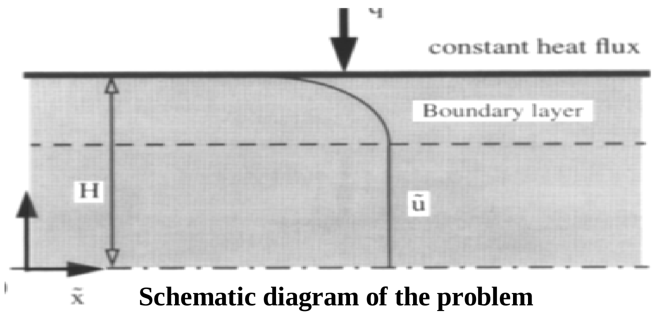
medium. There has been considerable interest in modeling the effect of thermal dispersion in porous media. Several such applications has been recorded by Plumb [1] Plumb and Whitaker [2], [3] Cheng [4] and Nakayama [5], Nield and Bejan [6]. In case of a two dimensional flow in an isotropic, homogeneous porous medium, it is necessary to account for an apparent increase of thermal conductivity in longitudinal and transverse directions. In porous media, generally longitudinal heat conduction was neglected by Cheng *et al* [7], Nakayama *et al*. [8], Vafi and Kim [9]. Such an assumption can be strictly justified for large Peclet numbers [10], because in this case $\frac{\partial^2 T}{\partial x^2}$ is negligible when compared with $\frac{\partial^2 T}{\partial y^2}$, where T is a temperature, x is stream wise coordinate and y is the transverse coordinate. When the Peclet numbers are very large the influence of longitudinal dispersion is found to be very negligible due to the reason that it is multiplied by a negligible term. Under the assumptions of Plumb [1], it is assumed that the thermal dispersion in longitudinal direction is negligible when compared to a thermal dispersion in transverse direction. By assuming that the effective thermal conductivity consisting of stagnet and dispersion conductivity, a detailed analysis was made by Amiri and Vafi [11], [12] Proceeding from the results obtained by them the effective thermal conductivity in the transverse direction can be expressed as

$$k_{eff} = k_m + C k_f \Pr \frac{\varepsilon f \tilde{u}_f d_p}{\mu_f} \tag{1}$$



Schematic diagram of the problem

Where the stagnant thermal conductivity of the porous matrix and k_m, is given by the following equation:

$$k_m = \varepsilon k_f + (1-\varepsilon) k_s \tag{2}$$

The equations of motion are presented in the dimensional form as

$$-\frac{d\tilde{p}}{d\tilde{x}} + \mu_{eff} \frac{d^2 \tilde{u}_f}{d\tilde{y}^2} - \frac{\mu_f}{K}\tilde{u}_f - \frac{\rho_f c_F}{K^{1/2}} \tilde{u}_f^{\,2} = 0, \quad 0 \le \tilde{y} \le H \tag{3}$$

$$\rho_f c_f \tilde{u}_f \frac{\partial \tilde{T}}{\partial \tilde{x}} = \frac{\partial}{\partial y}\left[\left(k_m + C k_f \Pr \frac{\rho_f \tilde{u}_f d_p}{\mu_f}\right)\frac{\partial \tilde{T}}{\partial \tilde{y}}\right], \quad 0 \le y \le H \tag{4}$$

Where c_f is the specific heat of the fluid in space c_F is the Forchheimer coefficient, H is half the distance between the plates of the channel, K is the permeability of the porous medium, $\tilde{p}$ is the pressure, and μ_{eff} is the effective dynamic viscosity in the Brinkman, term. Equation (3) is the momentum equation for the flow in the porous medium which accounts for both Brinkman and Forchheimer extensions of the Darcy law. The second term in this equation $\mu_{eff} \frac{d^2 \tilde{u}_f}{d\tilde{y}^2}$ is the Brinkman (viscous) term which makes it possible to impose the no - slip boundary conditions at the walls of the channel. The fourth term in equation (3), $-\frac{\rho_f c_F}{K^{1/2}} \tilde{u}_f^{\,2}$, is the Forchheimer (quadratic drag) term which makes it possible to account for deviation from linearity, which generally occurs for large filtration velocities [6].

Equation (4) is the energy equation. Equations (3) and (4) must be solved subject to the following boundary conditions:

$$\frac{\partial \tilde{u}_f}{\partial \tilde{y}} = 0, \quad \frac{\partial \tilde{T}}{\partial \tilde{y}} = 0 \quad \text{at } \tilde{y} = 0, \tag{5}$$

$$\tilde{u}_f = 0, \left(k_m + C k_f \Pr \frac{\rho_f \tilde{u}_f d_p}{\tilde{u}_f} \right) \frac{\partial \tilde{T}}{\partial \tilde{y}} = q'' \quad \text{at} \quad \tilde{y} = H, \tag{6}$$

Where q'' is the wall heat flux.

The region of the channel is assumed to be fully developed and is laminar and the walls of the channel are subject to a uniform heat flux, $\frac{\partial \tilde{T}}{\partial \tilde{x}}$ on the left – hand side of equation (4) must be constant (Bejan [9]). The value of $\frac{\partial \tilde{T}}{\partial \tilde{x}}$ can then be found from the following energy balance:

$$\rho_f c_f H \tilde{U} \frac{\partial \tilde{T}}{\partial \tilde{x}} = q'', \tag{7}$$

Where $\tilde{U}$ is the mean flow velocity

After introducing dimensionless variables, equations (3) and (4) can be presented as

$$1 + \gamma^2 \frac{d^2 u}{dy^2} - \frac{1}{Da} u = 0, \quad 0 \le y \le 1, \tag{8}$$

$$\frac{d}{dy}\left[\left(\frac{k_m}{k_f} + C \Pr \mathrm{Re}_p u \right) \frac{dT}{dy} \right] = -\frac{1}{2} Nu \frac{u}{U}, \quad 0 \le y \le 1, \tag{9}$$

$$\left.\begin{aligned} &\text{Where } u = \frac{\tilde{u}_f \mu_f}{G H^2}, \; U = \frac{\tilde{U} \mu_f}{G H^2}, \\ &y = \frac{\tilde{y}}{H}, \quad \gamma = \left(\frac{\mu_e ff}{\mu_f} \right)^{1/2} \end{aligned}\right\} \tag{10}$$

$$\left.\begin{aligned} &Da = \frac{K}{H^2}, \; T = \frac{\tilde{T} - T_w}{\tilde{T}_m - \tilde{T}_w}, \\ &\mathrm{Re}_p = \left(\frac{G H^2}{\mu_f} \right) \frac{d_p}{\nu_f} \end{aligned}\right\} \tag{11}$$

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In equations (10) and (11), $\frac{GH^2}{\mu_f}$ is the velocity scale, $G = -\frac{d\tilde{p}}{d\tilde{x}}$ is the applied pressure gradient, and ν_f is kinematic viscosity of the fluid phase.

Equations (8) and (9) must be solved subject to the following non dimensional boundary conditions:

$$\frac{du}{dy}=0, \frac{dT}{dy}=0, \text{ at } y=0, \tag{12}$$

$$u=0, \; T=0 \quad \text{at} \quad y=1. \tag{13}$$

SOLUTION OF THE PROBLEM:

The velocity distribution in the channel is obtained as follows:

$$u = Da\left(1-\frac{\cosh By}{\cosh B}\right) \tag{14}$$

Where $B^2 = \dfrac{1}{\gamma^2 Da}$

The temperature distribution can be obtained by integrating the energy equation (9) along with the boundary conditions given by equations (12) and (13)

$$T = \frac{Nu\left(1-y^2\right)}{4U\left(\frac{k_m}{k_f}+C \Pr \mathrm{Re}_p u\right)} \tag{15}$$

RESULTS AND CONCLUSIONS:

1. The influence of Darcy's numbers on the velocity profiles for a constant Prandtl number and for different values of γ (the ratio of μ_{eff} and μ_f) has been illustrated in fig - 1, fig - 2, fig - 3 and fig - 4. In each of these situations, it is observed that increase in the Darcy's number contributes to increase in the velocity field. Further, it is also noted that increase in γ contributes to decrease in the velocity field. It is seen that as γ increases, the fluid velocity decreases. However might be the pore size of the medium, the velocity profiles continues to maintain their parabolic nature.

2. Fig – 5, fig – 6, fig - 7 and fig - 8 illustrates the influence of Darcy's number on the velocity profiles, for Pr = 0.7 and different γ values. In each of these illustrations it is seen that as the pore size increases the velocity is found to be increasing. Further from fig – 5, fig – 6 and fig – 9 it is seen that the γ influences velocity profiles quiet significantly. Further as γ increases the velocity is found to be decreasing significantly. Similar such trend is seen in fig - 7 and fig - 8.

3. Fig - 10, fig - 11, fig - 12 and fig - 13 shows the influence of the Darcy's number on temperature profiles when Pr = 0.7 and Nu = 5, for different values of γ. In each of these situations it is seen that as the Darcy's number increases the temperature is found to be increasing.

4. On comparing fig - 10 and fig - 13 it is observed that while the nature of profiles remaining same the temperature is found to be decreasing. However, such a decrease is found to be very minute and is noted to be increasing in the value of γ. Further similar such trend is seen in fig – 11, fig – 14 and fig – 15.

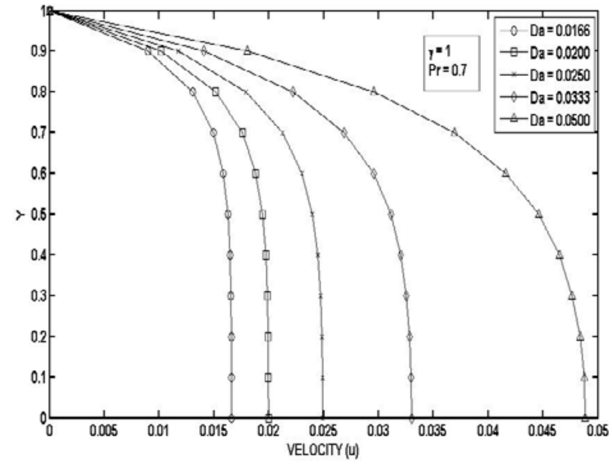

Fig.1: Effect of Darcy's number On Velocity

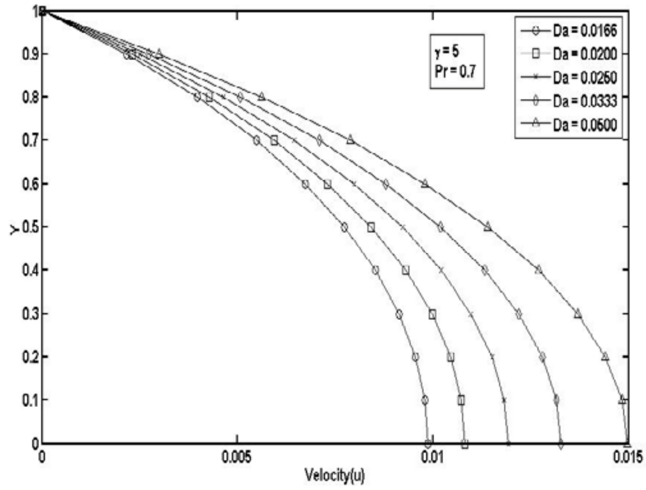

Fig.2: Effect of Darcy's number on Velocity

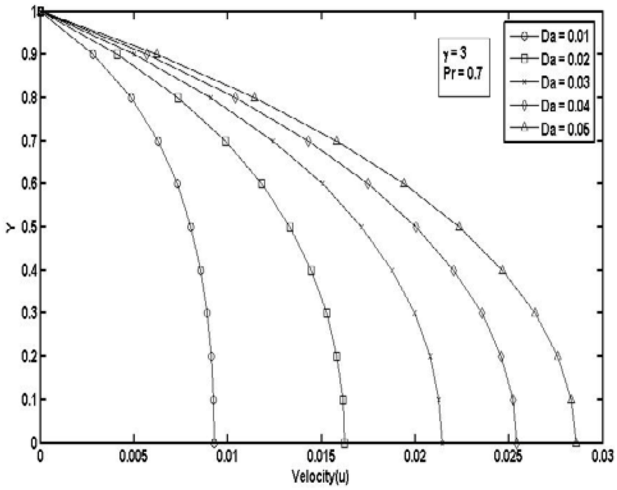

Fig.3: Effect of Darcy's number on Velocity

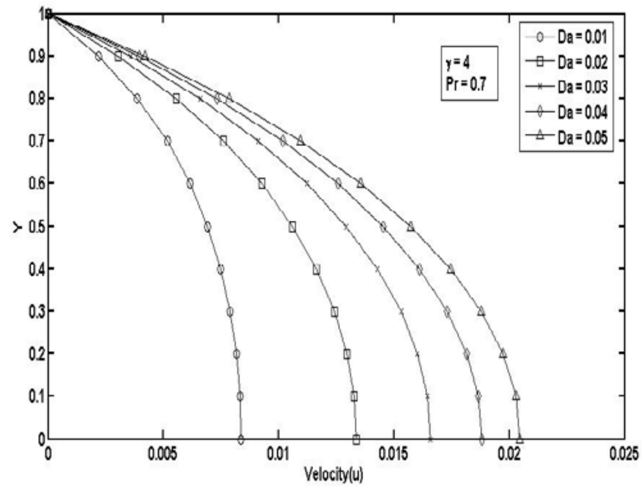

Fig.4: Effect of Darcy's number on Velocity

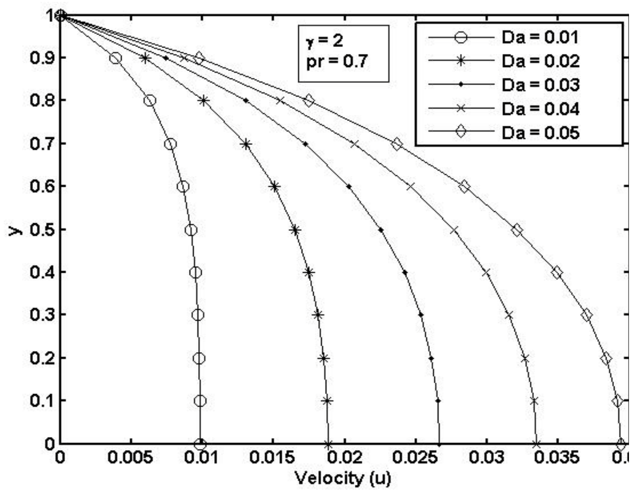

Fig – 5: Effect of Darcy number on Velocity

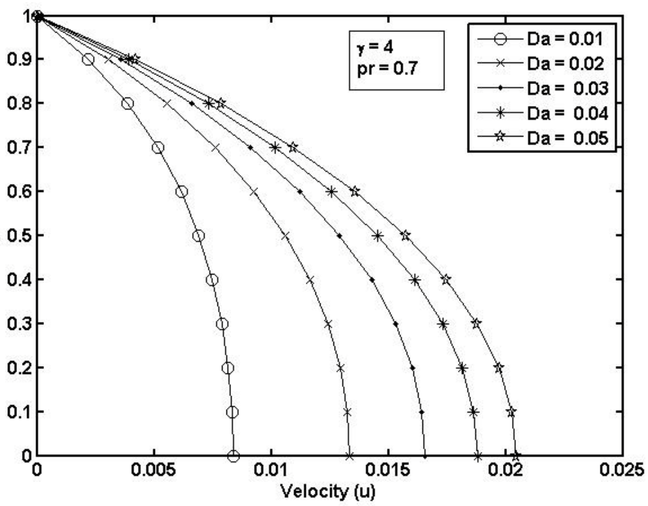

Fig – 6: Effect of Darcy number on velocity

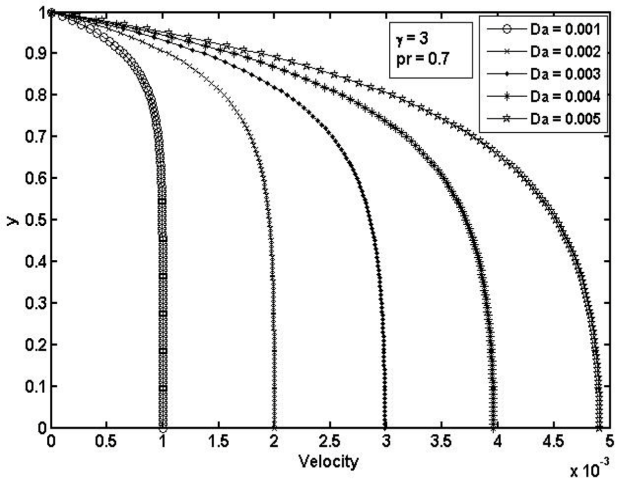

Fig – 7: Effect of Darcy number on velocity

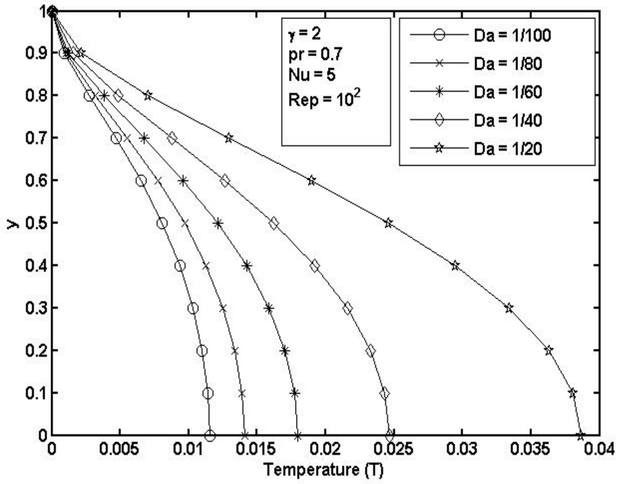

Fig – 10: Effect of Darcy number on Temperature

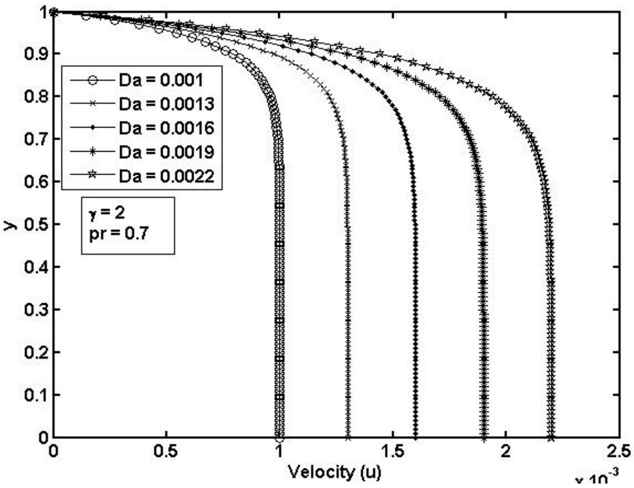

Fig – 8: Effect of Darcy number on velocity

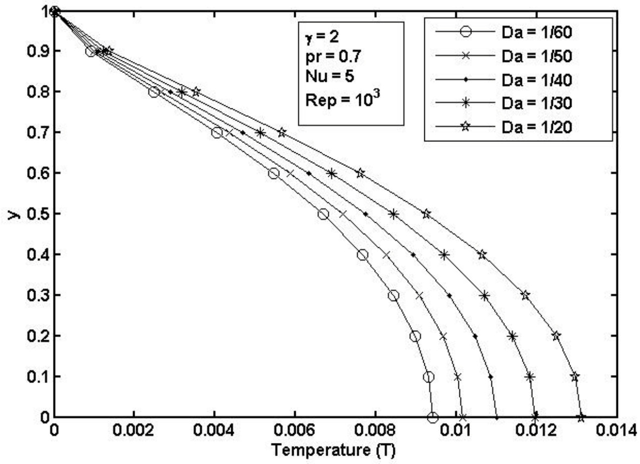

Fig – 11: Effect of Darcy number on Temperature

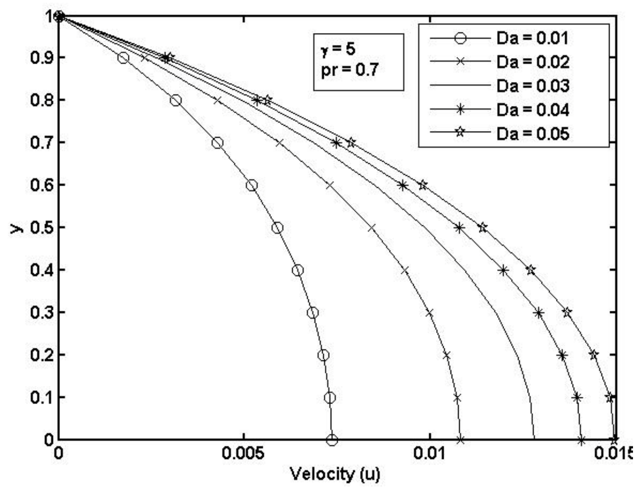

Fig – 9: Effect of Darcy number on velocity

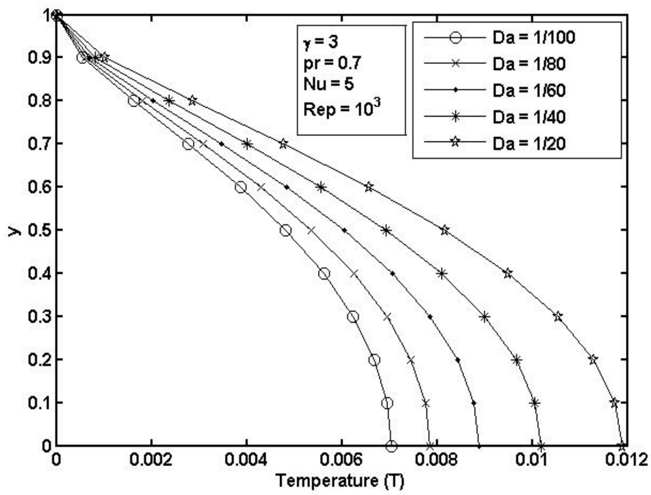

Fig – 12: Effect of Darcy number on Temperature

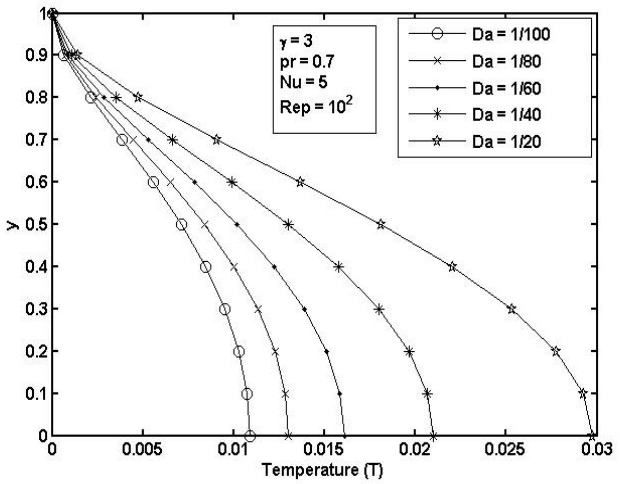

Fig – 13: Effect of Darcy number on Temperature

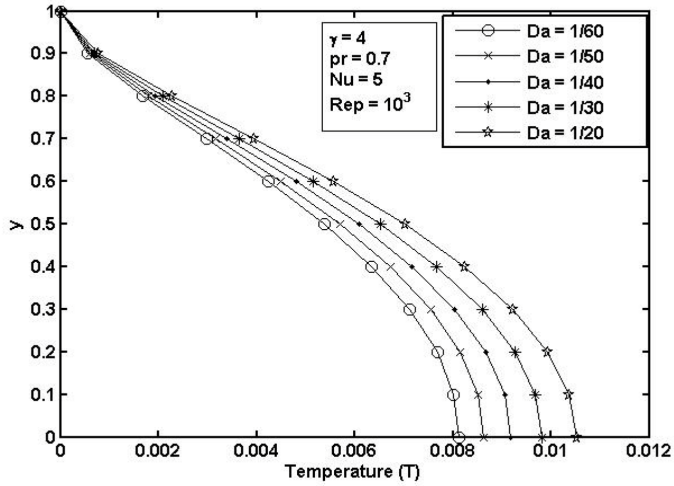

Fig – 14: Effect of Darcy number on Temperature

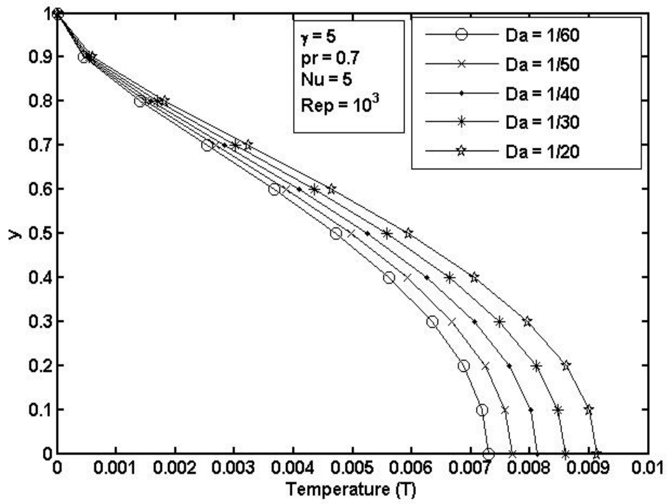

Fig - 15 Effect of Darcy number on Temperature

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