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n x 2 Specially Structured Flow Shop Scheduling With Transportation Time to Minimize the Rental Cost of Machines

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ABSTRACT

The objective of this paper is to develop a heuristic algorithm to minimize the utilization time and hence the rental cost of machines in n–jobs, 2–machines specially structured flow shop scheduling under specified rental policy in which the processing times of the machines are associated with their respective probabilities including the significant transportation time of the jobs. The processing time of the jobs are not completely random, but bear a well defined relationship to one another. A computer programme followed by a numerical illustration is given to justify the algorithm.

Keywords: *Rental Policy, Processing Time, Transportation Time, Utilization Time, Specially Structured Flow Shop Scheduling.*

1. INTRODUCTION:

In a general flowshop scheduling problem, n jobs are to be scheduled on m machines in order to optimize some measures of performance. All jobs have the same processing requirements so they need to be processed on all machines in the same order. Two-machine flowshop scheduling problem has been considered as a major subproblem due to its applications in real-life. There are cases when the processing time of jobs are not random but follow some well defined structural conditions. Further the transportation times (loading time, moving time and unloading time etc.) from one machine to other are not negligible and therefore must be included in the job processing. However, in some applications, transportation time have major impact on the performance measure considered for the scheduling problem so they need to be considered separately. One of the earliest results in flowshop scheduling theory is an algorithm given by Johnson's (1954) for scheduling jobs in a two machine flowshop to minimize the time at which all jobs are completed. Gupta, J.N.D (1975) gave an algorithm to find the optimal schedule for specially structured flowshop scheduling. Maggu & Das (1980) consider a two machine flowshop problem with transportation time of the jobs. Some of the noteworthy heuristic approaches are due to Ignall & Scharge (1965), Bagga (1969), Szware (1977), Singh, T.P (1985), Gupta, J.N.D (1988), Panwalker (1991), Narain & Bagga (1998), Chen & Lee (2001), Anup (2002), Singh, T.P. & Gupta, D. (2005), Chikhi (2008), Khodadadi (2008), Pandian & Rajendran (2010) and Gupta, D. & Sharma, S. (2011).

In this paper we addresses a specially structured flow shop scheduling model an alternative to the Johnson's (1954) algorithm to minimize the utilization time of the machines and hence their rental cost under specified rental policy in which the processing time are associated with probabilities which bear a well-defined relationship to one another including the transportation time. The proposed algorithm is more efficient as compared to Johnson's (1954) algorithm to minimize the utilization time of machines and hence their rental cost for specially structured flow shop scheduling.

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2. PRACTICAL SITUATION:

Many applied and experimental situations exist in our day-to-day working in factories and industrial production concerns etc. In many manufacturing companies different jobs are processed on various machines. These jobs are required to process in a machine shop A, B, C, ---- in a specified order. When the machines on which jobs are to be processed are planted at different places the transportation time (which include loading time, moving time and unloading time etc.) has a significant role in production concern. Various practical situations occur in real life when one has got the assignments but does not have one's own machine or does not have enough money or does not want to take risk of investing huge amount of money to purchase machine. Under such circumstances, the machine has to be taken on rent in order to complete the assignments. For example, In his starting career, we find a medical practitioner does not buy expensive machines say X-ray machine, the Ultra Sound Machine, Rotating Triple Head Single Positron Emission Computed Tomography Scanner, Patient Monitoring Equipment, and Laboratory Equipment etc., but instead takes on rent. Rental of medical equipment is an affordable and quick solution for hospitals, nursing homes, physicians, which are presently constrained by the availability of limited funds due to the recent global economic recession. Renting enables saving working capital, gives option for having the equipment, and allows upgradation to new technology.

3. NOTATIONS:

S	: Sequence of jobs 1, 2, 3,….,n
S_k	: Sequence obtained by applying Johnson's procedure, k = 1, 2, 3, -------
M_j	: Machine j, j= 1, 2
M	: Minimum makespan
a_{ij}	: Processing time of i^{th} job on machine M_j
p_{ij}	: Probability associated to the processing time a_{ij}
A_{ij}	: Expected processing time of i^{th} job on machine M_j
$t_{ij}(S_k)$	: Completion time of i^{th} job of sequence S_k on machine M_j
$I_{ij}(S_k)$	: Idle time of machine M_j for job i in the sequence S_k
$T_{i,j\to k}$	: Transportation time of i^{th} job from j^{th} machine to k^{th} machine
$U_j(S_k)$	: Utilization time for which machine M_j
$R(S_k)$	: Total rental cost for the sequence S_k of all machine
C_j	: Rental cost of machine M_j.

Definition: Completion time of i^{th} job on machine M_j is denoted by t_{ij} and is defined as

$$t_{ij} = max\ (t_{i-1,j}\ ,\ t_{i,j-1})\ + T_{i,(j-1)\to j} + a_{ij} \times p_{ij} \quad \text{for} \quad j \geq 2.$$

$$= max\ (t_{i-1,j}\ ,\ t_{i,j-1})\ + T_{i,(j-1)\to j} + A_{i.j}$$

where $A_{i,j}$= expected processing time of i^{th} job on machine j.

4. RENTAL POLICY (P):

The machines will be taken on rent as and when they are required and are returned as and when they are no longer required i.e. the first machine will be taken on rent in the starting of the processing the jobs and 2nd machine will be taken on rent at time when 1st job is completed on 1st machine and transported to the 2nd machine.

5. PROBLEM FORMULATION:

Let some job i (i = 1,2,……..,n) are to be processed on three machines M_j (j = 1,2) under the specified rental policy P. let a_{ij} be the processing time of i^{th} job on j^{th} machine and let p_{ij} be the probabilities associated with a_{ij}. Let A_{ij} be the expected processing time of i^{th} job on j^{th} machine and $T_{i,j\to k}$ be the transportation time of i^{th} job from j^{th} machine to k^{th} machine. Our aim is to find the sequence $\{S_k\}$ of the jobs which minimize the rental cost of all the machines. The mathematical model of the problem in matrix form can be stated as:

Jobs	Machine A		$T_{i,1\to 2}$	Machine B	
i	a_{i1}	p_{i1}		a_{i2}	p_{i2}
1	a_{11}	p_{11}	$T_{1,1\to 2}$	a_{12}	p_{12}
2	a_{21}	p_{21}	$T_{2,1\to 2}$	a_{22}	p_{22}
3	a_{31}	p_{31}	$T_{3,1\to 2}$	a_{32}	p_{32}
4	a_{41}	p_{41}	$T_{4,1\to 2}$	a_{42}	p_{42}
-	-	-	-	-	-
n	a_{n1}	p_{n1}	$T_{n,1\to 2}$	a_{n2}	p_{n2}

Table – 1

Mathematically, the problem is stated as:

Minimize $R(S_k) = \sum_{i=1}^{n} A_{i1} \times C_1 + U_2(S_k) \times C_2$

Subject to constraint: Rental Policy (P)

Our objective is to minimize rental cost of machines while minimizing the utilization time.

6.1 Theorem: If $A_{i1} \le A_{j2}$ *for all i, j,≠i j,* then $k_1, k_2 \ldots\ldots k_n$ is a monotonically decreasing sequence, where $k_n = \sum_{i=1}^{n} A_{i1} - \sum_{i=1}^{n-1} A_{i2}$.

Solution: Let $A_{i1} \le A_{j2}$ for all i, j , $i \ne j$ i.e., max $A_{i1} \le$ min A_{j2} for all i, j ; $i \ne j$

Let $k_n = \sum_{i=1}^{n} A_{i1} - \sum_{i=1}^{n-1} A_{i2}$

Therefore, we have $k_1 = A_{11}$

Also $k_2 = A_{11} + A_{21} - A_{12} = A_{11} + (A_{21} - A_{12}) \le A_{11}$ $(\because A_{21} \le A_{12})$

$\therefore k_1 \le k_2$

Now, $k_3 = A_{11} + A_{21} + A_{31} - A_{12} - A_{22}$
$= A_{11} + A_{21} - A_{12} + (A_{31} - A_{22}) = k_2 + (A_{31} - A_{22}) \le k_2$ $(\because A_{31} \le A_{22})$

Therefore, $k_3 \le k_2 \le k_1$ or $k_1 \ge k_2 \ge k_3$.

Continuing in this way, we can have $k_1 \ge k_2 \ge k_3 \ge \ldots\ldots \ge k_n$, a monotonically decreasing sequence.

Corollary: 1 The total rental cost of machines is same for all the sequences.

Proof: The total elapsed time

$T(S) = \sum_{i=1}^{n} A_{i2} + T_{1,1\to 2} + k_1 = \sum_{i=1}^{n} A_{i2} + T_{1,1\to 2} + A_{11}$ **= Constant for all sequences**.

It implies that under rental policy P there is no idle time on machine M_2. Therefore total rental cost of machines is same for all the sequences.

6.2 Theorem: If $A_{j1} \ge A_{j2}$ for all i, j, $i \ne j$, M_j: Machine j. j = 1, 2 then $k_1, k_2 \ldots\ldots k_n$ is a monotonically increasing sequence, where $k_n = \sum_{i=1}^{n} A_{i1} - \sum_{i=1}^{n-1} A_{i2}$.

Proof: Let $k_n = \sum_{i=1}^{n} A_{i1} - \sum_{i=1}^{n-1} A_{i2}$

Let $A_{i1} \geq A_{j2}$ for all i, j , $i \neq j$ i.e., min $A_{i1} \geq$ max A_{j2} for all i, j , $i \neq j$

Here $k_1 = A_{11}$

$k_2 = A_{11} + A_{21} - A_{12} = A_{11} + (A_{21} - A_{12}) \geq k_1$ ($\because$ $A_{21} \geq A_{j2}$)

Therefore, $k_2 \geq k_1$.

Also, $k3 = A_{11} + A_{21} + A_{31} - A_{12} - A_{22} = A_{11} + A_{21} - A_{12} + (A_{31} - A_{22})$
$= k_2 + (A_{31} - A_{22}) \geq k_2$ ($\because A_{31} \geq A_{22}$)

Hence, $k_3 \geq k_2 \geq k_1$.

Continuing in this way, we can have $k_1 \leq k_2 \leq k_3 \ldots\ldots \leq k_n$, a monotonically increasing sequence.

Corollary: 2 The total rental cost of machines is same for all the possible sequences.

Proof: The total elapsed time $=T(S) =$

$$= \sum_{i=1}^{n} A_{i2} + k_n + T_{n,1\to2} = \sum_{i=1}^{n} A_{i2} + \left(\sum_{i=1}^{n} A_{i1} - \sum_{i=1}^{n-1} A_{i2} \right) + T_{n,1\to2} = \sum_{i=1}^{n} A_{i1} + \left(\sum_{i=1}^{n} A_{i2} - \sum_{i=1}^{n-1} A_{i2} \right) + T_{n,1\to2}$$

$$= \sum_{i=1}^{n} A_{i1} + A_{n2} + T_{n,1\to2} = \text{Constant for all sequences.}$$

It implies that under rental policy P there is no idle time on machine M_2 is always constant. Therefore total rental cost of machines is same for all the sequences.

7. ALGORITHM:

Step 1: Calculate the expected processing times, $A_{ij} = a_{ij} \times p_{ij} \forall i, j.$

Step 2: Define the two fictitious machines G and H with processing time g_i and h_i defined as follows: $g_i = A_{i1} + T_{i,1\to2}, h_i = A_{i2} + T_{i,1\to2}$

Step 3: Obtain the job J_1 (say) having maximum processing time on 1^{st} machine.

Step 4: Obtain the job J_n (say) having minimum processing time on 2^{nd} machine.

Step 5: If $J_1 \neq J_n$ then put J_1 on the first position and J_n as the last position & go to step 8, otherwise go to step 6.

Step 6: Take the difference of processing time of job J_1 on M_1 from job J_2 (say) having next maximum processing time on M_1. Call this difference as G_1.Also, Take the difference of processing time of job J_n on M_2 from job J_{n-1}(say) having next minimum processing time on M_2. Call the difference as G_2.

Step 7: If $G_1 \leq G_2$ put J_n on the last position and J_2 on the first position otherwise put J_1 on 1^{st} position and J_{n-1} on the last position.

Step 8: Arrange the remaining (n-2) jobs between 1^{st} job & last job in any order, thereby we get the sequences $S_1, S_2 \ldots S_r$.

Step 9: Compute the total completion time $CT(S_k)$ k=1, 2…r.

Step 10: Calculate utilization time U_2 of 2^{nd} machine $U_2(S_k) = CT(S_k) - A_{11}(S_k)$; k=1,2,…. r

Step 11: Find rental cost $R(S_k) = \sum_{i=1}^{n} A_{i1}(S_k) \times C_1 + U_2 \times C_2$, where C_1 & C_2 are the rental cost per unit time of 1^{st} & 2^{nd} machine respectively.

8. PROGRAMME:

```
#include<iostream.h>
#include<stdio.h>
#include<conio.h>
#include<process.h>

int n;
float a1[16],b1[16],a11[16],b11[16],a2[16],b2[16];float macha[16],machb[16],maxv,u2;
int j[16],j1[16],j2[16],j3[16],T12[16];
float costa,costb,cost;
int main()
{
	clrscr();
	int a[16],b[16];float p[16],q[16],g1,g2;
	cout<<"How many Jobs (<=15) : ";cin>>n;
	if(n<1 || n>15)
	{
		cout<<endl<<"Wrong input, No. of jobs should be less than 15..\n Exitting";getch();exit(0);
	}
	for(int i=1;i<=n;i++)
		{
cout<<"\nEnter the processing time and its probability of "<<i<<" job for machine A and transportation time from A to B : ";cin>>a[i]>>p[i]>>T12[i];
cout<<"\nEnter the processing time and its probability of "<<i<<" job for machine B : ";cin>>b[i]>>q[i];
//Calculate the expected processing times of the jobs for the machines:
		a2[i] = a[i]*p[i];b2[i] = b[i]*q[i];j[i]=i;a1[i]=a2[i]+T12[i];b1[i]=b2[i]+T12[i];
		}
	cout<<"\n Enter the rental cost for Machine M1 & Machine M2 :";cin>>costa>>costb;
	cout<<endl<<"Expected processing time of machine A and B: \n";
	for(i=1;i<=n;i++)
	{
	cout<<"\n"<<j[i]<<"\t"<<a1[i]<<"\t"<<b1[i]<<"\t";cout<<endl;
	}
	for(i=1;i<=n;i++)
	{
	if((a1[i]>=b1[i])^(a1[i]<=b1[i]))
	  {
		a1[i]=a1[i],b1[i]=b1[i];
	  }
	else
	{
	cout<<"\n The data is not in standard form";getch();exit(0);
	}}
void sort(float [],int);// function declaration
for(i=1;i<=n;i++)
{
a11[i]=a1[i];
}
sort(a11,n);//fuction call
cout<<"\nSorted processing times in ascending order of Machine A :\n";
for(i=1;i<=n;i++)
{
j1[i]=j[i];cout<<"\n"<<j1[i]<<"\t"<<a11[i];
}
for(i=1;i<=n;i++)
{
b11[i]=b1[i];j[i]=i;
}
sort(b11,n);// function call
cout <<"\nSorted processing times in ascending order of Machine B :\n";
for(i=1;i<=n;i++)
{
```



```
j2[i]=j[i];cout<<"\n"<<j2[i]<<"\t"<<b11[i];
}
if(j1[n]!=j2[1])
{
j3[1]=j1[n];j3[n]=j2[1];
for(int k=2;k<=n-1;k++)
{
if(j1[k-1]!=j2[1])
{
j3[k]=j1[k-1];
}
else
{
if(j1[n-1]!=j2[1])
{
j3[k]=j1[n-1];
}}}}
else
{
g1=a11[j1[n]]-a11[j1[n-1]];g2=b11[j2[2]]-b11[j2[1]];
if(g1<=g2)
{
j3[1]=j1[n-1];j3[n]=j2[1];
for(int g=2;g<=n-1;g++)
{
j3[g]=j1[g-1];
}}
else
{
j3[1]=j1[n];j3[n]=j2[2];
for(int f=2;f<=n-1;f++)
{
j3[f]=j2[f+1];
}}}
macha[1]=a2[j3[1]];machb[1]=macha[1]+T12[j3[1]]+b2[j3[1]];
// displaying solution
cout<<"\n\n\t**************************************************************";
cout<<"\n\t"<<"optimal sequence is";
for(i=1;i<=n;i++)
{
cout<<"\t"<<j3[i];
}
float time =0.0;
cout<<endl<<endl<<"In-Out Table is"<<endl<<endl;
cout<<"Jobs"<<"\t"<<"Machine M1"<<"\t"<<"Machine M2"<<endl;
cout<<j3[1]<<"\t"<<time<<"--"<<macha[1]<<"\t"<<"\t"<<(macha[1]+T12[j3[1]])<<"--"<<machb[1]<<"\t"<<endl;
for(i=2;i<=n;i++)
{
macha[i]=macha[i-1]+a2[j3[i]];
if(machb[i-1]>macha[i]+T12[j3[i]])
{
maxv= machb[i-1];
}
else
{
maxv=macha[i]+T12[j3[i]];
}
machb[i]=maxv+b2[j3[i]];
cout<<j3[i]<<"\t"<<macha[i-1]<<"--"<<macha[i]<<"\t"<<"\t"<<maxv<<"--"<<machb[i]<<"\t"<<endl;
}
u2=machb[n]-macha[1]-T12[1];cost=macha[n]*costa+u2*costb;
cout<<"\n\nThe total rental cost of machines is:"<<cost;
cout<<"\n\n\t**************************************************************";
getch();return 0;
```



```
}
void sort(float x[],int n)// function declaration
{
float temp; int temp1;
//outer for loop to control no of passe
for(int k=1;k<=n;k++)
{
//inner for loop for making comparison per pass
for(int m=1;m<=n-k;m++)
{
        if(x[m]>x[m+1])
        {
        temp=x[m];temp1=j[m];x[m]=x[m+1];j[m]=j[m+1];x[m+1]=temp;j[m+1]=temp1;
        }
}}}
```

9. NUMERICAL ILLUSTRATION:

probabilities and significant transportation time are given. The rental costs per unit time for machines M_1 and M_2 are 10 units & 7 units respectively. Our objective is to obtain a sequence of jobs with minimum possible rental cost of the machines.

Jobs	Machine M_1		$T_{i,1\to 2}$	Machine M_2	
i	a_{i1}	p_{i1}		a_{i2}	p_{i2}
1	10	0.3	2	5	0.1
2	6	0.1	3	2	0.2
3	5	0.2	3	4	0.1
4	7	0.2	2	3	0.1
5	8	0.1	3	1	0.3
6	14	0.1	2	3	0.2

Table -2

Solution: The expected processing times for machines M_1 and M_2 are

Job	g_i	h_i
i	A_{i1}	A_{i2}
1	5.0	2.5
2	3.6	3.4
3	4.0	3.4
4	3.4	2.3
5	3.8	3.3
6	3.4	2.6

Table – 3

Here each $A_{i2} \geq A_{j1}$ for all *i, j*. Also, Max A_{i1} = 5.0 which is for job 1.i.e. J_1= 5 and Min A_{i2}= 2.3 which is for job 4.i.e. J_n = 4

.i.e. $J_1 \neq J_n$, therefore J_1 = 1 will be on 1st position and J_n=4 will be on the last position.

Therefore, the optimal sequences are:

S_1 = 1- 3- 6- 2- 5- 4, S_2 = 1- 6- 3- 5- 2- 4, S_3 = 1- 3- 5- 6- 2- 4,---------------.

The total elapsed time is same for all these possible 24 sequences S_1, S_2, S_3, S_4-------------,S_{24}.

The In- out table for any of these 24 sequences S_1, S_2, S_3, S_4------, S_{24} ; say for S_1 =1 - 3- 6- 2 - 5 - 4 is

Jobs	M_1	$T_{i,1\to 2}$	M_2
	In- out		In – out
1	0.0 – 3.0	2	5.0 – 5.5
3	3.0 – 4.0	3	7.0 – 7.4
6	4.0 – 5.4	2	7.4 – 8.0
2	5.4 – 6.0	3	9.0 – 9.4
5	6.0 – 6.8	3	9.8 – 10.1
4	6.8 – 8.2	2	10.2 – 10.5

Table – 4

Therefore, the total elapsed time =CT (S_1) =10.5 units and Utilization time for $M_2 = U_2(S_1)$ = 10.5 – 5.0 = 5.5 units.

Also, $\sum_{i=1}^{n} A_{i1}$ = 8.2 units. Therefore the total rental cost for each of the sequence (S_k), k = 1, 2, 3, -----------, 16 is

$R(S_k)$ = 8.2 × 10 + 5.5 × 7= 64 + 27.3 = 120.5 units

10. REMARKS:

If we solve the same problem by Johnson's (1954) methods we get the optimal sequence as S= 2 – 3 – 5 – 6 – 1 – 4. The in-out flow table is

Jobs	M_1	$T_{i,1\to 2}$	M_2
	In- out		In- out
2	0.0 – 0.6	3	3.6 – 4.0
3	0.6 – 1.6	3	4.6 – 5.0
5	1.6 – 2.4	3	5.4 – 5.7
6	2.4 – 3.8	2	5.8 – 6.4
1	3.8 – 6.8	2	8.8 – 9.3
4	6.8 – 8.2	2	9.8 – 10.1

Table – 5

Therefore, the total elapsed time =CT(S) =10.1 units and Utilization time for $M_2 = U_2(S)$ = 10.1- 3.6 = 6.5 units.

Also, $\sum_{i=1}^{n} A_{i1}$ = 8.2 units. Therefore the total rental cost for each of the sequence S is

R(S) = 8.2 × 10 + 6.5 × 7= 82 + 45.5 = 127.7 units

11. CONCLUSION:

The algorithm proposed in this paper for specially structured two stage flow shop scheduling problem in which the processing times are associated with probabilities including transportation time is more efficient and less time consuming as compared to the algorithm proposed by Johnson's(1954) to find an optimal sequence to minimize the rental cost of machines. Due to our rental policy the utilization time for second machine is always minimum and hence, rental cost will always be minimum.

12. REFERENCES:

[1] Johnson S. M., "Optimal two and three stage production schedule with set up times included", *Naval Research Logistic*, Vol.1, No.1, pp. 61-68,1954.

[2] Ignall, E., and Schrage, L., "Application of the branch and bound technique to some flow shop scheduling problems", *Operation Research*, Vol.13, pp.400-412, 1965.

[3] Bagga, P.C., "Sequencing in a rental situation", *Journal of Canadian Operation Research Society,* Vol.7, pp.152-153, 1969.

[4] Gupta, J.N.D., "Optimal Schedule for specially structured flow shop", *Naval Research Logistic*, Vol.22, No.2, pp. 255-269, 1975.

[5] Szware, W., "Special cases of the flow shop problems", *Naval Research Logistic*, Vol.22, No.3, pp. 483-492, 1977.

[6] Yoshida and Hitomi, “Optimal two stage production scheduling with set up times separated”, *AIIE Transactions*, Vol.11, No. 3,pp. 261-263, 1979.

[7] Maggu, P. L. & Das, “On n x 2 sequencing problem with transportation time of jobs”, *Pure and Applied Mathematika Sciences*, Vol. 3, pp.12-16, 1981.

[8] Gupta, J.N.D., “Two stage hybrid flow shop scheduling problem”, *J.Ope.Res.Soc.*, Vol. 39, No. 4, pp. 359 – 364, 1988.

[9] Singh T.P, “On n×2 shop problem involving job block. Transportation times and Break-down Machine times”, *PAMS*, Vol. XXI , No. 1-2, 1985.

[10] Chandrasekharan Rajendaran, “Two-Stage Flowshop Scheduling Problem with Bicriteria ” *O.R. Soc.* ,Vol. 43, No. 9, pp.871-84, 1992.

[11] Chander Sekharan, K. Rajendra & Deepak Chanderi, “An Efficient Heuristic Approach to the scheduling of jobs in a flow shop”, *European Journal of Operation Research*, Vol. 61, pp.318-325,1992.

[12] Panwalker, S. S., “Scheduling of a two machine flow-shop with travel time between machines”, *J. Opl. Res. Soc*, Vol. 42, No. 7, pp.609-613, 1999.

[13] Chen, Z.L. & Lee, C.Y., “Machine scheduling with transportation considerations” *USA; Journal of scheduling*, Vol. 4, pp.3-24, 2001.

[14] Anup, “On two machine flow shop problem in which processing time assumes probabilities and there exists equivalent for an ordered job block”, *JISSOR*, Vol. XXIII No. 1-4, pp. 41-44, 2002.

[15] Singh T. P., K. Rajindra & Gupta Deepak, “Optimal three stage production schedule the processing time and set times associated with probabilities including job block criteria”, *Proceedings of National Conference FACM-*2005,pp. 463-492, 2005..

[16] Narian L & Bagga P.C., “Two machine flow shop problem with availability constraint on each machine”, *JISSOR*, Vol. XXIV 1-4, pp.17-24,2005.

[17] Narian L & Bagga P.C., “Scheduling problems in Rental Situation”, *Bulletin of Pure and Applied Sciences E.Mathematics and Statistics*. Vol.24, ISSN: 0970-6577, 2005.

[18] Singh, T.P, Gupta Deepak, “Minimizing rental cost in two stage flow shop , the processing time associated with probabilies including job block”, *Reflections de ERA*, Vol 1, No.2, pp.107-120, 2005.

[19] Chikhi, N., **“**Two machine flow-shop with transportation time**”**, *Thesis of Magister*, Faculty of Mathematics, USTHB University, Algiers, 2008.

[20] Khodadadi, A., “Development of a new heuristic for three machine flow-shop scheduling with transportation time of job”, *World Applied Sciences Journal*, Vol. 5, No.5, pp.598- 601, 2008.

[21] P. Pandian & P. Rajendran, “Solving Constraint flow shop scheduling problems with three machines”, *Int. J. Contemp. Math. Sciences*, Vol.5, No. 19, pp.921-929, 2010.

[22] Gupta Deepak & Sharma Sameer, “Minimizing rental cost under specified rental policy in two stage flow shop , the processing time associated with probabilities including breakdown interval and Job-block criteria”, *European Journal of Business and Management*, Vol. 2, No. 3, pp.85-103, 2011.

[23] Gupta, D., Sharma, S. & Bala, S, “ Specially structured two stage flowshop scheduling to minimize the rental cost”, *International Journal of Emerging trends in Engineering and Development*, Vol. 1, Issue.2, pp.206-215.

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