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THERMAL RADIATION EFFECTS ON ROTATING FLUID PAST A VERTICAL PLATE WITH VARIABLE MASS DIFFUSION

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ABSTRACT

Theoretical study of thermal radiation effects on unsteady free convection and variable mass diffusion over a moving vertical plate in a rotating fluid is considered. An exact solution is obtained for the axial and transverse components of the velocity by defining a complex velocity. The effects of velocity, temperature and concentration for different parameters like radiation parameter, rotation parameter, Schmidt number, thermal Grashof number, mass Grashof number, Prandtl number and time on the plate are discussed.

Key words: *Radiation, gray, rotation, vertical plate, heat and mass transfer.*

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1. INTRODUCTION

The effect of coriolis force has wide applications in science and technology. The effect of radiation is quite significant at high temperature. Radiative heat and mass transfer play an important role in manufacturing industries for the design of reliable equipment. Developments in hypersonic flight, missile re-entry, combustion chambers in racket, power plants for interplanetary flight and gas cooled nuclear reactors have focused attention on thermal radiation as a mode of energy transfer.

Arpaci [2] studied the interaction between thermal radiation and laminar convection of heated vertical plate in a stagnant radiating gas. Bestman and Adjepong [3] studied the magnetohydrodynamic free convection flow, with radiative heat transfer, past an infinite moving plate in rotating incompressible, viscous and optically transparent medium. Das et al [4] have analyzed radiation effects on flow past an impulsively started infinite isothermal vertical plate. England and Emery [5] have studied the thermal radiation effects of an optically thin gray gas bounded by a stationary vertical plate. Muthucumaraswamy and Ganeshan [6] have considered radiation effects on flow past an impulsively started infinite vertical plate with variable temperature. Naroua et al. [7] considered the radiative free convective flow of an optically thin gray gas past a semi-infinite vertical plate. Raptis and Perdikis [8] considered the effects of thermal radiation and free convection flow past a moving vertical plate. . Again, Raptis and Perdikis [9] investigated free convection and mass transfer effects on optically thin gray gas past an infinite moving vertical plate.
Singh [10] studied the effects of coriolis as well as magnetic force on the flow field of an electrically conducting fluid past an impulsively started infinite vertical plate. Soundalgakar and Takhar [11] considered the radiative free convictive flow of an optically thin gray gas past a semi-infinite vertical plate. The governing equations were solved analytically.

However, heat and mass transfer effects on a moving infinite vertical plate in a rotating fluid in the presence of thermal radiation is not studied in the literature. It is proposed to study thermal radiation effects on flow past an impulsively started infinite isothermal vertical plate with variable mass diffusion, in a rotating fluid. The dimensionless governing equations are solved by Laplace transform technique.\

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2. BASIC EQUATIONS

Consider the three dimensional flow of a viscous incompressible fluid past an impulsively started infinite vertical isothermal plate with variable mass diffusion in a rotating fluid singh [10], Bestman and Adjepong[3]. On this plate, the x′-axis is taken along the plate in the vertically upward direction and the y′-axis is taken normal to x′-axis in the plane of the plate and z′-axis is normal to it. Both the fluid and the plate are in a state of rigid rotation with uniform angular velocity Ω' about the z′-axis. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. Initially, the plate and fluid are at rest with the temperature T'_∞ and concentration C'_∞ every where. At time $t' > 0,$ the plate is given an impulsive motion in the vertical direction against gravitational field with constant velocity u_0 in a fluid, in the presence of thermal radiation. At the same time the plate temperature is raised to T'_w and the concentration to $C'_w,$ which are there after maintained constant. Since the plate occupying the plane z′ = 0 is of infinite extent, all the physical quantities depend only on z′ and t'. Then by usual Boussinesq's approximation, the unsteady flow is governed by the following equations:

$$\frac{\partial u'}{\partial t'} - 2\Omega' v' = g\beta(T' - T'_\infty) + g\beta^*(C^{'} - C^{'}{}_{\infty}) + \nu\frac{\partial^2 u'}{\partial z'^2} \tag{2.1}$$

$$\frac{\partial v'}{\partial t'} - 2\Omega' u' = \nu\frac{\partial^2 v'}{\partial z'^2} \tag{2.2}$$

$$\rho C_p \frac{\partial T'}{\partial t'} = k\frac{\partial^2 T'}{\partial z'^2} - \frac{\partial q_r}{\partial z'} \tag{2.3}$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial z'^2} \tag{2.4}$$

The term $\frac{\partial q_r}{\partial z'}$ represents the change in the radiative flux with distance normal to the plate with the following initial and boundary conditions:

$$\begin{array}{lllll} \boldsymbol{t'} \le 0: & \boldsymbol{u'} = 0, & \boldsymbol{T'} = \boldsymbol{T'_\infty}, & \boldsymbol{C'} = \boldsymbol{C'_\infty} & \text{for all } \boldsymbol{z'} \\ \boldsymbol{t'} > 0: & \boldsymbol{u'} = \boldsymbol{u}_0, & \boldsymbol{T'} = \boldsymbol{T'_w}, & \boldsymbol{C'} = \boldsymbol{C'_w} + (\boldsymbol{C}_w{}' - \boldsymbol{C}_\infty{}')\boldsymbol{A}\boldsymbol{t} & \text{at} \quad \boldsymbol{z'} = 0 \\ & \boldsymbol{u} = 0, & \boldsymbol{T} \to \boldsymbol{T}_\infty, & \boldsymbol{C'} \to \boldsymbol{C'_\infty} & \text{as} \quad \boldsymbol{z'} \to \infty. \end{array} \tag{2.5}$$

where $\boldsymbol{A} = \frac{\boldsymbol{u}_0^2}{\nu}$. By Rosseland approximation (Raptis and Perdikis (1999)) , radiative heat flux of an optically thin gray gas is expressed by

$$\frac{\partial q_r}{\partial z'} = -4a^*\sigma(T'^4_\infty - T'^4) \tag{2.6}$$

It is assume that the temperature differences within the flow are sufficiently small such that T'^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T'^4 in a Taylor series about T'_∞ and neglecting higher-order terms, thus

$$T'^4 \cong 4T'^3_\infty T' - 3T'^4_\infty \tag{2.7}$$

By using equations (2.6) and (2.7), equation (2.3) reduces to

$$\rho C_p \frac{\partial T'}{\partial t'} = k\frac{\partial^2 T'}{\partial y'^2} + 16a^*\sigma\, T'^3_\infty (T'_\infty - T') \tag{2.8}$$

On introducing the following dimensionless quantities:

$$(u,v)=\frac{(u',v')}{u_0},\quad t=\frac{t'u_0'^2}{\nu},\quad z=\frac{z'u_0}{\nu},\quad \theta=\frac{T'-T'_\infty}{T'_w-T'_\infty},$$
$$Gr=\frac{g\beta\nu(T'_w-T'_\infty)}{u_0'^3},\quad C=\frac{C'-C'_\infty}{C'_w-C'_\infty},\quad Gc=\frac{\nu g\beta^*(C'_w-C'_\infty)}{u_0^3}, \tag{2.9}$$
$$Pr=\frac{\mu C_p}{k},\quad \Omega=\frac{\Omega'\nu}{u_0'^2},\quad R=\frac{16a^*\nu^2\sigma T_\infty'^3}{ku_0'^2}.$$

and the complex velocity q = u + iv, $i=\sqrt{-1}$ in equations (2.1) to (2.5), the equations relevant to the problem reduces to

$$\frac{\partial q}{\partial t}+2i\Omega=Gr\theta+GcC+\frac{\partial^2 q}{\partial z^2}, \tag{2.10}$$
$$\frac{\partial\theta}{\partial t}=\frac{1}{Pr}\frac{\partial^2\theta}{\partial z^2}-\frac{R}{Pr}\theta \tag{2.11}$$
$$\frac{\partial C}{\partial t}=\frac{1}{Sc}\frac{\partial^2 C}{\partial z^2} \tag{2.12}$$

The initial and boundary conditions in non-dimensional form are

$$\begin{array}{lllll} & \boldsymbol{q}=0, & \theta=0, & \boldsymbol{C}=0, & \text{for all}\quad \boldsymbol{z}\le 0\ \&\ \boldsymbol{t}\le 0\\ \boldsymbol{t}>0: & \boldsymbol{q}=1, & \theta=1, & \boldsymbol{C}=\boldsymbol{t}, & \boldsymbol{at}\quad \boldsymbol{z}=0\\ & \boldsymbol{q}=0, & \theta\to 0, & \boldsymbol{C}\to 0, & \boldsymbol{as}\quad \boldsymbol{z}\to\infty. \end{array} \tag{2.13}$$

All the physical variables are defined in the nomenclature. The solutions are obtained for the equations (2.10) to (2.12), subject to the boundary conditions (2.13), by Laplace-transform technique and the solutions are derived as follows:

$$\theta=\frac{1}{2}\left[\exp(2\eta\sqrt{R})erfc\left(\eta\sqrt{\Pr}+\sqrt{at}\right)+\exp(-2\eta\sqrt{R})erfc\left(\eta\sqrt{\Pr}-\sqrt{at}\right)\right] \tag{2.14}$$

$$\text{C}=\text{t}\left[(1+2\eta^2\text{Sc})\ \text{erfc}(\eta\sqrt{\text{Sc}})-\frac{2}{\sqrt{\pi}}\eta\sqrt{\text{Sc}}\ \exp(-\eta^2\text{Sc})\right] \tag{2.15}$$

$$q=\frac{1}{2}\left(1+\frac{Gr}{b(1-Pr)}+\frac{Gc}{c^2(1-Sc)}+\frac{Gc\,t}{c(1-Sc)}\right)\left[\exp(-2\eta\sqrt{mt})\ erfc(\eta-\sqrt{mt})+\exp(2\eta\sqrt{mt})\ erfc(\eta+\sqrt{mt})\right]$$
$$-\frac{Gr\exp(bt)}{2b(1-Pr)}\left[\exp(-2\eta\sqrt{(b+m)}t\ erfc(\eta-\sqrt{(b+m)t}\ )+\exp(2\eta\sqrt{(b+m)t}\ )\ erfc(\eta+\sqrt{(b+m)t}\ )\right]$$
$$-\frac{Gc\ \ \eta}{2c(1-Sc)}\sqrt{\frac{t}{m}}\left[\exp(-2\eta\sqrt{mt})\ erfc(\eta-\sqrt{mt})-\exp(2\eta\sqrt{mt})\ erfc(\eta+\sqrt{mt})\right]$$
$$-\frac{Gc\exp(ct)}{2c^2(1-Sc)}\left[\exp(-2\eta\sqrt{(c+m)t}\ )erfc(\eta-\sqrt{(c+m)t}\ )+\exp(2\eta\sqrt{(c+m)t}\ )\ erfc(\eta+\sqrt{(c+m)t}\ )\right]$$
$$+\frac{Gr\exp(bt)}{2b(1-Pr)}\left[\begin{array}{l}\exp(-2\eta\sqrt{Pr(b+a)t})erfc(\eta\sqrt{Pr}\ -\sqrt{(a+b)t}\ )\\ +\exp(2\eta\sqrt{Pr(b+a)t}\ )\ erfc(\eta\sqrt{Pr}\ +\sqrt{(a+b)t}\ )\end{array}\right]$$

$$-\frac{Gr}{2b(1-Pr)}\left[\exp(-2\eta\sqrt{Rt})\ erfc(\eta\sqrt{Pr}-\sqrt{at}\,)+\exp(2\eta\sqrt{Rt})\ erfc(\eta\sqrt{Pr}+\sqrt{at}\,)\right]-\frac{Gc}{c^2(1-Sc)}erfc\left(\eta\sqrt{Sc}\right)$$
$$-\frac{Gc\,t}{c(1-Sc)}\left[(1+2\eta^2\mathrm{Sc})\ \mathrm{erfc}(\eta\sqrt{\mathrm{Sc}}\,)-\frac{2}{\sqrt{\pi}}\eta\sqrt{\mathrm{Sc}}\ \exp(-\eta^2\mathrm{Sc})\right]$$
$$+\frac{Gc\exp(ct)}{2c^2(1-Sc)}\left[\exp(-2\eta\sqrt{Sc\,ct})\ erfc(\eta\sqrt{Sc}-\sqrt{ct}\,)+\exp(2\eta\sqrt{Sc\,ct})\ erfc(\eta\sqrt{Sc}+\sqrt{ct}\,)\right] \qquad (2.16)$$

Where $a=\dfrac{R}{Pr},\quad b=\dfrac{R-m}{1-Pr}\;,\; \eta=\dfrac{z}{2\sqrt{t}}$ and $m=2i\Omega.$

In equation (2.16), the argument of the complementary error function and error function is complex. Hence in order to obtain the u and v components of the velocity, we have used the following formula due to Abramowitz and stegun [1]:

$$erf(a+ib)=erf(a)+\frac{\exp(-a^2)}{2a\pi}\left[1-\cos(2ab)+i\sin(2ab)\right]$$
$$+\frac{2\exp(-a^2)}{\pi}\sum\frac{\exp(-n^2/4)}{n^2+4a^2}[\,f_n(a,b)+i\,g_n(a,b)]+\varepsilon(a,b)$$

Where

$$f_n=2a-2a\cosh(nb)\cos(2ab)+n\sinh(nb)\sin(2ab)$$

$$g_n=2a\cosh(nb)\sin(2ab)+n\sinh(nb)\cos(2ab)$$

$$\left|\varepsilon(a,b)\right|\approx 10^{-16}\left|erf(a+ib)\right|.$$

3. DISCUSSION OF RESULTS

Using the above formula, expressions for u, v are obtained but they are omitted here to save the space. In order to get a physical view of the problem, these expressions are used to obtain the numerical values of u, v, for different values of the various parameter like rotation, radiation, Schmidt number, thermal Grashof number and mass Grashof number.

The temperature profiles for air (Pr = 0.71) are calculated for different values of thermal radiation parameter from Equation (2.14) and these are shown in Fig. 1. The effect of thermal radiation parameter is important in temperature profiles. It is observed that the temperature increases with decreasing radiation parameter as well as the time.

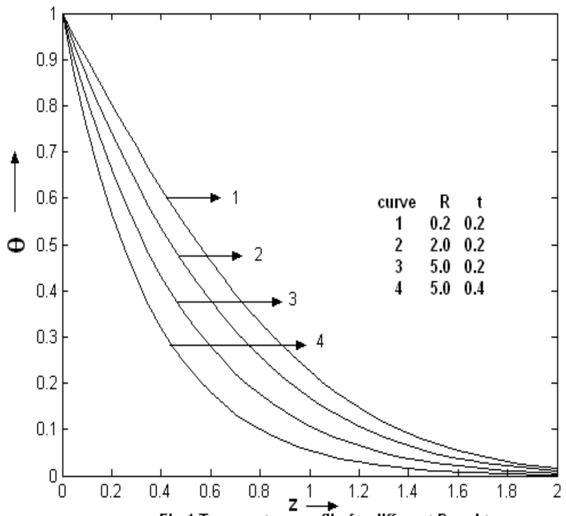

Fig.1.Temperature profile for different R and t

Fig. 2, depicts the concentration profiles for different values of the Schmidt number (Sc = 0.16, 0.24, 0.6, 0.78, 2.01) at time t = 0.2. The wall concentration increases with decreasing Schmidt number. It is observed that there is a fall in concentration due to increasing the values of the Schmidt number.

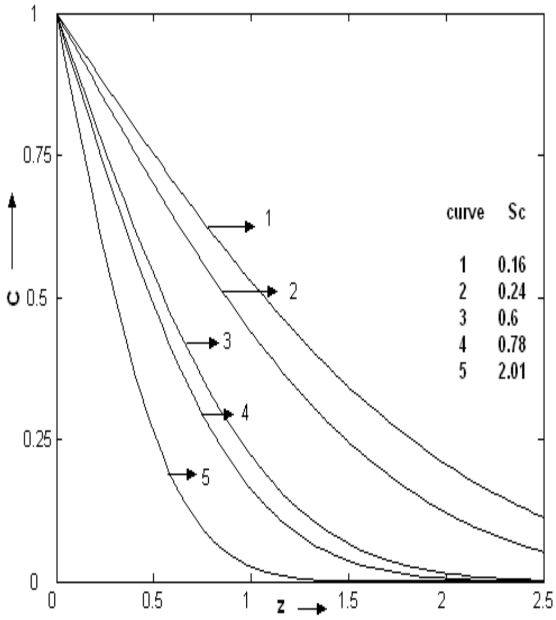

Fig.2. Concentration profiles for different Sc

The primary velocity profiles of air for different values of the radiation parameter (R = 0.2, 25.0), Gr = 5, Gc = 5, Sc = 0.6, t = 0.2, Pr = 0.71 and rotation parameter (Ω = 0.5, 2, 3) are shown in Fig. 3. It is observed that the primary velocity increases with decreasing radiation parameter R as well as the rotation parameter Ω in cooling of the plate. This shows that primary velocity decreases in the presence of high thermal radiation and rotation.

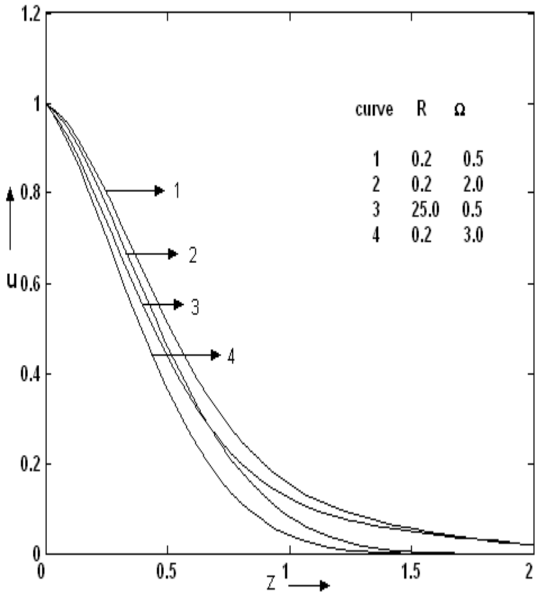

Fig.3.Primary velocity profiles for Different R and Ω

The secondary velocity profiles of air for different values of the radiation parameter (R = 0.2, 25.0), Gr = 5, Gc = 5, Sc = 0.6, t = 0.2, Pr = 0.71 and rotation parameter (Ω = 0.5, 2, 3) are shown in Fig. 4, the effect of radiation increases the secondary velocity v. But the effect of rotation on v is just reverse to that of radiation parameter.

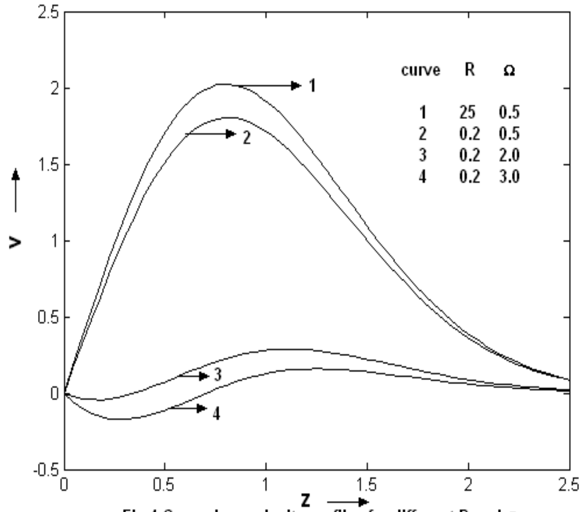

Fig.4 Secondary velocity profiles for different R and Ω

The primary velocity profiles for different thermal Grashof number (Gr = 2, 5), mass Grashof number (Gc = 2, 10), Sc = 0.6 and time t = 0.2 are shown in Fig. 5. It is clear that the primary velocity increases with increasing thermal Grashof number and decreases with mass Grashof number near the plate and the trend changes faraway from the plate.

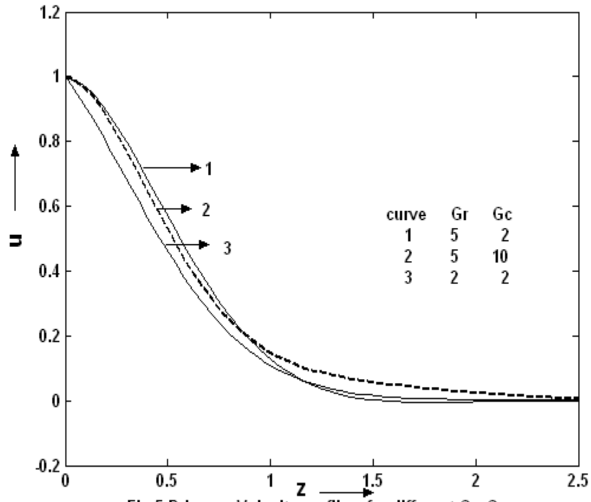

Fig.5.Primary Velocity profiles for different Gr, Gc

The secondary velocity profiles for different thermal Grashof number (Gr = 2, 5), mass Grashof number (Gc = 2, 10), Sc = 0.6 and time t = 0.2 are shown in Fig. 6. It shows the reverse trend for both thermal Grashof number and mass Grashof number.

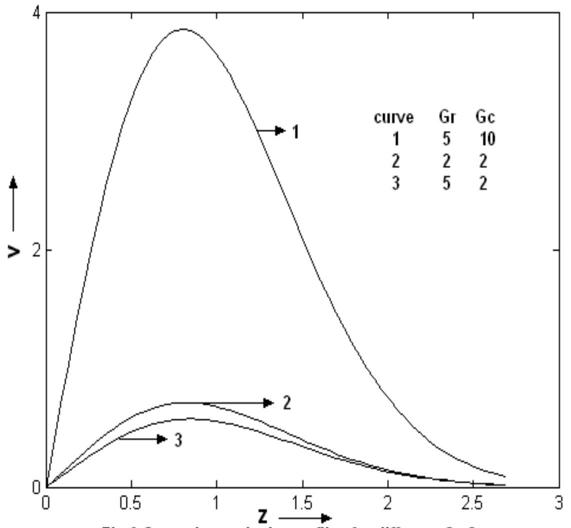

Fig.6.Secondary velocity profiles for different Gr,Gc

4. CONCLUSIONS

Theoretical analysis is performed to study flow past an impulsively started infinite isothermal vertical plate with variable mass diffusion, in the presence of thermal radiation in a rotating fluid. The dimensionless governing equations are solved by Laplace-transform technique. The conclusions of the study are as follows:

Temperature is enhanced with the decreasing radiation parameter as well as the time.

Concentration falls with the raise in Schmidt number.

The influence of the radiation or rotation parameter on primary flow has a retarding effect for cooling of the plate.

The secondary velocity is enhanced with the raise in thermal radiation and opposite phenomenon occurs with the rotation parameter.

Greater cooling enhances primary velocity and lowers the secondary velocity. But the trend is reversed with mass Grashof number.

Nomenclature

a^*	absorption coefficient
C'	concentration
C	dimensionless concentration
C_p	specific heat at constant pressure
D	mass diffusion coefficient
g	acceleration due to gravity
Gr	thermal Grashof number
Gc	mass Grashof number
k	thermal conductivity of the fluid
Pr	Prandtl number
q_r	radiative heat flux in the y-direction
R	radiation parameter
Sc	Schmidt number
T'_∞	temperature of the fluid far away from the plate
T'_w	temperature of the plate
T'	temperature of the fluid near the plate
t'	time
t	dimensionless time
u'	velocity of the fluid in the x'-direction
u_0	velocity of the plate
u	dimensionless velocity
v'	velocity of the fluid in the y'-direction
v	dimensionless velocity
y'	coordinate axis normal to x'-axis
z'	coordinate axis normal to the plate
z	dimensionless coordinate axis normal to the plate

Greek Symbols

β	volumetric coefficient of thermal expansion
β^*	volumetric coefficient of expansion with concentration
μ	coefficient of viscosity
ν	kinematic viscosity
Ω'	rotation parameter
Ω	dimensionless rotation parameter
ρ	density
τ	dimensionless skin-friction
σ	Stefan-Boltzman constant
θ	dimensionless temperature
erfc	complementary error function

Subscripts

w	conditions on the wall
∞	free stream conditions

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