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COMMON FIXED POINT THEOREM FOR SIX WEAKLY-COMPATIBLE MAPPINGS IN INTUITIONISTIC FUZZY METRIC SPACE

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ABSTRACT

The Purpose of this paper is to obtain a common fixed point theorem for six weakly compatible mappings in intuitionistic fuzzy metric space. We extend some earlier results.

Keyword: *Intuitionistic fuzzy metric space, R-commuting maps, weak-compatible maps, common fixed point.*

2000 Mathematics Subject Classification: *47H10, 54H25.*

INTRODUCTION

As a generalization of fuzzy sets introduced by Zadeh [11], Atanassav [2] introduced the concept of intuitionistic fuzzy sets. Recently, using the idea of intuitionistic fuzzy sets, Park [6] introduced the notion of intuitionistic fuzzy metric spaces with the help of continuous t-norms and continuous t-conorms as a generalization of fuzzy metric spaces due to George and Veeramani [3] and introduced the notionof Cauchy sequences in an intuitionistic fuzzy metric space. Turkoglu et al. [9], gave generalization of Jungck's common fixed point theorem [4] to intuitionistic fuzzy metric spaces. Recently, many authors have studied fixed point theory in intuitionistic fuzzy metric spaces (See [1], [5], [6], [9], [10]).

In this paper, we prove a common fixed point theorem for six self maps in intuitionistic fuzzy metric space under the assumption of weak compatibility of maps.

PRELIMINARIES

Definition 1[8]: A binary operation $*$: $[0, 1] \times [0, 1] \to [0, 1]$ is continuous t-norm if $*$ is satisfying the following conditions:
(i) $*$ is commutative and associative;
(ii) $*$ is continuous;
(iii) $a * 1 = a$ for all $a \in [0, 1]$;
(iv) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0, 1]$.

Definition 2[8]: A binary operation $\Diamond$: $[0, 1] \times [0, 1] \to [0, 1]$ is continuous t-conorm if $\Diamond$ is satisfying the following conditions:
(i) $\Diamond$ is commutative and associative;
(ii) $\Diamond$ is continuous;
(iii) $a \Diamond 0 = a$ for all $a \in [0, 1]$;
(iv) $a \Diamond b \geq c \Diamond d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0, 1]$.

Definition 3[1]: A 5-tuple $(X, M, N, *, \Diamond)$ is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm, $\Diamond$ is a continuous t-conorm and M, N are fuzzy sets on $X^2 \times (0,\infty)$ satisfying the following conditions:
(i) $M(x, y, t) + N(x, y, t) \leq 1$ for all $x, y \in X$ and $t > 0$;
(ii) $M(x, y, 0) = 0$ for all $x, y \in X$;

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(iii) M(x, y, t) = 1 for all x, y ∊ X and t > 0 if and only if x = y;
(iv) M(x, y, t) = M(y, x, t) for all x, y ∊ X and t > 0;
(v) M(x, y, t) * M(y, z, s) ≤ M(x, z, t + s) for all x, y, z ∊ X and s, t > 0;
(vi) For all x, y ∊ X, M(x, y, .): [0, ∞)→[0, 1] is continuous;
(vii) $\lim_{t\to\infty} M$(x, y, t) = 1 for all x, y ∊ X and t > 0;
(viii) N(x, y, 0) = 1 for all x, y ∊ X;
(ix) N(x, y, t) = 0 for all x, y ∊ X and t > 0 if and only if x = y;
(x) N(x, y, t) = N(y, x, t) for all x, y ∊ X and t > 0;
(xi) N(x, y, t) ◊ N(y, z, s) ≥ N(x, z, t + s) for all x, y, z ∊ X and s, t > 0;
(xii) For all x, y ∊ X, N(x, y, .) : [0, ∞)→[0, 1] is continuous;
(xiii) $\lim_{t\to\infty} N$(x, y, t) = 0 for all x, y in X;

Then (M, N) is called an intuitionistic fuzzy metric on X. The functions M(x, y, t) and N(x, y, t) denote the degree of nearness and the degree of non-nearness between x and y with respect to t, respectively.

Remark 1: Every fuzzy metric space (X, M, *) is an intuitionistic fuzzy metric space of the form (X, M, 1-M, *, ◊) such that t-norm * and t-conorm ◊ are associated as x ◊ y = 1- ((1-x) * (1-y)) for all x, y ∊ X.

Example 1[6]: Let (x, d) be a metric space, define t-norm a * b = min{a, b} and t-conorm a ◊ b = max{a, b} and for all x, y ∊ X and t > 0,

$$M_d(x, y, t) = \frac{t}{t + d(x,\ y)},\ N_d(x, y, t) = \frac{d(x,\ y)}{t + d(x,\ y)}$$

Then (X, M, N, *,◊) is an intuitionistic fuzzy metric space. We call this intuitionistic fuzzy metric (M, N) induced by the metric d the standard intuitionistic fuzzy metric.

Definition 4[1]: Let (X, M, N, *, ◊) be an intuitionistic fuzzy metric space. Then
(a) a sequence $\{x_n\}$ in X is said to be Cauchy sequence if, for all t > 0 and p > 0,
$\lim_{n\to\infty} M(x_{n+p}, x_n, t) = 1$, $\lim_{n\to\infty} N(x_{n+p}, x_n, t) = 0$.

(b) a sequence $\{x_n\}$ in X is said to be convergent to a point x ∊ X if, for all t > 0,
$\lim_{n\to\infty} M(x_n, x, t) = 1$, $\lim_{n\to\infty} N(x_n, x, t) = 0$.

Since * and ◊ are continuous, the limit is uniquely determined from (v) and (xi) of definition (3), respectively.

Definition 5[1]: An intuitionistic fuzzy metric space (X, M, N, *,◊) is said to be Complete if and only if every Cauchy sequence in X is convergent.

Definition 6[7]: Let A and B be mappings from an intuitionistic fuzzy metric space (X, M, N, *,◊) into itself. The maps A and B are said to be compatible if, for all t > 0,

$$\lim_{n\to\infty} M\ (ABx_n, BAx_n, t) = 1 \text{ and } \lim_{n\to\infty} N\ (ABx_n, BAx_n, t) = 0$$

whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = x$ for some x∊ X.

Definition 7[7]: Let A and B be mappings from an intuitionistic fuzzy metric space (X, M, N,◊)*,into itself. The maps A and B are said to be semi-compatible if and only if

$\lim_{n\to\infty} M\ (ABx_n, Bx, t) = 1$ and $\lim_{n\to\infty} N\ (ABx_n, Bx, t) = 0$ for all t > 0,

whenever $\{x_n\}$ ∊ X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = x$, for all x∊ X.

Definition 8: Two self maps A and B in a intuitionistic fuzzy metric space (X,M,N, *, ◊) is said to be weak compatible if they commute at their coincidence points.

Lemma 1[1]: In intuitionistic fuzzy metric space X, M(x, y, .) is non-decreasing and N(x, y, .) is non-increasing for all x, y∊ X.

Lemma 2[7]: Let (X, M, N, *, ◊) be an intuitionistic fuzzy metric space. If there exists k ∊(0, 1) such that

M(x, y, kt) ≥ M(x, y, t) and N(x, y, kt) ≤ N(x, y, t) for x, y ∊X. then x = y.

Theorem: Let A, B, S, T, P and Q are self maps on a complete intuitionistic fuzzy metric space $(X, M, N, *,\Diamond)$ with t-norm * and t-conorm $\Diamond$ defined by $a*b = \min\{a, b\}$ and $a\Diamond b = \max\{a, b\}$ for all $a, b \in[0, 1]$. Satisfying:

(i) $P(X) \subseteq ST(X)$, $Q(X) \subseteq AB(X)$
(ii) AB=BA, ST=TS, PB=BP, QT=TQ
(iii) Either AB or P is continuous;
(iv) (P, AB) is compatible and (Q, ST) is weakly compatible;
(v) There exists $k \in(0, 1)$ such that

$$M(Px, Qy,kt)\geq \text{Min}\{M(ABx, Px, t), M(STy, Qy, t), M(STy, Px, \beta t), M(ABx, Qy, (2-\beta)t), M(ABx, STy, t)\}$$

and $$N(Px, Qy,kt) \leq \text{Max}\{N(ABx, Px, t), N(STy, Qy, t), N(STy, Px, \beta t), N(ABx, Qy, (2-\beta)t), N(ABx, STy, t)\}$$

For all $x, y \in X$, $\beta \in (0, 2)$ and $x, y>0$

Then A, B, S, T, P and Q have a unique common fixed point in X.

Proof: Let $x_0 \in X$, from condition (1) there exists $x_1, x_2 \in X$ such that $Px_0 =STx_1 =y_0$ and $Qx_1 =ABx_2 = y_1$ Inductively we can construct sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$Px_{2n} = STx_{2n+1} = y_{2n} \text{ and } Qx_{2n+1} = ABx_{2n+2} = y_{2n+1} \text{ for all } n= 0, 1, 2 \ldots$$

Step 1: Putting $x = x_{2n}$, $y = x_{2n+1}$ for all $x, y> 0$ and $\beta=1- q$ with $q \in(0, 1)$ in (5) we get,

$$M(Px_{2n}, Qx_{2n+1},kt) \geq \text{Min}\{M(ABx_{2n}, Px_{2n}, t), M(STx_{2n+1}, Qx_{2n+1}, t),M(STx_{2n+1}, Px_{2n}, \beta t), M(ABx_{2n}, Qx_{2n+1}, (2-\beta)t), M(ABx_{2n}, STx_{2n+1}, t)\}$$

$$M(y_{2n}, y_{2n+1}, kt) \geq \text{Min}\{M(y_{2n-1}, y_{2n}, t), M(y_{2n}, y_{2n+1}, t), 1, M(y_{2n-1}, y_{2n+1}, (1+q)t), M(y_{2n-1}, y_{2n}, t)\}$$

$$\geq \text{Min}\{M(y_{2n-1}, y_{2n}, t), M(y_{2n}, y_{2n+1}, t), M(y_{2n-1}, y_{2n}, t), M(y_{2n}, y_{2n+1}, t)\}$$

$$\geq \text{Min}\{M(y_{2n-1}, y_{2n}, t), M(y_{2n-1}, y_{2n}, t), M(y_{2n}, y_{2n+1}, t), M(y_{2n}, y_{2n+1}, t)\}$$

and $$N(Px_{2n}, Qx_{2n+1} ,kt) \leq \text{Max}\{N(ABx_{2n}, Px_{2n}, t), N(STx_{2n+1}, Qx_{2n+1}, t), N(STx_{2n+1}, Px_{2n}, \beta t), N(ABx_{2n}, Qx_{2n+1}, (2-\beta)t), N(ABx_{2n}, STx_{2n+1}, t)\}$$

$$N(y_{2n}, y_{2n+1}, kt) \leq \text{Max}\{N(y_{2n-1}, y_{2n}, t), N(y_{2n}, y_{2n+1}, t), 0, N(y_{2n-1}, y_{2n+1}, (1+q)t), N(y_{2n-1}, y_{2n},t)\}$$

$$\leq \text{Max}\{N(y_{2n-1}, y_{2n}, t), N(y_{2n}, y_{2n+1}, t), N(y_{2n-1}, y_{2n}, t), N(y_{2n}, y_{2n+1}, t)\}$$

$$\leq \text{Max}\{N(y_{2n-1}, y_{2n}, t), N(y_{2n-1}, y_{2n}, t), N(y_{2n}, y_{2n+1}, t), N(y_{2n}, y_{2n+1}, t)\}$$

As t-norm and t-conorm are continuous, letting $q\rightarrow 1$, we get,

$$M(y_{2n}, y_{2n+1}, kt) \geq \text{Min}\{M(y_{2n-1}, y_{2n}, t), M(y_{2n}, y_{2n+1}, t), M(y_{2n}, y_{2n+1}, t)\}$$

$$\geq \text{Min}\{M(y_{2n-1}, y_{2n}, t), M(y_{2n}, y_{2n+1}, t)\}$$

and $$N(y_{2n}, y_{2n+1}, kt) \leq \text{Max}\{N(y_{2n-1}, y_{2n}, t), N(y_{2n}, y_{2n+1}, t), N(y_{2n}, y_{2n+1}, t)\}$$

$$\leq \text{Man}\{N(y_{2n-1}, y_{2n}, t), N(y_{2n}, y_{2n+1}, t)\}$$

Hence, $M(y_{2n}, y_{2n+1}, kt) \geq \text{Min}\{M(y_{2n-1}, y_{2n}, t), M(y_{2n}, y_{2n+1}, t)\}$

and $N(y_{2n}, y_{2n+1}, kt) \leq \text{Max}\{N(y_{2n-1}, y_{2n}, t), N(y_{2n}, y_{2n+1}, t)\}$

Similarly, $M(y_{2n+1}, y_{2n+2}, kt) \geq \text{Min}\{M(y_{2n}, y_{2n+1}, t), M(y_{2n}, y_{2n+1}, t)\}$

and $N(y_{2n+1}, y_{2n+2}, kt) \leq \text{Max}\{N(y_{2n}, y_{2n+1}, t), N(y_{2n}, y_{2n+1}, t)\}$

Therefore, for all n even or odd we have,

$M(y_n, y_{n+1}, kt) \geq \text{Min}\{M(y_{n-1}, y_n, t), M(y_n, y_{n+1}, t)\}$ and $N(y_n, y_{n+1}, kt) \leq \text{Max}\{N(y_{n-1}, y_n, t), N(y_n, y_{n+1}, t)\}$

Consequently, $M(y_n, y_{n+1}, t) \geq \text{Min}\{M(y_{n-1}, y_n, k^{-1}t), M(y_n, y_{n+1}, k^{-1}t)\}$

and $N(y_n, y_{n+1}, t) \leq \text{Max}\{N(y_{n-1}, y_n, k^{-1}t), N(y_n, y_{n+1}, k^{-1}t)\}$

by repeated application of inequality, we get,

$$M(y_n, y_{n+1}, t) \geq \text{Min}\{M(y_{n-1}, y_n, k^{-1}t), M(y_n, y_{n+1}, k^{-m}t)\}$$

and $N(y_n, y_{n+1}, t) \leq \text{Max}\{N(y_{n-1}, y_n, k^{-1}t), N(y_n, y_{n+1}, k^{-m}t)\}$

Since $M(y_n, y_{n+1}, k^{-m}t)\to 1$ and $N(y_n, y_{n+1}, k^{-m}t)\to 0$ as $m\to\infty$, it follows that

$M(y_n, y_{n+1}, kt) \geq M(y_{n-1}, y_n, t)$ and $N(y_n, y_{n+1}, kt) \leq N(y_{n-1}, y_n, t)$ for all $n \in N$ and $x, y \in X$.

Therefore by lemma (2), $\{y_n\}$ is a Cauchy sequence in X. which is complete. Hence $\{y_n\}\to z \in X$. Also its subsequences converge as follows.

$\{Qx_{2n+1}\} \to z$ and $\{STx_{2n+1}\} \to z$ (3.1)

$\{Px_{2n}\} \to z$ and $\{ABx_{2n+1}\} \to z$ (3.2)

Case I: AB is continuous. As AB is continuous, $(AB)^2x_{2n} \to ABz$ and $(AB)Px_{2n} \to ABz$. As (P, AB) is compatible, we have $P(AB)x_{2n} \to ABz$.

Step 2: Putting $x = ABx_{2n}$, $y = x_{2n+1}$ with $\beta = 1$ in condition (5), we get

$$M(PABx_{2n}, Qx_{2n+1}, kt) \geq \text{Min}\{M(ABAx_{2n}, PABx_{2n}, t), M(STx_{2n+1}, Qx_{2n+1}, t), M(STx_{2n+1}, PABx_{2n}, t), M(ABABx_{2n}, Qx_{2n+1}, t), M(ABABx_{2n}, STx_{2n+1}, t)\}$$

and $N(PABx_{2n}, Qx_{2n+1}, kt) \leq \text{Max}\{N(ABAx_{2n}, PABx_{2n}, t), N(STx_{2n+1}, Qx_{2n+1}, t), N(STx_{2n+1}, PABx_{2n}, t), N(ABABx_{2n}, Qx_{2n+1}, t), N(ABABx_{2n}, STx_{2n+1}, t)\}$

Letting $n \to\infty$, we get,

$$M(ABz, z, kt) \geq \text{Min}\{M(ABz, ABz, t), M(z, z, t), M(z, ABz, t), M(ABz, z, t), M(ABz, z, t)\}$$

and $N(ABz, z, kt) \leq \text{Max}\{N(ABz, ABz, t), N(z, z, t), N(z, ABz, t), N(ABz, z, t), N(ABz, z, t)\}$

i.e. $M(ABz, z, kt)\geq M(ABz, z, t)$ and $N(ABz, z, kt)\leq N(ABz, z, t)$

Therefore by lemma (2), we get $ABz = z$. (3.3)

Step 3: Putting $x = z$, $y = x_{2n+1}$ with $\beta = 1$ in condition (5), we get,

$$M(Pz, Qx_{2n+1}, kt) \geq \text{Min}\{M(ABz, Pz, t), M(STx_{2n+1}, Qx_{2n+1}, t), M(STx_{2n+1}, Pz, t), M(ABz, Qx_{2n+1}, t), M(ABz, STx_{2n+1}, t)\}$$

and $N(Pz, Qx_{2n+1}, kt) \leq \text{Max}\{N(ABz, Pz, t), N(STx_{2n+1}, Qx_{2n+1}, t), N(STx_{2n+1}, Pz, t), N(ABz, Qx_{2n+1}, t), N(ABz, STx_{2n+1}, t)\}$

Letting $n\to\infty$, we get

$$M(Pz, z, kt) \geq \text{Min}\{M(z, Pz, t), M(z, z, t), M(z, Pz, t), M(Pz, z, t), M(Pz, z, t)\}$$

and $N(Pz, z, kt) \leq \text{Max}\{N(z, Pz, t), N(z, z, t), N(z, Pz, t), N(Pz, z, t), N(Pz, z, t)\}$

i.e. $M(Pz, z, kt)\geq M(Pz, z, t)$ and $N(Pz, z, kt) \leq N(Pz, z, t)$

Which gives $Pz = z$. Therefore $ABz = Pz = z$.

Step 4: Putting $x = Bz$, $y = x_{2n+1}$ with $\beta = 1$ in condition (5), we get,

$$M(PBz, Qx_{2n+1}, kt) \geq \text{Min}\{M(ABBz, PBz, t), M(STx_{2n+1}, Qx_{2n+1}, t), M(STx_{2n+1}, PBz, t), M(ABBz, Qx_{2n+1}, t)\}$$

and $N(PBz, Qx_{2n+1}, kt) \leq \text{Max}\{N(ABBz, PBz, t), N(STx_{2n+1}, Qx_{2n+1}, t), N(STx_{2n+1}, PBz, t), N(ABBz, Qx_{2n+1}, t)\}$

As BP = PB, AB = BA so we have P(Bz) = B(Pz) = Bz and AB(Bz) = B(ABz) = Bz.

Letting $n\to\infty$, we get, $M(Bz, z, kt) \geq \text{Min}\{M(Bz, z, t), M(z, z, t), M(z, Bz, t), M(Bz, z, t), M(Bz, z, t)\}$

$$\text{and } N(Bz, z, kt) \leq \text{Max}\{N(Bz, z, t), N(z, z, t), N(z, Bz, t), N(Bz, z, t), N(Bz, z, t)\}$$

i.e. $M(Bz, z, kt) \geq M(Bz, z, t)$ and $N(Bz, z, kt) \leq N(Bz, z, t)$

which gives Bz = z and ABz = z implies Az = z. Therefore Az = Bz = Pz = z. (3.4)

Step 5: $P(X) \subseteq ST(X)$, there exists $v \in X$ such that z=Pz=STv. Putting $x=x_{2n}$, $y=v$ with $\beta=1$ in condition (5), we get,

$$M(Px_{2n}, Qv, kt) \geq \text{Min}\{M(ABx_{2n}, Px_{2n}, t), M(STv, Qv, t), M(STv, Px_{2n}, t), M(ABx_{2n}, Qv, t), M(ABx_{2n}, STv, t)\}$$

and $N(Px_{2n}, Qv, kt) \leq \text{Max}\{N(ABx_{2n}, Px_{2n}, t), N(STv, Qv, t), N(STv, Px_{2n}, t), N(ABx_{2n}, Qv, t), N(ABx_{2n}, STv, t)\}$

Letting $n\to\infty$ and using eq^n. (3.2), we get,

$$M(z, Qz, kt) \geq \text{Min}\{M(z, z, t), M(z, Qv, t), M(z, z, t), M(z, Qz, t), M(z, z, t)\}$$

$$\text{and } N(z, Qz, kt) \leq \text{Max}\{N(z, z, t), N(z, Qv, t), N(z, z, t), N(z, Qz, t), N(z, z, t)\}$$

i.e. $M(z, Qz, kt) \geq M(z, Qz, t)$ and $N(z, Qz, kt) \leq N(z, Qz, t)$.

Therefore by lemma (2), Qv = z. Hence STv = Qv.

As (Q, ST) is weakly compatible, we have STQv = QSTv. Thus STz = Qz.

Step 6: Putting $x = x_{2n}$, $y = z$ with $\beta=1$ in condition (5), we get,

$$M(Px_{2n}, Qz, kt) \geq \text{Min}\{M(ABx_{2n}, Px_{2n}, t), M(STz, Qz, t), M(STz, Px_{2n}, t), M(ABx_{2n}, Qz, t), M(ABx_{2n}, STz, t)\}$$

and $N(Px_{2n}, Qz, kt) \leq \text{Max}\{N(ABx_{2n}, Px_{2n}, t), N(STz, Qz, t), N(STz, Px_{2n}, t), N(ABx_{2n}, Qz, t), N(ABx_{2n}, STz, t)\}$

Letting $n\to\infty$ and using eq^n. (3.1) and Step (5), we get,

$$M(z, Qz, kt) \geq \text{Min}\{M(z, z, t), M(Qz, Qz, t), M(Qz, z, t), M(z, Qz, t), M(z, Qz, t)\}$$

$$\text{and } N(z, Qz, kt) \leq \text{Max}\{N(z, z, t), N(Qz, Qz, t), N(Qz, z, t), N(z, Qz, t), N(z, Qz, t)\}.$$

i.e. $M(z, Qz, kt) \geq M(z, Qz, t)$ and $N(z, Qz, kt) \leq N(z, Qz, t)$

Hence z = Qz.

Step 7: Putting $x = x_{2n}$, $y = z$ with $\beta = 1$ in condition (5), we get,

$M(Px_{2n}, QTz, kt) \geq \text{Min}\{M(ABx_{2n}, Px_{2n}, t), M(STTz, QTz, t), M(STTz, Px_{2n}, t), M(ABx_{2n}, QTz, t), M(ABx_{2n}, STTz, t)\}$

and $N(Px_{2n}, QTz, kt) \leq \text{Max}\{N(ABx_{2n}, Px_{2n}, t), N(STTz, QTz, t), N(STTz,Px_{2n}, t), N(ABx_{2n},QTz, t),N(ABx_{2n}, STTz, t)\}$

As QT = TQ and ST = TS we have QTz = TQz = Tz and ST(Tz) = T(STz) = Tz.

Letting $n\to\infty$, we get,

$$M(z, Tz, kt) \geq \text{Min}\{M(z, z, t), M(Tz, Tz, t), M(Tz, z, t), M(z, Tz, t), M(z, Tz, t)\}$$

$$\text{and } N(z, Tz, kt) \leq \text{Max}\{N(z, z, t), N(Tz, Tz, t), N(Tz, z, t), N(z, Tz, t), N(z, Tz, t)\}$$

i.e. $M(z, Tz, kt) \geq M(z, Tz,t)$ and $N(z, Tz, kt) \leq N(z, Tz, t)$. Therefore by lemma (2), Tz = z.

Now STz = Tz = z implies Sz = z. Hence Sz = Tz = Qz = z. (3.5)

Combining (3.4) and (3.5), we get, Az = Bz = Pz = Qz = Tz = Sz = z.

Hence, the six self maps have a common fixed point in this case also.

Case II: P is continuous. As P is continuous, $P^2x_{2n} \to Pz$ and $P(ABx_{2n}) \to Pz$. As (P, AB) is compatible, we have

$$(AB)Px_{2n} \to Pz.$$

Step 8: Putting $x=Px_{2n}$, $y=x_{2n+1}$ with β= 1 in condition (5), we get,

$$M(PPx_{2n}, Qx_{2n+1}, kt) \geq \text{Min}\{M(ABPx_{2n}, PPx_{2n}, t), M(STx_{2n+1}, Qx_{2n+1}, t), M(STx_{2n+1}, PPx_{2n}, t), M(ABPx_{2n}, Qx_{2n+1}, t), M(ABPx_{2n}, STx_{2n+1}, t)\}$$

and $N(PPx_{2n}, Qx_{2n+1}, kt) \leq \text{Max}\{N(ABPx_{2n}, PPx_{2n}, t), N(STx_{2n+1}, Qx_{2n+1}, t), N(STx_{2n+1}, PPx_{2n}, t), N(ABPx_{2n}, Qx_{2n+1}, t), N(ABPx_{2n}, STx_{2n+1}, t)\}$.

Letting $n \to \infty$, we get,

$$M(Pz, z, kt) \geq \text{Min}\{M(Pz, z, t), M(z, z, t), M(z, Pz, t), M(Pz, z, t), M(Pz, z, t)\}$$

$$\text{and } N(Pz, z, kt) \leq \text{Max}\{N(Pz, z, t), N(z, z, t), N(z, Pz, t), N(Pz, z, t), N(Pz, z, t)\}$$

i.e. $M(Pz, z, kt) \geq M(Pz, z, t)$ and $N(Pz, z, kt) \leq N(Pz, z, t)$.

which gives Pz = z. now using Step (5) and (7) gives us Qz = STz = Sz = Tz = z.

Step 9: As $Q(X) \subseteq AB(X)$ there exists $w \in X$ suh that z=Qz=ABw. Putting x=w, $y=x_{2n+1}$ with β=1 in condition (5), we get,

$$M(Pw, Qx_{2n+1}, kt) \geq \text{Min}\{M(ABw, Pw, t), M(STx_{2n+1}, Qx_{2n+1}, t), M(STx_{2n+1}, Pw, t), M(ABw, Qx_{2n+1}, t), M(ABw, STx_{2n+1}, t)\}$$

and $N(Pw, Qx_{2n+1}, kt) \leq \text{Max}\{N(ABw, Pw, t), N(STx_{2n+1}, Qx_{2n+1}, t), N(STx_{2n+1}, Pw, t), N(ABw, Qx_{2n+1}, t), N(ABw, STx_{2n+1}, t)\}$

Letting $n \to \infty$, we get,

$$M(Pw, z, kt) \geq \text{Min}\{M(z, Pw, t), M(z, z, t), M(z, Pw, t), M(z, z, t), M(z, z, t)\}$$

and $N(Pw, z, kt) \leq \text{Max}\{N(z, Pw, t), N(z, z, t), N(z, Pw, t), N(z, z, t), N(z, z, t)\}$

i.e. $M(Pw, z, kt) \geq M(Pw, z, t)$ and $N(Pw, z, kt) \leq N(Pw, z, t)$

which gives Pw= z= ABw. As (P, AB) is weakly compatible.

We have Pz=ABz. Also Bz=z follows from Step 4.

Thus, Az=Bz=Pz=z and we obtain that z is the common fixed point of the six self maps in this case also.

Step 10: (Uniqueness) let u be another common fixed point of A, B, P, Q, S and T.

Then Au=Bu=Pu=Tu=Qu=Su=u. Putting x=z, y=u with β=1 in condition (5), we get,

$$M(Pz, Qu, kt) \geq \text{Min}\{M(ABz, Pz, t), M(STu, Pu, t), M(STu, Au, t), M(ABz, Qu, t), M(ABz, STu, t)\}$$

and $N(Pz, Qu, kt) \leq \text{Max}\{N(ABz, Pz, t), N(STu, Pu, t), N(STu, Au, t), N(ABz, Qu, t), N(ABz, STu, t)\}$

i.e. $M(z, u, kt) \geq M(z, u, t)$ and $N(z, u, kt) \leq N(z, u, t)$

which gives z=w. Therefore z is a unique common fixed point of A, B, P, Q, S and T.

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