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COMMUTATIVITY OF NONASSOCIATIVE RINGS WITH SOME IDENTITIES IN THE CENTER

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ABSTRACT

Let R be a nonassociative ring with center Z(R). In this paper, it is shown that a 2-torsion free nonassociative ring R with unity is commutative if it satisfies any one of the following identities:
(i) $(xy)^2+x^2y^2 \in Z(R)$
(ii) $(xy)^2+y^2x^2 \in Z(R)$
(iii) $(xy)^2+(xy^2)x \in Z(R)$
(iv) $x^2y^2+x(y^2x) \in Z(R)$
(v) $(yx^2)y-(xy)x \in Z(R)$
(vi) $(yx^2)y-x(yx) \in Z(R)$
(vii) $x^2y^2-x^2y-xy^2-xy \in Z(R)$
(viii) $x^2y^2-x^2y-xy^2-yx \in Z(R)$
(ix) $(xy)^2-x^2y-y^2x-x^2y^2 \in Z(R)$
(x) $(xy)^2-x^2y-y^2x-y^2x^2 \in Z(R)$ *for all x, y in R. Also we prove that a 2-torison free nonassociative ring R is commutative if it satisfies* $((xy)z)^2+(x^2y^2)z^2 \in Z(R)$ *or* $((xy)z)^2+(xy)^2z^2 \in Z(R)$ *for all x, y, z in R.*

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***Key words**: nonassociative ring, 2-torsion free ring, center.*

INTRODUCTION

R. D. Giri and A. K. Modi [1] have proved that if R is a 2-torison free nonassociative ring with unity satisfying the condition $(xy)^2-xy \in Z(R)$ for all x, y in R, then R is commutative. We generalize this by proving that if R is a 2-torsion free nonassociative ring with unity satisfying $(xy)^2+x^2y^2 \in Z(R)$ or $(xy)^2+y^2x^2 \in Z(R)$ for all x, y in R, then R is commutative. Also we prove that if R is a 2-torsion free nonassociative ring with unity satisfying $((xy)z)^2+(x^2y^2)z^2 \in Z(R)$ or $((xy)z)^2+(xy)^2z^2 \in Z(R)$ for all x, y, z in R. Then R is commutative. R.D. Giri and R.R. Rakhunde [2] have proved that if R is a nonassociative semi-simple ring with unity satisfying $x^2y^2-x^2y-xy^2+xy \in Z(R)$ for x,y in R, then R is commutative. We generalise this result as if R is a 2-torsion free nonassociative ring with unity satisfying $(xy)^2-x^2y-y^2x-x^2y^2 \in Z(R)$ or $(xy)^2-x^2y-y^2x-y^2x^2 \in Z(R)$ for all x, y in R, then R is commutative.

PRELIMINARIES

Throughout this paper, R represents a nonassociative ring with unity. The center Z(R) is defined as
$Z(R)=\{z \in R/[z,R]=0\}$ and a ring R is said to be n-torsion free if nx=0 implies x=0.

MAIN RESULTS

Lemma 1: Let R be a 2-torsion free nonassociative ring with unity satisfying $(xy)^2 \in Z(R)$ for all x,y in R then R is Commutative.

Proof: By hypothesis, $(xy)^2 \in Z(R)$ (1)
Replace x by x+1 in (1), we get
$\left((x+1)y\right)^2 \in Z(R)$
$(xy+y)^2 \in Z(R)$
$(xy)^2+y^2+(xy)y+y(xy) \in Z(R)$

Using (1) we get,
$y^2+(xy)y+y(xy) \in Z(R)$ (2)

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Replace x by x+1, we get
$y^2 + ((x+1)y)y + y((x+1)y) \in Z(R)$
$y^2 + (xy+y)y + y(xy+y) \in Z(R)$
$y^2 + (xy)y + y^2 + y(xy) + y^2 \in Z(R)$

Using (2) we get
$2y^2 \in Z(R)$

Since R is 2-torsion free, $y^2 \in Z(R)$ (3)

Replace y by y+1 and using (3), we get
$2y \in Z(R)$

Since 2-torsion free, we get $y \in Z(R)$

Hence R is Commutative.

Theorem 1: Let R be a 2-Torsion free nonassociative ring with unity satisfying
i) $(xy)^2 + x^2y^2 \in Z(R)$
ii) $(xy)^2 + y^2x^2 \in Z(R)$
iii) $(xy)^2 + (xy^2)x \in Z(R)$
iv) $x^2y^2 + x(y^2x) \in Z(R)$
v) $(yx^2)y - (xy)x \in Z(R)$
vi) $(yx^2)y - x(yx) \in Z(R)$
vii) $x^2y^2 - x^2y - xy^2 - xy \in Z(R)$
viii) $x^2y^2 - x^2y - xy^2 - yx \in Z(R)$
(ix) $(xy)^2 - x^2y - xy^2 - x^2y^2 \in Z(R)$
x) $(xy)^2 - x^2y - xy^2 - y^2x^2 \in Z(R)$ for all x, y in R then R is Commutative.

Proof: i) By hypothesis
$(xy)^2 + x^2y^2 \in Z(R)$ (4)

Replace x by x+1, we get
$((x+1)y)^2 + (x+1)^2y^2 \in Z(R)$
$(xy)^2 + y^2 + (xy)y + y(xy) + x^2y^2 + y^2 + 2xy^2 \in Z(R)$

Using (4), we get
$2y^2 + (xy)y + y(xy) + 2xy^2 \in Z(R)$ (5)

Replace x by x+1 in (5), we get
$2y^2 + ((x+1)y)y + y((x+1)y) + 2(x+1)y^2 \in Z(R)$
$2y^2 + (xy+y)y + y(xy+y) + 2xy^2 + 2y^2 \in Z(R)$
$2y^2 + (xy)y + y^2 + y(xy) + y^2 + 2xy^2 + 2y^2 \in Z(R)$

Using (5), we get
$4y^2 \in Z(R)$

Since R is 2-torsion free, we get
$y^2 \in Z(R)$ (6)

Replace y by xy in (6), we get
$(xy)^2 \in Z(R)$ for all x, y in R

By lemma 1, R is Commutative.

ii) By hypothesis
$(xy)^2 + y^2x^2 \in Z(R)$ (7)

Replace x by x+1, we get
$((x+1)y)^2 + y^2(x+1)^2 \in Z(R)$
$(xy)^2 + y^2 + (xy)y + y(xy) + y^2x^2 + y^2 + 2y^2x \in Z(R)$

Using (7), we get

$$2y^2 + (xy)y + y(xy) + 2y^2x \in Z(R) \quad (8)$$

Replace x by x+1, we get

$$2y^2 + ((x+1)y)y + y((x+1)y) + 2y^2(x+1) \in Z(R)$$
$$2y^2 + (xy+y)y + y(xy+y) + 2y^2x + 2y^2 \in Z(R)$$
$$2y^2 + (xy)y + y^2 + y(xy) + y^2 + 2y^2x + 2y^2 \in Z(R)$$

Using (8), we get

$$4y^2 \in Z(R)$$

Applying same argument as in i), R is commutative.

iii) By hypothesis

$$(xy)^2 + (xy^2)x \in Z(R) \quad (9)$$

Replace x by x+1, we get

$$((x+1)y)^2 + \left((x+1)y^2\right)(x+1) \in Z(R)$$
$$(xy+y)^2 + (xy^2+y^2)(x+1) \in Z(R)$$
$$(xy)^2 + y^2 + (xy)y + y(xy) + (xy^2)x + xy^2 + y^2x + y^2 \in Z(R)$$

Using (9), we get

$$2y^2 + (xy)y + y(xy) + xy^2 + y^2x \in Z(R) \quad (10)$$

Replace x by x+1, we get

$$2y^2 + ((x+1)y)y + y((x+1)y) + (x+1)y^2 + y^2(x+1) \in Z(R)$$
$$2y^2 + (xy)y + y^2 + y(xy) + y^2 + xy^2 + y^2 + y^2x + y^2 \in Z(R)$$

Using (10), we get

$$4y^2 \in Z(R)$$

Applying same argument as in i), R is commutative.

iv) By hypothesis

$$x^2y^2 + x(y^2x) \in Z(R) \quad (11)$$

Replace x by x+1, we get

$$(x+1)^2y^2 + (x+1)(y^2(x+1)) \in Z(R)$$
$$x^2y^2 + y^2 + 2xy^2 + x(y^2x) + xy^2 + y^2x + y^2 \in Z(R)$$

Using (11), we get

$$2y^2 + 2xy^2 + \ xy^2 + y^2x \in Z(R) \quad (12)$$

Replace x by x+1, we get

$$2y^2 + 2(x+1)y^2 + \ (x+1)y^2 + y^2(x+1) \in Z(R)$$
$$2y^2 + 2xy^2 + 2y^2 + xy^2 + y^2 + y^2x + y^2 \in Z(R)$$

Using (12), we get

$$4y^2 \in Z(R)$$

Applying same argument as in i), R is commutative.

v) By hypothesis $(yx^2)y - (xy)x \in Z(R)$ (13)

Replace x by x+1, we get

$$(y(x+1)^2)y - \left((x+1)y\right)(x+1) \in Z(R)$$
$$(yx^2 + y + yx + yx)y - (xy+y)(x+1) \in Z(R)$$
$$(yx^2)y + y^2 + (yx)y + (yx)y - (xy)x - xy - yx - y \in Z(R)$$

Using (13), we get

$$y^2 + (yx)y + (yx)y - xy - yx - y \in Z(R) \quad (14)$$

Replace x by x+1, we get
$y^2 + (y(x+1))y + (y(x+1))y - (x+1)y - y(x+1) - y \in Z(R)$
$y^2 + (yx)y + y^2 + (yx)y + y^2 - xy - y - yx - y - y \in Z(R)$

Using (14), we get
$2y^2 - 2y \in Z(R)$

Since R is 2-Torsion free, we get
$y^2 - y \in Z(R)$ (15)

Replace y by y+1, we get
$2y \in Z(R)$

Since R is 2-Torsion free, we get
$y \in Z(R)$ (16)

Using (15) and (16), we get
$y^2 \in Z(R)$ (17)

Replace x by xy in (17), we get
$(xy)^2 \in Z(R)$ for all x,y in R

By lemma 1, R is Commutative.

vi) By hypothesis $(yx^2)y - (xy)x \in Z(R)$

Applying the Same argument as in v), R is commutative.

vii) By hypothesis $x^2y^2 - x^2y - xy^2 - xy \in Z(R)$ (18)

Replace x by x+1, we get
$(x+1)^2y^2 - (x+1)^2y - (x+1)y^2 - (x+1)y \in Z(R)$
$x^2y^2 + y^2 + xy^2 + xy^2 - x^2y - y - xy - xy - xy^2 - y^2 - xy - y \in Z(R)$

Using (18), we get
$2(xy^2 - xy - y) \in Z(R)$

Since R is 2-Torsionfree ring, we get
$xy^2 - xy - y \in Z(R)$ (19)

Replace x by x+1, we get
$(x+1)y^2 - (x+1)y - y \in Z(R)$
$xy^2 + y^2 - xy - y - y \in Z(R)$

Using (19), we get
$y^2 - y \in Z(R)$ (20)

Replace y by y+1, we get
$(y+1)^2 - (y+1) \in Z(R)$
$2y \in Z(R)$

Since R is 2-torsionfree ring, we get
$y \in Z(R)$

Hence R is commutative.

viii) By hypothesis $x^2y^2 - x^2y - xy^2 - xy \in Z(R)$

Applying the Same argument as in vii), R is commutative

ix) By hypothesis $(xy)^2 - x^2y - xy^2 - x^2y^2 \in Z(R)$ (21)

Replace x by x+1, we get
$((x+1)y)^2 - (x+1)^2y - (x+1)y^2 - (x+1)^2y^2 \in Z(R)$
$(xy+y)^2 - (x^2+1+2x)y - xy^2 - y^2 - (x^2+1+2x)y^2 \in Z(R)$
$(xy)^2 + y^2 + (xy)y + y(xy) - x^2y - y - 2xy - xy^2 - y^2 - x^2y^2 - y^2 - 2xy^2 \in Z(R)$

Using (21), we get
$(xy)y + y(xy) - y - 2xy - y^2 - 2xy^2 \in Z(R)$ (22)

Replace x by x+1, we get
$\big((x+1)y\big)y + y\big((x+1)y\big) - y - 2(x+1)y - y^2 - 2(x+1)y^2 \in Z(R)$
$(xy)y + y^2 + y(xy) + y^2 - y - 2xy - 2y - y^2 - 2xy^2 - 2y^2 \in Z(R)$

Using (22), we get
$-2y \in Z(R)$
$2y \in Z(R)$

Since R is 2-torsion free, we get
$y \in Z(R)$ (23)

Hence R is commutative.

x) By hypothesis $(xy)^2 - x^2y - xy^2 - y^2x^2 \in Z(R)$

Applying the Same argument as in ix), R is commutative.

Theorem 2: Let R be a 2-torsion free nonassociative ring with unity satisfying xi) $\big((xy)z\big)^2 + (xy)^2z^2 \in Z(R)$ xii) $\big((xy)z\big)^2 + (x^2y^2)z^2 \in Z(R)$ for all x, y, z in R then R is commutativity.

Proof: xi) By hypothesis $\big((xy)z\big)^2 + (xy)^2z^2 \in Z(R)$ (24)

Replace z by z+1 in (24), we get
$\big((xy)(z+1)\big)^2 + (xy)^2(z+1)^2 \in Z(R)$
$((xy)z + xy)^2 + (xy)^2(z^2+1+2z) \in Z(R)$
$\big((xy)z\big)^2 + (xy)^2 + \big((xy)z\big)(xy) + (xy)\big((xy)z\big) + (xy)^2z^2 + (xy)^2 + 2(xy)^2z \in Z(R)$

Using (24), we get
$2(xy)^2 + \big((xy)z\big)(xy) + (xy)\big((xy)z\big) + 2(xy)^2z \in Z(R)$ (25)

Replace z by z+1 in (25), we get
$2(xy)^2 + \big((xy)(z+1)\big)(xy) + (xy)\big((xy)(z+1)\big) + 2(xy)^2(z+1) \in Z(R)$
$2(xy)^2 + \big((xy)z + xy\big)(xy) + (xy)\big((xy)z + xy\big) + 2(xy)^2z + 2(xy)^2 \in Z(R)$
$2(xy)^2 + \big((xy)z\big)(xy) + (xy)(xy) + (xy)\big((xy)z\big) + (xy)(xy) + 2(xy)^2z + 2(xy)^2 \in Z(R)$

Using (25), we get
$(xy)(xy) + (xy)(xy) + 2(xy)^2 \in Z(R)$
$4(xy)^2 \in Z(R)$ (26)

Since R is 2-torsion free, we get
$(xy)^2 \in Z(R)$

By using Lemma 1, R is commutative.

xi) By hypothesis $\big((xy)z\big)^2 + (x^2y^2)z^2 \in Z(R)$ (27)

Replace z by z+1 in (27), we get
$\big((xy)(z+1)\big)^2 + (x^2y^2)(z+1)^2 \in Z(R)$
$((xy)z + xy)^2 + (x^2y^2)(z^2+1+2z) \in Z(R)$
$\big((xy)z\big)^2 + (xy)^2 + \big((xy)z\big)(xy) + (xy)\big((xy)z\big) + (x^2y^2)z^2 + x^2y^2 + 2(x^2y^2)z \in Z(R)$

Using (24), we get

$$(xy)^2 + \big((xy)z\big)(xy) + (xy)\big((xy)z\big) + x^2y^2 + 2(x^2y^2)z \in Z(R) \tag{28}$$

Replace z by z+1 in (28), we get

$$(xy)^2 + \big((xy)(z+1)\big)(xy) + (xy)\big((xy)(z+1)\big) + x^2y^2 + 2(x^2y^2)(z+1) \in Z(R)$$
$$(xy)^2 + \big((xy)z + xy\big)(xy) + (xy)\big((xy)z + xy\big) + x^2y^2 + 2(x^2y^2)z + 2(x^2y^2) \in Z(R)$$
$$(xy)^2 + \big((xy)z\big)(xy) + (xy)(xy) + (xy)\big((xy)z\big) + (xy)(xy) + x^2y^2 + 2(x^2y^2)z + 2(x^2y^2) \in Z(R)$$

Using (28), we get

$$(xy)(xy) + (xy)(xy) + 2(x^2y^2) \in Z(R)$$
$$2(xy)^2 + 2(x^2y^2) \in Z(R)$$

Since R is 2-torsion free, we get

$$(xy)^2 + (x^2y^2) \in Z(R)$$

Applying same argument as in i), R is commutative.

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