International Journal of Mathematical Archive-3(9), 2012, 3501-3503
Available online through www.ijma.info ISSN 2229 – 5046

CONDITION FOR HAMILTONIANCITY

Mushtaq Ahmad Shah*
Research Scholar, Department of mathematics CMJ University, Meghalaya, Shillong- 793003, India

(Received on: 20-08-12; Revised & Accepted on: 20-09-12)

ABSTRACT

A Hamiltonian path is a spanning path in a graph i.e. a path through every vertex. The problem of determining if a graph is Hamiltonian is well known to be NP-complete while there are several necessary condition for a hamiltonianicity, the search continue for sufficient conditions in this paper I shall prove that every simple graph of order (2n+1) and degree of each vertex is 2n respectively $\forall\, n \in N$ then G is Hamiltonian. In fact it has n edge-disjoint Hamiltonian circuits.

Keywords: *Degree, Vertex Hamiltonian Cycle.*

1. INTRODUCTION AND PREVIOUS WORK

A graph is said to be Hamiltonian if it contains a spanning cycle. The spanning cycle is called a Hamiltonian cycle of G and G is said to be a Hamiltonian graph. A Hamiltonian path is a path that contains all the vertices in V(G) but does not return to the vertex in which it began. No characterization of Hamiltonian graph exists, yet these many sufficient conditions (see [3] pp 30-37) see [5] PP (18-24). We begin our investigation of sufficient conditions for hamiltonianicity with early results like Dirac and ore. Both results consider this intuitive fact the more edges a graph has the more likely it is that a Hamiltonian cycle will exists.

Theorem 1.1: (Dirac, 1952 [2]) if G is a graph of order $n \geq 3$ such that $\delta \geq \frac{n}{2}$ then G is Hamiltonian

As an illustration of Dirac's theorem consider the Graph on three vertices (fig 1.1) in this graph $\delta = 2 \geq \frac{3}{2}$, so it is Hamiltonian, traversing the vertices in numerical order 1-3 and back to 1 yields a Hamiltonian cycle

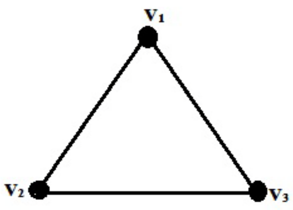

Figure: 1.1

Theorem 1.2: (Ore, 1960 [4]) if g is a graph of order $n \geq 3$ such that for all distinct non-adjacent pairs of vertices U and V deg(u) + deg(v) $\geq$ n then G is Hamiltonian.

As an Illustration of ore, theorem consider a graph of six vertices in figure (1.2)

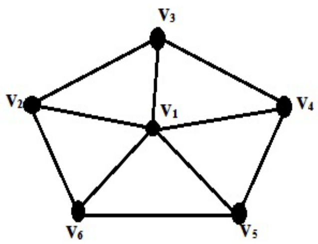

Figure: 1.2

Corresponding author: *Mushtaq Ahmad Shah**
Research Scholar, Department of mathematics CMJ University, Meghalaya, Shillong- 793003, India

The sum of the degree of non adjacent vertices i.e {deg (V_2) +deg (V_3), deg (V_3) + deg (V_6) deg (V_3) + deg (V_5) etc} is the order of the Graph.

Theorem 1.3: (Bondy-chvatu 1979[1]) if G is a simple graph with n vertices, then G is Hamiltonian if and only if its closure is Hamiltonian.

The Hamiltonian closure of a graph G, denoted by C(G) is the super graph of G on V(G) obtained by it iteratively adding edges between pairs of nonadjacent vertices whose degree sum is at least n, until no such pair remains. Fortunately the closure does not depend on the order in which we choose to add edges more than one is available i.e. the closure of G is well defined

Theorem 1.4: (Nash-Williams, 1971) let G be a 2 connected graph of order n with $\delta(G) \geq \left\{\frac{n+2}{3}, \beta\right\}$

Then G is Hamiltonian.

As an illustration of Nash-Williams theorem consider to following graph.

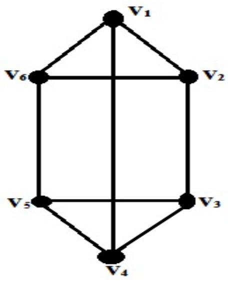

Figure: 1.3

The graph in figure demonstrates the Nash-Williams result in this connected graph on six vertices

$\delta = 3$, $\beta = 3$ and $\delta \geq \left[\frac{6+2}{3}, 2\right]$ implying Hamiltoncity.

Theorem 1.5 [3] In a complete graph with n vertices there are $\frac{n-1}{2}$ edge-disjoint-Hamiltonian circuits if n is an odd number ≥ 3.

2. THE MAIN RESULT

Theorem 2.1

Statement: Every simple graph G of order (2n + 1) and degree of each vertex is 2n respectively $\forall$ n $\in$ N then G is Hamiltonian in fact it is exactly n edge – disjoint Hamiltonian circuits

Proof: For the proof we first show that graph must be simple in considering the existence of a Hamiltonian circuit we need only consider simple graphs. This is because Hamiltonian circuit travels every vertex exactly once. Hence it can't include a self loop or a set of parallel edges. Thus for looking weather the given graph is Hamiltonian we have to remove parallel edges and self loops for the purpose to make a graph simple.

Now first of all I shall prove that order of G must be 2n + 1 $\forall$ n $\in$ N if it is less than 2n + 1 then graph is not Hamiltonian consider a graph of order two

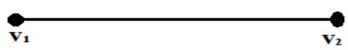

Figure: 2.1

Graph in Figure 2.1 is not Hamiltonian thus order musty be 2n + 1 $\forall n \in N$

It is not necessary that if the order of graph is 2n + 1 it is Hamiltonian for this considers the following figures 2.2 order of graph is 3, it is not Hamiltonian.

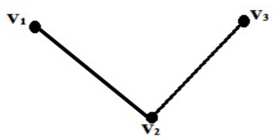

Figure: 2.2

To prove that if the order of graph is 2n + 1 and degree 2n respectively $\forall$ n $\in$ N then G is Hamiltonian and contain n disjoint-Hamiltonian cycles. For this purpose we first show that it is Hamiltonian contain at least one Hamiltonian cycle

Let G = (V E) be a connected Graph with 2n + 1 vertices and let P be a longest path in G if P is contained in a cycle then P is a Hamiltonian path We choose this path from any vertex say V_r Let us suppose that P ={U = U_0 , U_1 ,U_2 . . . U_t =V} of length t and p is contained in a cycle C = { U = U_0 ,U_1 . . . U_t = V} since vertices of "C" and vertices of "P" are equal i.e., V(C) = V(P) since otherwise p would be a part of a longest path a contradiction assume for the sake of contradiction t < 2n i.e. P is not Hamiltonian path since G is connected there must be an edge of the form ($\alpha\beta$) such that α $\in$ V(p) = V(c) and β $\in$ V(G) – V(C)

Let $\alpha \in$ Ui then there is a path P' = { β, α = Ui, Ui+1, . . . U1 U2 . . . Ui-1} with length t + 1 which is contradiction, since P is the longest path in G. since degree of each vertex is 2n i.e. every vertex has a path to reach other vertices, because the degree of each vertex is at most 2n thus the given path p can be reached to the beginning vertex thus path p is Hamiltonian cycle in G thus G is Hamiltonian

Now we shall prove that the graph G has exactly N edged disjoint Hamiltonian circuits. Since the order of graph is 2n + 1 and degree is 2n. As in a graph of order 2n + 1 a vertex can be joined to at most 2n other vertices therefore the maximum degree a vertex can have is 2n in our case the degree of each vertex is 2n thus there exists an edge between every pair of vertices. Thus the numbers of edge disjoint Hamiltonian circuits in G are the sub graph of a complete graph of 2n+1 vertices is a Hamiltonian circuit. Keeping the vertices fixed on circle; rotate the polygonal pattern clockwise by

$\frac{360}{2n}, \frac{2\cdot 360}{2n}, \frac{3\cdot 360}{2n} \cdots \frac{360}{2n}$ Degrees. We observe that each rotation produces the Hamiltonian circuit that has no edge in common with any of the previous lines. Thus we have $\left(\frac{2n+1-1}{2}\right) = \frac{2n}{2} = n$ new Hamiltonian circuits. That proves the theorem

CONCLUSION

In this paper I present a new condition for a graph to possess a Hamiltonian cycle in fact it has n edge disjoint Hamiltonian circuits. My condition in this form of theorem 2.1 seems to be significant and interesting the condition I present explores a new idea since the condition is applied on the degree of vertex and finally I have proved that every simple graph G of order (2n +1) and degree of each vertex is 2n respectively $\forall\ N \in\ N$ than G is Hamiltonian in fact it has n edge disjoint Hamiltonian circuits.

REFERENCES

[1] J.A Bondy and V. Chvatal. A method in gaph theory Disscr. Math 15(1979). Pp 111-136

[2] G.A Dirac, some theorems on abstract graphs proc. Lond. Math soc. 2(1952). PP69-81

[3] NARSINGH DEO Graph theorey with applications prentice- Hall of India private limited new dehli 11000/2005 pp (30-37)

[4] O.ore, note on Hamiltonian circuits, Am. Mat mothly 67(1960) PP55

[5] Factors and cycles in graph PhD thesis Jakub Teska Zapadoceska Univerzita V Plazni (pp-18-24).

Source of support: Nil, Conflict of interest: None Declared