International Journal of Mathematical Archive-3(2), 2012, Page: 767-780
Available online through www.ijma.info ISSN 2229 – 5046

HOMOTOPY ANALYSIS METHOD AND DIFFUSION-CONVECTION EQUATION

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(Received on: 20-09-11; Accepted on: 06-10-11)

ABSTRACT

In this paper, the Homotopy Analysis method (HAM) is employed to find a suitable solution for Diffusion-Convection equation. This method is a strong and easy-to-use analytic tool for investigating linear and nonlinear problems, which do not need small parameters. Homotopy Analysis method (HAM) contains the auxiliary parameter $\hbar$, which provides us with a simple way to adjust and control the convergence region of solution series. By suitable choice of auxiliary parameter $\hbar$, we can obtain reasonable solutions for large modulus. In this study, we compare obtained results through (HAM) with the exact solutions. This type of equations governs on numerous scientific and engineering experimentations.

Keywords: *Homotopy Analysis method, linear and non-linear diffusion-convection problems, approximate solution, exact solution.*

1. INTRODUCTION

Nonlinear equations are difficult to solve, especially analytically. Perturbation techniques [1-12] are widely used in science and engineering, and do great contribution to help us understand many nonlinear phenomena. However, it is well known that perturbation methods are strongly dependent upon small/large physical parameters, such as the Lynapunov's artificial small parameter method [13], the δ-expansion method [14, 15], Adomain's decomposition method [16-19], and so on, are formally independent of small/ large physical parameters. But, all of these traditional non-perturbation methods cannot ensure the convergence conditions of the solution series: they are in fact only valid for weekly nonlinear problems, too.

The homotopy analysis method (HAM) [20-27] is a general analytic approach to get series solutions of various types of linear and nonlinear equations, including algebraic equations, ordinary differential equations, partial differential equations and coupled equations of them. Unlike perturbation method, the HAM is independent of small/ large physical parameters and thus is valid no matter whether a nonlinear problem contains small/ large physical parameters or not. More importantly, different from all perturbation and traditional non-perturbation methods, the HAM provides us a simple way to ensure the convergence of solution series, and therefore, the HAM is valid even for strongly nonlinear problems. Besides, different from all perturbation and pervious non-perturbation methods, the HAM provides us with great freedom to choose proper base functions to approximate a nonlinear problem [21-26]. Many researchers have been successfully applying this method to various nonlinear problems in science and engineering, such as the viscous flows of non-Newtonian fluids [28-38], the KdV-type equations [39-43], nonlinear heat transfer equations [44-46], finance problems [47,48], Riemann problems related to nonlinear shallow water equations [49], projectile motion [50], Glauert-jet flow [51], nonlinear water waves [52], ground water flow [53], Burgers-Huxley equation [54], time-dependent Emden-Fowler type equations [55], differential-difference equation [56], Laplace equation with Dirichlet and Neumann boundary conditions [57], thermal-hydraulic network [58], boundary layer flows over a stretching surface with suction and injection [59], Three dimensional diffusion equation [60], Fractional equations [61], MHD mixed convection flow [62], Travelling solutions [63], Lattice systems [64], Inverse problems [65] and so on. Also HAM is also combined with well defined Pade approximations to produce highly effective results [66]. This shows the great potential of the HAM for strongly nonlinear problems in science and engineering. In this paper we apply Homotopy Analysis method (HAM) to solve linear and nonlinear Diffusion Convection equations. These equations have special importance in science and engineering and constitute a good model for many systems in various fields. The non-homogeneous equation is

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effectively solved by employing the phenomena of self-canceling noise terms whose sum vanishes in the limit. Some special cases of the equation are solved as examples to illustrate ability and reliability of the method.

2. BASIC IDEA OF HOMOTOPY ANALYSIS METHOD (HAM)

In this paper, we apply the HAM to the five problems to be discussed. In order to show the basic idea of HAM, consider the following differential equation:

$$N[u(x,t)] = 0, \tag{1}$$

where N is a nonlinear operator, x and t denote the independent variables and u is an unknown function. For simplicity, we ignore all boundary or initial conditions, which can be treated in the similar way. By means of the HAM, we first construct the so-called zeroth-order deformation equation.

$$(1-q)\,L[\phi(x,t;q) - u_0(x,t)] = q\ \hbar\, H(x,t)\, N[\phi(x,t;q)] \tag{2}$$

where $q \in [0,1]$ is the embedding parameter, $\hbar \neq 0$ is an auxiliary parameter, L is an auxiliary linear operator, $\phi(x,t;q)$ is an unknown function, $u_0(x,t)$ is an initial guess of $u(x,t)$ and $H(x,t)$ denotes a nonzero auxiliary function. It is obvious that when the embedding parameter $q = 0$ and $q = 1$, equation (2) becomes

$$\phi(x,t;0) = u_0(x,t), \phi(x,t;0) = u(x,t),$$

respectively. Thus as q increases form 0 to1, the solution $\phi(x,t;q)$ varies from the initial guess $u_0(x,t)$ to the solution $u(x,t)$. Expanding $\phi(x,t;q)$ in Taylor series with respect to q, one has

$$\phi(x,t;q) = u_0(x,t) + \sum_{m=1}^{\infty} u_m(x,t)\, q^m, \tag{3}$$

where

$$u_m(x,t) = \frac{1}{m!}\frac{\partial^m \phi(x,t;q)}{\partial q^m}\Big|q = 0. \tag{4}$$

The convergence of the series (3) depends upon the auxiliary parameter $\hbar$. If it is convergent at $q = 1$, one has

$$u(x,t) = u_0(x,t) + \sum_{m=1}^{\infty} u_m(x,t), \tag{5}$$

which one of the solutions of the original nonlinear equation, as proven by Liao [22]. Define the vectors

$$\vec{u}_n = \{u_0(x,t),\, u_1(x,t), \ldots,\, u_n(x,t)\}. \tag{6}$$

Differentiating the zeroth-order deformation equation (2) m-times with respect to q and then dividing them by $m!$ and finally setting $q = 0$, we get the following mth-order deformation equation:

$$L[u_m(x,t) - \chi_m u_{m-1}(x,t)] = \hbar\, \Re_m(\vec{u}_{m-1}), \tag{7}$$

where

$$\Re_m(\vec{u}_{m-1}) = \frac{1}{(m-1)!}\frac{\partial^{m-1} N[\phi(x,t;q)]}{\partial q^{m-1}}\Big|q = 0, \tag{8}$$

and

$$\chi_m = \begin{cases} 0, & m \le 1, \\ 1, & m > 1. \end{cases} \tag{9}$$

It should be emphasized that $u_m(x,t)$ for $m \ge 1$ is governed by the linear equation (7) with linear boundary conditions that comes from the original problem, which can be easily solved by the symbolic computation softwares such as Maple, Mathematica and Matlab.

3. APPLICATIONS

In this section, we will present the solutions of the linear and nonlinear Diffusion-Convection equations with variable coefficients investigated by Y.Liu, X.Zhao [67], S. Momani [68] and M.Ghasemi, M.T.Kajani [69] to assess the efficiency of the homotopy analysis method. For all of these equations, we choose the solution expressed by the base function of the form $\left\{ t^{an+b} \middle| a > 0; b > 0; n = 0,1,2,... \right\}$ (10)

The rule of solution expression together with the initial condition in (2) suggest the initial approximation

$$u_0(x,t) = t \tag{11}$$

The rule of solution expression also suggests that we define the linear operator *L* by

$$L[\phi(x,t;q)] = \frac{\partial \phi(x,t;q)}{\partial t} \tag{12}$$

with the property

$$L[c_1] = 0 \tag{13}$$

Example: 3.1 Consider the Kolomogrov-Petrovsly-Piskunov (KPP) equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} - u(x,t) \tag{14}$$

with the initial conditions $u(x,0) = x + e^{-x}$

According to the style of the solution and the initial condition, we take the initial guess as

$$u_0(x,t) = x + e^{-x}$$

The nonlinear operator is

$$N[\phi(x,t;q)] = \frac{\partial \phi(x,t;q)}{\partial t} - \frac{\partial^2 \phi(x,t;q)}{\partial x^2} + \phi(x,t;q) \tag{15}$$

and thus

$$\Re_m(\vec{u}_{m-1}) = \frac{\partial u_{m-1}(x,t)}{\partial t} - \frac{\partial^2 u_{m-1}(x,t)}{\partial x^2} + u_{m-1}(x,t) \tag{16}$$

The m^{th} -order deformation equation is given by

$$L[u_m(x,t) - \chi_m u_{m-1}(x,t)] = \hbar \Re_m(\vec{u}_{m-1}) \tag{17}$$

Solving above equation (17) under the initial conditions $u_m(x,0) = 0,\ m = 1,2,3...$ we get

$$u_1(x,t) = \hbar xt$$

$$u_2(x,t) = \hbar(1+\hbar)\,xt + \frac{\hbar^2 x t^2}{2}$$

$$u_3(x,t) = \hbar(1+\hbar)^2 xt + \hbar^2(1+\hbar)xt^2 + \frac{\hbar^3 xt^3}{6} \tag{18}$$

$$u_4(x,t) = \hbar(1+\hbar)^3 xt + \hbar^2(1+\hbar)^2 xt^2 + \frac{\hbar^3(1+\hbar)xt^3}{6} + \frac{\hbar^4 xt^4}{24}$$

$$\vdots$$

and so on

Taking $\hbar = -1$, the approximate solution is given by

$$u(x,t) = \sum_{r=0}^{m-1} u_r(x,t) = e^{-x} + xe^{-t} \tag{19}$$

which is an exact solution and is same as obtained by Y.Liu, X.Zhao [67], S. Momani [68] and M.Ghasemi, M.T.Kajani [69].

Example: 3.2 Consider the following diffusion-convection problem

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \left(-1 + \cos x - \sin^2 x\right) u$$

with the initial condition $u(x,0) = \frac{1}{10} e^{\cos x - 11}$ (20)

According to the style of the solution and the initial condition, we take the initial guess as

$$u_0(x,t) = \frac{1}{10} e^{\cos x - 11} \tag{21}$$

The nonlinear part is

$$N[\phi(x,t;q)] = \frac{\partial \phi(x,t;q)}{\partial t} - \frac{\partial^2 \phi(x,t;q)}{\partial x^2} + (1 - \cos x + \sin^2 x)\, \phi(x,t;q) \tag{22}$$

and thus

$$\Re_m(\vec{u}_{m-1}) = \frac{\partial u_{m-1}(x,t)}{\partial t} - \frac{\partial^2 u_{m-1}(x,t)}{\partial x^2} + (1 - \cos x + \sin^2 x)\, u_{m-1}(x,t) \tag{23}$$

The m^{th} -order deformation equation is given by

$$L[u_m(x,t) - \chi_m u_{m-1}(x,t)] = \hbar \Re_m(\vec{u}_{m-1}) \tag{24}$$

solving above equation (24) under the initial conditions $u_m(x,0) = 0,\ m = 1,2,3\ldots$ we get

$$u_1(x,t) = \frac{1}{10} e^{\cos x - 11} \hbar t$$

$$u_2(x,t) = \frac{\hbar(1+\hbar)\, e^{\cos x - 11}\, t}{10} + \frac{\hbar^2 e^{\cos x - 11}\, t^2}{20}$$

$$u_3(x,t) = \frac{\hbar(1+\hbar)^2 e^{\cos x - 11}\, t}{10} + \frac{\hbar^2(1+\hbar) e^{\cos x - 11}\, t^2}{20} + \frac{\hbar^3 e^{\cos x - 11}\, t^3}{60} \tag{25}$$

$$u_4(x,t) = \frac{\hbar(1+\hbar)^3 e^{\cos x - 11}\, t}{10} + \frac{\hbar^2(1+\hbar)^2 e^{\cos x - 11}\, t^2}{20} + \frac{\hbar^3(1+\hbar) e^{\cos x - 11}\, t^3}{60} + \frac{\hbar^4 e^{\cos x - 11}\, t^4}{240}$$

$$\vdots$$

and so on

Taking $\hbar = -1$, the approximate solution is given by

$$u(x,t) = \frac{1}{10} e^{\cos x - 11}\left(1 - t + \frac{t^2}{2!} - \frac{t^3}{3!} + \ldots\right) = \frac{1}{10} e^{\cos x - 11 - t} \tag{26}$$

which is an exact solution and is same as obtained by Y.Liu, X.Zhao [67], S. Momani [68] and M.Ghasemi, M.T.Kajani [69].

Example: 3.3 Consider the following diffusion-convection problem

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}-\frac{1}{4}u, \qquad x,t\in R$$

with the initial condition $u(x,0)=\frac{1}{2}x+e^{-x/2}$ (27)

According to the style of the solution and the initial condition, we take the initial guess as

$$u_0(x,t)=\frac{1}{2}x+e^{-x/2} \tag{28}$$

The nonlinear part is

$$N[\phi(x,t;q)]=\frac{\partial\phi(x,t;q)}{\partial t}-\frac{\partial^2\phi(x,t;q)}{\partial x^2}+\frac{1}{4}\phi(x,t;q) \tag{29}$$

and thus

$$\Re_m(\vec{u}_{m-1})=\frac{\partial u_{m-1}(x,t)}{\partial t}-\frac{\partial^2 u_{m-1}(x,t)}{\partial x^2}+\frac{1}{4}u_{m-1}(x,t) \tag{30}$$

The m^{th} -order deformation equation is given by

$$L[u_m(x,t)-\chi_m u_{m-1}(x,t)]=\hbar\Re_m(\vec{u}_{m-1}) \tag{31}$$

solving above equation (31) under the initial conditions $u_m(x,0)=0,\ m=1,2,3\ldots$ we get

$$\begin{aligned}
u_1(x,t)&=\frac{1}{8}\hbar t\\
u_2(x,t)&=\frac{\hbar(1+\hbar)xt}{8}+\frac{\hbar^2xt^2}{64}\\
u_3(x,t)&=\frac{\hbar(1+\hbar)^2xt}{8}+\frac{\hbar^2(1+\hbar)xt^2}{32}+\frac{\hbar^3xt^3}{768}\\
u_4(x,t)&=\frac{\hbar(1+\hbar)^3xt}{8}+\frac{3\hbar^2(1+\hbar)^2xt^2}{64}+\frac{3\hbar^3(1+\hbar)xt^3}{768}+\frac{\hbar^4xt^4}{12288}\\
&\vdots
\end{aligned} \tag{32}$$

and so on

Taking $\hbar=-1$, the approximate solution is given by

$$u(x,t)=e^{-x/2}+\frac{x}{2}\left(1+\frac{\left(-t/4\right)}{1!}+\frac{\left(-t/4\right)^2}{2!}+\frac{\left(-t/4\right)^3}{3!}+\ldots\right)=e^{-x/2}+\frac{x}{2}e^{-t/4} \tag{33}$$

which is an exact solution and is same as obtained by Y.Liu, X.Zhao [67], S. Momani [68] and M.Ghasemi, M.T.Kajani [69].

Example 3.4 Consider the following nonlinear diffusion-convection problem

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}-\frac{\partial u}{\partial x}+\xi(u), \quad 0\le x\le 1,\, t>0$$

where $\xi(u)=-u^2+u\,u_{xx}+u$ (34)

with the initial condition $u(x,0)=e^x$ (35)

According to the HAM, the initial guess is taken as $u_0(x,t)=e^x$ (36)

The nonlinear part is

$$N[\phi(x,t;q)]=\frac{\partial\phi(x,t;q)}{\partial t}-\frac{\partial^2\phi(x,t;q)}{\partial x^2}+\frac{\partial\phi(x,t;q)}{\partial x}-\phi^2(x,t;q)-\phi(x,t;q)\frac{\partial^2\phi(x,t;q)}{\partial x^2}+\phi(x,t;q) \tag{37}$$

and thus

$$\Re_m(\vec{u}_{m-1})=\frac{\partial u_{m-1}(x,t)}{\partial t}-\frac{\partial^2 u_{m-1}(x,t)}{\partial x^2}-\frac{\partial u_{m-1}(x,t)}{\partial x}-u_{m-1}(x,t)$$
$$+\sum_{r=0}^{m-1}u_r(x,t).u_{m-1-r}(x,t)+u_r(x,t).\frac{\partial^2 u_{m-1-r}(x,t)}{\partial x^2} \tag{38}$$

The m^{th} -order deformation equation is given by

$$L[u_m(x,t)-\chi_m u_{m-1}(x,t)]=\hbar\,\Re_m(\vec{u}_{m-1}) \tag{39}$$

solving above equation (39) under the initial conditions $u_m(x,0)=0,\ m=1,2,3...$ we get

$$\begin{aligned}
u_1(x,t)&=-\hbar e^x t\\
u_2(x,t)&=-\hbar(1+\hbar)e^x t+\frac{\hbar^2 e^x t^2}{2}\\
u_3(x,t)&=-\hbar(1+\hbar)^2 e^x t+\frac{\hbar^2(1+\hbar)e^x t^2}{2}-\frac{\hbar^3 e^x t^3}{6}\\
u_4(x,t)&=-\hbar(1+\hbar)^3 e^x t+\frac{\hbar^2(1+\hbar)^2 e^x t^2}{2}-\frac{\hbar^3(1+\hbar)e^x t^3}{6}+\frac{\hbar^4 e^x t^4}{24}\\
&\ \vdots
\end{aligned} \tag{40}$$

and so on

Taking $\hbar=-1$, the approximate solution is given by

$$u(x,t)=e^x\left(1+t+\frac{t^2}{2}+\frac{t^3}{6}+...\right)=e^{x+t} \tag{41}$$

which is an exact solution and is same as obtained by Y.Liu, X.Zhao [67], S. Momani [68] and M.Ghasemi, M.T.Kajani [69].

If we denote the approximation of k^{th} terms by ψ_k, then 4 -terms approximation is denoted by $\psi_4=\sum_{i=0}^{3}u_i(x,t)$.

The error between exact and approximate solution is given in Table 1.

Example: 3.5 Consider the following nonlinear diffusion-convection problem

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial x} + \xi(u) + g(x,t), \quad 0 \le x \le 1,\, t > 0$$

where $\xi(u) = \dfrac{\partial\, f(u)}{\partial t}$, $\quad f(u) = u\, u_x \quad$ and $\quad g(x,t) = e^{-t}\cos x + e^{-2t}\sin 2x$ (42)

with the initial condition $u(x,0) = \sin x$ (43)

According to the HAM, the initial guess is taken as $u_0(x,t) = \sin x$ (44)

for simplicity we take approximation by using double Maclaurin series representation

$$e^{-t} \approx 1 - t + \frac{t^2}{2}, \;\; \sin x \approx x - \frac{x^3}{3}, \quad \cos x \approx 1 - \frac{x^2}{2},$$

so that

$$g(x,t) = \left(1 - t + \frac{t^2}{2}\right)\left(1 - \frac{x^2}{2}\right) + 2\left(1 - 2t + 2t^2\right)\left(x - \frac{x^3}{3}\right) \tag{45}$$

The nonlinear part is

$$N[\phi(x,t;q)] = \frac{\partial\phi(x,t;q)}{\partial t} - \frac{\partial^2\phi(x,t;q)}{\partial x^2} + \frac{\partial\phi(x,t;q)}{\partial x} - \frac{\partial}{\partial t}\left(\phi(x,t;q)\frac{\partial\phi(x,t;q)}{\partial x}\right) + g(x,t) \tag{46}$$

and thus

$$\Re_m(\vec{u}_{m-1}) = \frac{\partial u_{m-1}(x,t)}{\partial t} - \frac{\partial^2 u_{m-1}(x,t)}{\partial x^2} - \frac{\partial u_{m-1}(x,t)}{\partial x} - u_{m-1}(x,t)\,\frac{\partial}{\partial t}\left(\sum_{r=0}^{m-1} u_r(x,t)\frac{\partial u_{m-1-r}}{\partial x}\right) - g(x,t)\left(1 - \chi_m\right) \quad 47)$$

The m^{th} -order deformation equation is given by

$$L[u_m(x,t) - \chi_m u_{m-1}(x,t)] = \hbar\,\Re_m(\vec{u}_{m-1}) \tag{48}$$

$$u_1(x,t) = -\hbar\,(x+t) + \frac{\hbar\, x^2\, t}{2} + \frac{\hbar\, t^2}{2} - \frac{\hbar\, x^2\, t^2}{4} - 2\hbar\, x\, t + \hbar\, x^3\, t + 2\, x\,\hbar\, t^2$$
$$-\hbar\, x^3 t^2 - \frac{\hbar\, t^3}{6} - \frac{4\hbar x t^3}{3} + \frac{\hbar\, x^2\, t^3}{12} + \frac{2\hbar\, x^3\, t^3}{3}$$

$$u_2(x,t) = \frac{\hbar^2\left(-8\,x\,(x^2-1) - 3\,x^2\right)t}{2} + \frac{\hbar^2\left(12\,x^5 + 10\,x^4 - 14\,x^3 - 15\,x^2 - 14\,x - 12\right)t^2}{4}$$
$$+ \frac{\hbar^2\left(-72x^5 - 45x^4 + 154x^3 + 186x^2 - 30x - 24\right)t^3}{12} + \frac{\hbar^2\left(168x^5 + 80x^4\right)t^4}{24}$$
$$+ \frac{\hbar^2\left(-441x^3 - 180x^2 + 187x + 55\right)t^4}{24} + \frac{\hbar^2\left(-240x^5 - 75x^4 + 635x^3\right)t^5}{60}$$
$$+ \frac{\hbar^2\left(180x^2 - 310x - 60\right)t^5}{60} + \frac{\hbar^2\left(576x^5 + 20x^4 - 255x^3 - 48x^2 - 126x - 16\right)t^6}{72}$$
$$\vdots$$

and so on

Taking $\hbar = -1$ and sum the series up to 9-term we finds the noise terms are carry same and opposite sign which are cancelled out and remaining terms will satisfy the equation, therefore the solution in closed form is given by

$$u(x,t) = e^{-t}\sin x \tag{49}$$

which is an exact solution and is same as obtained by Y.Liu, X.Zhao [67], S. Momani [68] and M.Ghasemi, M.T.Kajani [69]. The error between exact and approximate solution is given in Table 2.

4. RESULTS AND DISCUSSIONS

We present the comparison of the analytical result between the 8^{th} -order HAM and others semi-analytical methods, particularly for the solutions of the diffusion-convection equation in section 3. Also HAM provides to adjust and control the convergence rate of the solution in the particular region with $\hbar$. Fig.2-5 shows that the HAM solution has the same shape as the exact solution and approximate solution even for larger range of t, *i.e* $t = \{[0,5],[0,15]\}$ given at $\hbar = -1$. The particular value of $\hbar$, $-2 < \hbar < 2$ is in the convergent region as shown in the Fig 1, indicates that the solution is convergent and tends to exact for larger values of *t* for *x=1* and *t=1*. The 4^{th} -order approximation of the $\hbar$ -curve is converges constantly in the given region, 6^{th} -order approximation converges fast in the region $-2 < \hbar < 0.5$ and then tends to slowly change in the interval $0.5 < \hbar < 2$, denotes that the solution is convergent and remain same as the exact solution for the same order. Also 8^{th} -order approximation denotes that the solution converges very fast to the exact solution. Also form the tables 1 and 2 it has been observed that the errors between the exact and approximate solutions are very small and are negligible.

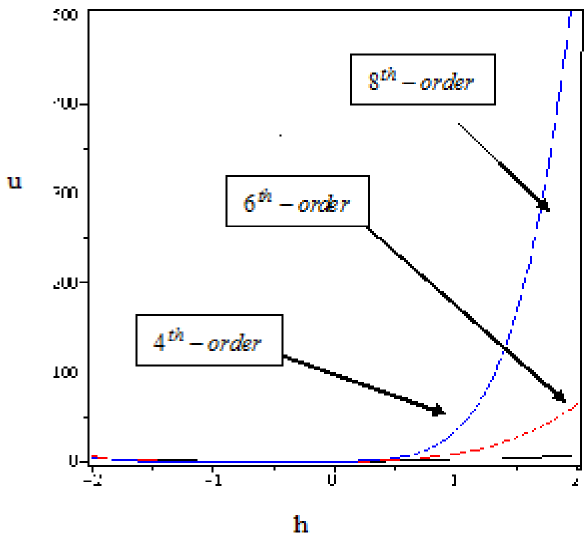

Fig.1: The $\hbar$ -curve of the 4^{th} , 6^{th} , 8^{th} order approximation for the diffusion-convection equation t $x = 1$, $t = 1$.

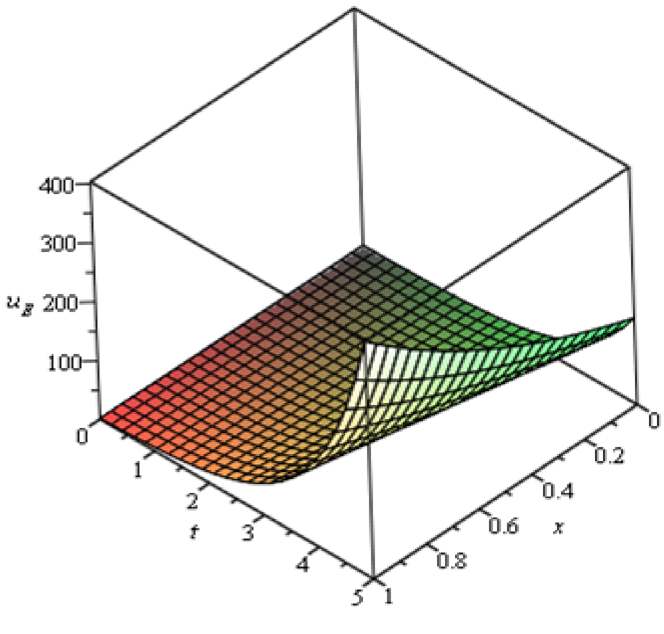

Fig.2: Exact solution graph of diffusion-convection equation for t =0 to t =5

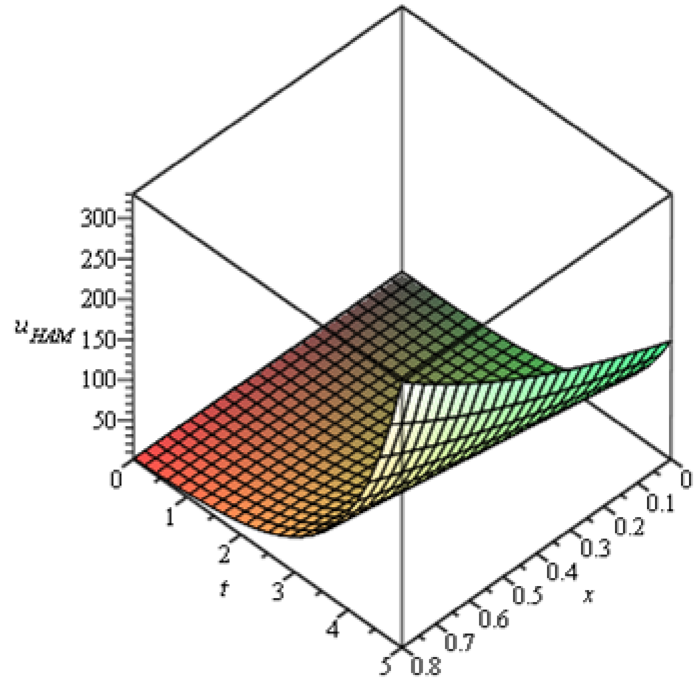

Fig 3: Approximate solution graph of diffusion-convection equation for t =0 to t =5

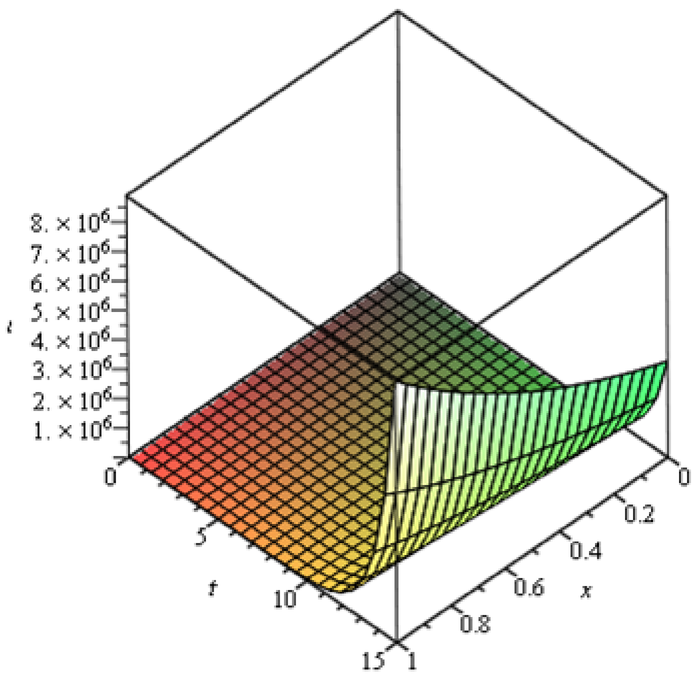

Fig.4: Exact solution graph of diffusion-convection equation for t =0 to t =15

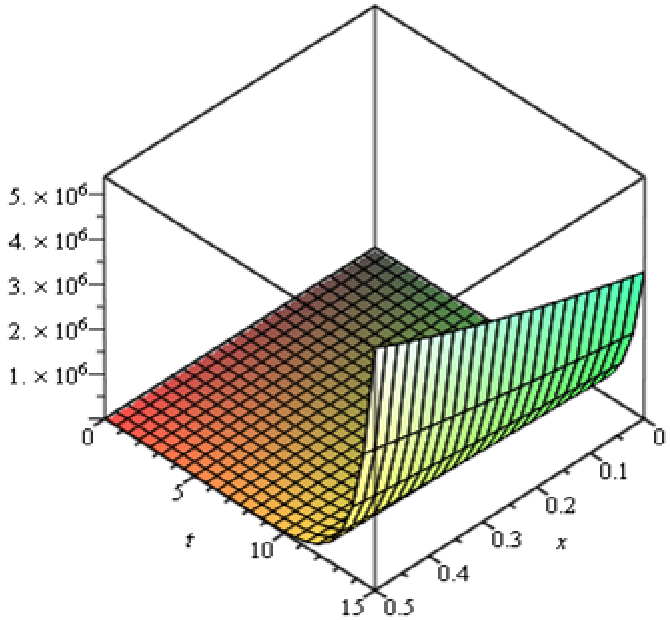

Fig 5: Approximate solution graph of diffusion-convection equation for t =0 to t =15

Table: 1
Comparison of the exact solution with 4-term HAM taking $\hbar$ = -1 solution of Ex. 4

(x_i, t_i)	Exact Solution $u(x,t)$	Approximate Solution $\psi_4(x,t)$	Error $\lvert u(x,t)-\psi_4(x,t)\rvert$
(0.01, 0.01)	1.020201	1.020201	8.43103E-13
(0.02, 0.02)	1.040811	1.040811	2.72962E-11
(0.03, 0.03)	1.061837	1.061837	2.09715E-10
(0.04, 0.04)	1.083287	1.083287	8.94114E-10
(0.05, 0.05)	1.105171	1.105171	2.76066E-09
(0.06, 0.06)	1.127497	1.127497	6.95001E-09
(0.07, 0.07)	1.150274	1.150274	1.51984E-08
(0.08, 0.08)	1.173511	1.173511	2.99799E-08
(0.09, 0.09)	1.197217	1.197217	5.46597E-08
(0.1, 0.1)	1.221403	1.221403	9.36547E-08

Table: 2
Comparison of the exact solution with 4-term HAM taking $\hbar$ = -1 solution of Ex. 5

(x_i, t_i)	Exact Solution $u(x,t)$	Approximate Solution $\psi_4(x,t)$	Error $\lvert u(x,t)-\psi_4(x,t)\rvert$
(0.01, 0.01)	0.009903	0.009903	3.21026E-15
(0.02, 0.02)	0.019603	0.019603	9.35652E-14
(0.03, 0.03)	0.029109	0.029109	2.35860E-14

(0.04, 0.04)	0.038421	0.038421	0.000000000
(0.05, 0.05)	0.047542	0.047542	0.000000000
(0.06, 0.06)	0.056472	0.056472	6.95001E-15
(0.07, 0.07)	0.065214	0.065214	0.000000000
(0.08, 0.08)	0.073771	0.073771	2.76066E-12
(0.09, 0.09)	0.082413	0.082413	4.01235E-12
(0.1, 0.1)	0.090333	0.090333	8.02354E-12

5. CONCLUSIONS

In this paper the HAM is used to obtain the exact solutions of the various linear and nonlinear Diffusion-Convection equations. The comparison is made between the solutions obtained by HAM with other semi-analytical methods such as the Adomain decomposition method (ADM), the Variational iteration method (VIM) and the Homotopy perturbation method (HPM), shows that HAM is more effective than others. Further, for all of the discussed examples, it was found that there was no error in obtaining the exact solutions using HAM. Hence it may be conclude that this method is a powerful an efficient technique in finding the exact solution for wider class of problems. This paper also illustrated the validity and the great potential of the HAM for solving nonlinear problems in science and engineering. It is also worth mentioning at this end that the advantage of this method is the fast convergent of the solutions by means of the auxiliary parameter $\hbar$ .In this paper, Numerical computations has been done by Maple-13 software package.

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