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CYLINDRICALLY SYMMETRIC INHOMOGENEOUS COSMIC STRINGS AND DOMAIN WALLS IN BIMETRIC RELATIVITY

S.D. Deo

Department of Mathematics N. S. Sc. And Arts College, Bhadrawati. Dist. - Chandrapur. (M. S.), India
Email: s_deo01@yahoo.in
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ABSTRACT

Cylindrically symmetric inhomogeneous universe is studied with thick domain walls and cosmic strings in Bimetric Theory of Relativity .It is observed that, in this theory, thick domain walls and cosmic strings do not exist. As we know that, at the early stage of evolution of the universe domain walls as well as cosmic strings do appear which lead to formation of galaxies .Thus, it may be said that the bimetric relativity does not help to describe the early era of the universe.

Key Words: *- cylindrically, inhomogeneous, domain walls, string, bimetric.*

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1. Introduction:

Now days, there has been a lot of interest in the study of large scale structure of the universe. The fact that the origin of structure in the universe is one of the cosmological mysteries .Phase transitions in the early universe can give rise to various forms of topological defects which are domain walls, cosmic strings, monopoles, textures etc. Kibble (1), Linde (2) and Mermin (3) have studied domain walls and cosmic strings in general relativity. And as we know that, the formation of galaxies is due to domain walls and cosmic strings (Hill et al 4, Vilenkin 5). Since today, ther is a considerable amount of work has been done on domain walls and cosmic strings.Vilenkin (6), Isper and Sikivie (7), Widraw (8), Goetz (9), Mukherji(10) and Wang (11) have investigated several aspects of domain walls in general relativity .

Rehaman et al (12), Rehaman (13) and Rehaman and Mukhareji (14) have studied plane symmetric thick domain walls in Lyra (15) Geometry.

Letelier (16) has studied spherical, plane and a particular case of cylindrical symmetry and given the general solutions to Einstein's field equations for cosmic cloud strings. After that in 1983 , he has solved Einstein's field equations for cloud of massive strings and obtained cosmological models in Bianchi - I and Kantowski – Sachs space – time (17).

Banerjee et al (18) have investigated Bianchi – I string cosmological models with and without a source free magnetic field in general relativity. Further, Krori et al (19) have studied Bianchi – II, VI_0, VIII and IX space – time with cosmic strings and obtained the exact solutions in Einstein's General Theory of Relativity.
In 1991, Nevin (20) has solved Einstein's field equations for spherically or static cylindrical symmetry with string dust source. H. Baysal et al (21) have studied strings cosmological models in cylindrically symmetric inhomogeneous universe in general relativity and various physical and geometrical properties of the model have been discussed. Recently , Deo (22,23,24) have studied four and higher dimensional static plane symmetric space time with cosmic strings in Bimetric Relativity and observed that the resulting space – time can be reduced to the conformal one. They (25) further studied the cosmic strings in cylindrically symmetric cosmological model in bimetric relativity and observed that the model does not exist in this theory. In 2003, Mahurpawar and Deo (26) have obtained the nil contribution of cosmic strings coupled with Maxwell's field in axially symmetric Bianchi – I model in bimetric relativity .In 2004, Deo (27) has obtained the nil contribution of cosmic strings to plane symmetric space – time in bimetric relativity.

Here, Cylindrically symmetric inhomogeneous universe is studied with thick domain walls and cosmic strings in Bimetric Theory of Relativity . And it is observed that , in this theory , thick domain walls and cosmic strings do not exist . Hence , vacuum solutions can be obtained .

2. Field equations of bimetric relativity:

Field equations of bimetric relativity formulated by Rosen N (28,29) are-

$$K_i^j = N_i^j - \tfrac{1}{2} N g_i^j = -8 \pi k T_i^j \tag{2.1}$$

Where, $$N_i^j = \tfrac{1}{2} \gamma^{\alpha\beta} [g^{hj} g_{hi|\alpha}]_{|\beta} \tag{2.2}$$

$$N = N_\alpha^\alpha,\ k = (g/\gamma)^{1/2},\ g = \det.(g_{ij}),\ \gamma = \det.(\gamma_{ij}) \tag{2.3}$$

And a vertical bar (|) denotes the covariant differentiation with respect to γ_{ij}.

T_i^j is the energy momentum tensor for matter fields.

Corresponding author: S.D. Deo
E-mail: s_deo01@yahoo.in

3. Cylindrically symmetric nhomogeneous cosmic strings Solutions:

Let us consider the Cylindrically Symmetric Inhomogeneous Cosmic model in the form

$$ds^2 = A^2\,(dx^2 - dt^2) + B^2\,dy^2 + C^2\,dz^2 \qquad (3.1)$$

Where A, B and C are the functions of x and t.
and the background metric corresponding to equation (3.1) is

$$d\sigma^2 = -\,dt^2 + dx^2 + dy^2 + dz^2 \qquad (3.2)$$

The energy momentum tensor T_i^j for cosmic cloud strings is given by

$$T_i^j = T^j_{i\,strings} = \rho\, v_i\, v^j - \lambda\, x_i x^j \qquad (3.3)$$

Here ρ is the rest energy density for a cloud with particle attached along the extension , thus $\rho = \rho_p + \lambda$, where ρ_p is the particle energy density , λ is the tension density of the strings and vi the flow vector of matter. The flow of the matter is taken orthogonal to the hyper – surface of homogeneity so that $v_4 v^4 = -1$ and x^i representing the direction vector of anisotropy.

i.e. x – axis => $x_1 x^1 = 1$ and $v_i x^i = 0$.

Using the equations (2.1) – (2.3) with (3.1) - (3.3) we have

$$\tfrac{1}{2}\left[\left(\frac{B''}{B}-\frac{B'^2}{B^2}\right)-\left(\frac{B^{--}}{B}-\frac{B^{-2}}{B^2}\right)+\left(\frac{C''}{C}-\frac{C'^2}{C^2}\right)-\left(\frac{C^{--}}{C}-\frac{C^{-2}}{C^2}\right)\right]$$

$$== 8\pi k\lambda \qquad (3.3)$$

$$\tfrac{1}{2}\left[\left(\frac{B''}{B}-\frac{B'^2}{B^2}\right)-\left(\frac{B^{--}}{B}-\frac{B^{-2}}{B^2}\right)-\left(\frac{C''}{C}-\frac{C'^2}{C^2}\right)+\left(\frac{C^{--}}{C}-\frac{C^{-2}}{C^2}\right)\right]$$
$$-\left(\frac{A''}{A}-\frac{A'^2}{A^2}\right)+\left(\frac{A^{--}}{A}-\frac{A^{-2}}{A^2}\right)=0 \qquad (3.4)$$

$$-\tfrac{1}{2}\left[\left(\frac{B''}{B}-\frac{B'^2}{B^2}\right)-\left(\frac{B^{--}}{B}-\frac{B^{-2}}{B'^2}\right)+\left(\frac{C''}{C}-\frac{C'^2}{C^2}\right)+\left(\frac{C^{--}}{C}-\frac{C^{-2}}{C^2}\right)\right]$$
$$-\left(\frac{A''}{A}-\frac{A'^2}{A^2}\right)+\left(\frac{A^{--}}{A}-\frac{A^{-2}}{A^2}\right)=0 \qquad (3.5)$$

$$\tfrac{1}{2}\left[\left(\frac{B''}{B}-\frac{B'^2}{B^2}\right)-\left(\frac{B^{--}}{B}-\frac{B^{-2}}{B^2}\right)+\left(\frac{C''}{C}-\frac{C'^2}{C^2}\right)-\left(\frac{C^{--}}{C}-\frac{C^{-2}}{C^2}\right)\right]$$

$$= -\,8\pi k\rho \qquad (3.6)$$

where

$A^{\cdot} = \partial A / \partial t$, $A^{\cdot\cdot} = \partial^2 A / \partial t^2$, $A' = \partial A / \partial x$, $A'' = \partial^2 A / \partial x^2$ etc.

In cylindrically symmetric inhomogeneous universe, the rotation ω^2 is identically zero, the expansion Θ , sheer scalar 6^2 , acceleration vector $v^{\cdot}_i$ and proper volume V^3 are found respectively to have the following expressions:

$$\Theta = v_i^i = A^{-1}\left(\frac{A^-}{A}+\frac{B^-}{B}+\frac{C^-}{C}\right) \qquad (3.7)$$

$$6^2 = \tfrac{1}{2}\,6_{ij}6^{ij} = \tfrac{1}{3}\Theta^2 - A^{-2}\left(\frac{A'B'}{AB}+\frac{A^-C^-}{AC}+\frac{B^-C^-}{BC}\right) \qquad (3.8)$$

Where $6_{ij} = \tfrac{1}{2}\,(v_{i;j} + v_{j;i}) - \tfrac{1}{3}\,\Theta\,(g_{ij} - v_i v_j)$ (3.9)

And $v^{\cdot}_i = v_{i\,;\,j}\, v^j = \left(\frac{A'}{A}, 0,0,0\right)$ (3.10)

Also $V^3 = \sqrt{(-g)} = A^2 BC$ (3.11)

Using the Einstein's equations of General Relativity we have –

$$G_{ij} = R_{ij} - \tfrac{1}{2} R\, g_{ij} = -8\pi k T_{ij} \qquad (3.12)$$

For the metric (3.1) with the equations (3.7)and (3.8) one may obtain the Raychoudhuri's equation as –

$$\Theta^{\cdot} = v^{\cdot i}_{\ ;\,i} - \tfrac{1}{3}\,\Theta^2 - 2\,6^2 - \tfrac{1}{2}\,\rho_p \qquad (3.13)$$

Where $R_{ij}\, v^i v^j = \tfrac{1}{2}\,\rho_p$ (3.14)

For the metric (3.1) , the Bianchi Identity $T^{ij}{}_{;\,j} = 0$ gives us –

$$\rho' - \left(\frac{A^-}{A}\right)\lambda + \left(\frac{A^-}{A}+\frac{B^-}{B}+\frac{C^-}{C}\right)\rho = 0 \qquad (3.15)$$

And $\lambda' - \left(\frac{A'}{A}\right)\rho + \left(\frac{A'}{A}+\frac{B'}{B}+\frac{C'}{C}\right)\lambda = 0$ (3.16)

Thus due to all the three strong, weak ,and dominant energy conditions , one may find $\rho \geq 0$ and $\rho_p \geq 0$, together with the fact that the sign of λ is unrestricted , it may be positive , negative or zero .

Now using the equations (3.3) and (3.6) we get –

$$\rho + \lambda = 0 \tag{3.17}$$

But in view of reality conditions $\rho > 0$, $\lambda > 0$ we have

$$\rho = 0 = \lambda \tag{3.18}$$

Thus, nil contribution of cosmic strings in cylindrically symmetric inhomogeneous universe in Bimetric Relativity is obtained .

4. Cylindrically symmetric inhomogeneous domain walls solutions:

The energy momentum tensor T_i^j for domain walls is given by

$$T_{ij} = \rho\,(g_{ij} + \omega_i\,\omega_j\,) + p\,\omega_i\,\omega_j \tag{4.1}$$

With $\omega_1\,\omega^1 = -1$

Were ρ is the energy density of the wall , p is the pressure in the direction normal to the plane of the wall and ω_i is a unit space like vector in the same direction .

Using the equations (2.1) – (2.3) with (3.1) ,(3.2) and (4.1) we have

$$\tfrac{1}{2}\left[\left(\frac{B''}{B}-\frac{B'^2}{B^2}\right)-\left(\frac{\ddot{B}}{B}-\frac{\dot{B}^2}{B^2}\right)+\left(\frac{C''}{C}-\frac{C'^2}{C^2}\right)-\left(\frac{\ddot{C}}{C}-\frac{\dot{C}^2}{C^2}\right)\right] = 8\pi k p \tag{4.2}$$

$$\tfrac{1}{2}\left[\left(\frac{B''}{B}-\frac{B'^2}{B^2}\right)-\left(\frac{\ddot{B}}{B}-\frac{\dot{B}^2}{B^2}\right)+\left(\frac{C''}{C}-\frac{C'^2}{C^2}\right)+\left(\frac{\ddot{C}}{C}-\frac{\dot{C}^2}{C^2}\right)\right] -\left(\frac{A''}{A}-\frac{A'^2}{A^2}\right)+\left(\frac{\ddot{A}}{A}-\frac{\dot{A}^2}{A^2}\right) = -8\pi k p \tag{4.3}$$

$$-\tfrac{1}{2}\left[\left(\frac{B''}{B}-\frac{B'^2}{B^2}\right)-\left(\frac{\ddot{B}}{B}-\frac{\dot{B}^2}{B'^2}\right)+\left(\frac{C''}{C}-\frac{C'^2}{C^2}\right)+\left(\frac{\ddot{C}}{C}-\frac{\dot{C}^2}{C^2}\right)\right] -\left(\frac{A''}{A}-\frac{A'^2}{A^2}\right)+\left(\frac{\ddot{A}}{A}-\frac{\dot{A}^2}{A^2}\right) = -8\pi k p \tag{4.4}$$

$$-\tfrac{1}{2}\left[\left(\frac{B''}{B}-\frac{B'^2}{B^2}\right)-\left(\frac{\ddot{B}}{B}-\frac{\dot{B}^2}{B^2}\right)+\left(\frac{C''}{C}-\frac{C'^2}{C^2}\right)-\left(\frac{\ddot{C}}{C}-\frac{\dot{C}^2}{C^2}\right)\right] = -8\pi k \rho \tag{4.5}$$

where $\dot{A} = \partial A / \partial t$, $\ddot{A} = \partial^2 A / \partial t^2$, $A' = \partial A / \partial x$, $A'' = \partial^2 A / \partial x^2$ etc.

Using equations (4.2) and (4.5) we get the same result as equation (3.17) i.e.

$$p + \rho = 0 \tag{4.6}$$

But in view of reality conditions $\rho > 0$, $p > 0$ we have

$$p = 0 = \rho \tag{4.7}$$

Thus, one can state that the nil contribution of domain walls in cylindrically symmetric inhomogeneous universe in Bimetric Relativity .

CONCLUSION:

Here we have shown that Cylindrically Symmetric Inhomogeneous Universe does not accommodate cosmic strings and domain walls in Rosen's (1973) bimetric theory of relativity. As we know that, at the early stage of evolution of the universe domain walls as well as cosmic strings do appear which lead to formation of galaxies .Thus, it may be said that the bimetric relativity does not help to describe the early era of the universe.

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