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KALUZA-KLEIN BULK VISCOUS STRING COSMOLOGICAL MODELS IN LYRA MANIFOLD

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ABSTRACT

In this paper five dimensional string Cosmological models in presence of Cosmic string with constant Bulk viscous fluid in Lyra Manifold are constructed and some physical and kinematical behavior of these models are discussed.

***Key Words:** Five dimensions, Cosmic Strings, Lyra Manifold, Bulk viscous coefficient.*

1. INTRODUCTION

At present relativists are interested in theories with more than four dimensional space times. Alvarez et al ., [1], Randjbar – Daemi et al., [2] and Marciano [3] Suggested that the experimental detection of time variation of fundamental constants could provide strong evidence for the existence of extra dimensions. Kibble [4], Zeldovich [5] and Vilenkin [6] believed that strings may be one source of density that are required for the formation of large scale structures up the universe. It is a subject of considerable interest of cosmologists to study cosmic strings in the frame work of general relativity. For a physically realistic string models it is desirable that either strings fade away at a certain epoch of cosmic evolution or it has a particle dominated future asymptote with barely visible strings. This fact has attracted many researches Cchodos and Detweiler [7], Youshimura [8] and Chatterjee [9] to the field of higher dimensional cosmic string in various theories of gravitation. These higher dimensional theories constitute interesting candidates for unification of all interactions, including gravitation in the frame work of general relativity. Various authors Rahaman et al., 2003 [10], Reddy 2005 [11] constructed string Cosmological Models in Lyra's geometry. Singh et al.,[12] obtained exact solutions of the field equations for a five dimensional Cosmological models with bulk viscosity in Lyra geometry. The study of string theory is important in the early stages of the evolution of the universe before the particle creation. Cosmic strings have received considerable attention in cosmology as they are believed to give rise to density perturbations leading to the formation of galaxies. Rahman er al., [13] obtained exact solutions of the field equations for a five dimensional space time in Lyra manifold when the source of gravitation in massive strings, Mohanty et al., [14] constructed locally rotationally symmetric five dimensional string cosmological model with bulk viscous fluid in Lyra manifold and discussed some physical and geometrical properties of the model. Now a days it is conjectured that material distribution behaves like a viscous fluid during an early phase of the evolution of the universe when galaxies formed.G.F.R Ellis [15], Minser [16] studied the effect of viscosity in the evolution of the universe. Various aspects of this theory have been studied by authors like Sahu and Panigrahi [17] and recently by Panigrahi and Sahu [18]. The analog of Einstein's field equations based on Lyra's manifold as proposed by Sen [19] and Sen and Dunn [20] are

$$\mathrm{R}_{ij} - \frac{1}{2} g_{ij}\, \mathrm{R} + \frac{3}{2}\phi_i\, \phi_j - \frac{3}{4} g_{ij}\, \phi_m\, \phi^m = -\chi\, \mathrm{T}_{ij} \tag{1}$$

where ϕ is the displacement vector and other symbols have their usual meanings as in the Riemannian geometry.

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2. FIELD EQUATION AND THE COSMOLOGICAL MODEL

Here we consider the five dimensional spherically Symmetric metric in the form

$$ds^2 = dt^2 - e^{\lambda}(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta\, d\phi^2) - e^{\mu} dy^2 \tag{2}$$

where λ and μ are the function of time coordinate only.

We assume here that the coordinates to be commoving

$$\text{i.e., } u^0 = 1 \text{ and } u^1 = u^2 = u^3 = u^4 = 0. \tag{3}$$

Further we consider the displacement vector ϕ_i in the form $\phi_i = (\beta,0,0,0,0)$, where β is constant. (4)

The energy momentum tensor T_{ij} for cosmic string with bulk viscosity is

$$T_{ij} = \rho\, u_i u_j - \lambda_s x_i x_j - \xi\theta\,(u_i u_j - g_{ij}) \tag{5}$$

where ρ is the particle density, λ_s is the string tension density, ξ is the bulk viscous coefficient, u^i is the five velocity vector, g_{ij} is the covariant fundamental tensor, x^i is the direction of anisotropy of cosmic string satisfying

$$u_i u^i = -x_i x^i = 1 \tag{6}$$

$$\text{and } u_i x^i = 0. \tag{7}$$

Further the expansion scalar is given by

$$\theta = u^{\lambda};\lambda. \tag{8}$$

Using equation (4) (5) and (6) the explicit form of field equation (1) for the line element (2) are obtained as

$$\frac{3\dot{\lambda}^2}{4} + \frac{3}{4}\dot{\lambda}\dot{\mu} - \frac{3}{4}\beta^2 + \Lambda = \chi\,\rho, \tag{9}$$

$$\frac{3\dot{\lambda}^2}{4} + \ddot{\lambda} + \frac{\dot{\lambda}\dot{\mu}}{2} + \frac{\ddot{\mu}}{2} + \frac{\dot{\mu}^2}{4} + \frac{3}{4}\beta^2 = \chi\,\xi\,\theta \tag{10}$$

and

$$\frac{3\ddot{\lambda}}{2} + \frac{3\dot{\lambda}^2}{2} + \frac{3}{4}\beta^2 = \chi\,(\lambda_s + \xi\,\theta) \tag{11}$$

where over head dot denotes differentiation w.r.t. 't'. In the following section we intend to derive the exact solution of the field equations using β (constant) and ξ (constant) in order to overcome the difficulties due to non linear nature of the field equations.

3. COSMOLOGICAL SOLUTIONS

Here there are five unknowns viz., $\lambda, \mu, \rho, \theta$ and λ_s involved in three field equators $(9)-(11)$. In order to avoid the insufficiency of field equations for solving five unknowns through three field equations, we consider

$$\mu = a\lambda \tag{12}$$

where $a(\neq 0)$ is a parameter

Case I: cloud string ($\rho + \lambda_s = 0$)

The equation of state is given by

$$\rho + \lambda_s = 0. \tag{13}$$

Solving the equations (9), (10), (11) and (12) we get

$$\exp(\lambda) = \exp\left[\frac{1}{D}\left\{t - \frac{2}{A}\log\left(1 - C_1 e^{At}\right)\right\}\right] \tag{14}$$

and $$\exp(\mu) = \exp\left[\frac{a}{D}\left\{t - \frac{2}{A}\log\left(1 - C_1 e^{At}\right)\right\}\right] \tag{15}$$

where $$D = \frac{1}{\beta\left(\sqrt{\frac{3(a-1)}{2(a+2)(3-a)}}\right)}$$

and $$A = \frac{\beta}{(a-1)}\sqrt{3(a+2)(3-a)}. \tag{16}$$

$$\rho = \frac{9\,(1+a)\beta^2}{4\chi\,(2+a)(3-a)}\left(\frac{1+C_1 e^{At}}{1-C_1 e^{At}}\right)^2 + \frac{3}{4\chi}\beta^2, \tag{17}$$

$$\lambda_s = \frac{-9\,(1+a)\beta^2}{4\chi\,(2+a)(3-a)}\left(\frac{1+C_1 e^{At}}{1-C_1 e^{At}}\right)^2 + \frac{3}{4\chi}\beta^2 \tag{18}$$

and $$\theta = \left(\frac{3+a}{2}\right)\dot{\lambda} = \left(\frac{3+a}{2}\right)\frac{1}{D}\left(\frac{1+C_1 e^{At}}{1-C_1 e^{At}}\right)^2. \tag{19}$$

Case II: Geometric String $\rho = \lambda_s$ (Letelier[21])

In this case, due to paucity of one equation, an additional constraint relating these parameters is required to obtain explicit exact solution of the system of field equation. Therefore we take $\mu = a\lambda, a \neq 0$ is a parameter and the equation of state i.e. $\rho = \lambda_s$ Solving the equation (9), (10), (11) and (12) we get

$$\lambda = \beta\sqrt{\frac{3}{2(a^2+5a+12)}}\left\{t - \frac{2}{A}\log\left(1 - C_1 e^{At}\right)\right\},$$

$$\mu = a\,\beta\sqrt{\frac{3}{2(a^2+5a+12)}}\left\{t - \frac{2}{A}\log\left(1 - C_1 e^{At}\right)\right\}$$

$$\rho = \lambda_s = \frac{9\,\beta^2}{8\chi\,(a^2+5a+12)}\left(\frac{1+C_1 e^{At}}{1-C_1 e^{At}}\right)^2 [1+a] - \frac{3\beta^2}{4}$$

and $$\theta = \left(\frac{3+a}{2}\right)\dot{\lambda} = \left(\frac{3+a}{2}\right)\sqrt{\frac{3}{a(a^2+5a+12)}}\left\{\frac{1+C_1 e^{At}}{1-C_1 e^{At}}\right\}$$

Case III: P-String $\rho = (1+\omega)\lambda_s$

In this case P-String doesn't exist.

4. DISCUSSION

Some Physical and geometrical properties of the models:

(a) The anisotropy σ is defined as (Raychaudhuri, 1955)

$$\sigma^2 = \frac{1}{2}\left[\left(\frac{g_{00,0}}{g_{00}} - \frac{g_{11,0}}{g_{11}}\right)^2 + \left(\frac{g_{11,0}}{g_{11}} - \frac{g_{22,0}}{g_{22}}\right)^2 + \left(\frac{g_{22,0}}{g_{22}} - \frac{g_{33,0}}{g_{33}}\right)^2 + \left(\frac{g_{33,0}}{g_{33}} - \frac{g_{44,0}}{g_{44}}\right)^2 + \left(\frac{g_{44,0}}{g_{44}} - \frac{g_{00,0}}{g_{00}}\right)^2\right]$$

i.e., $\sigma^2 = \dot{\lambda}^2$

i.e., $\sigma = \dot{\lambda} = \frac{1}{D}\left(1 + \frac{2C_1 e^{At}}{1 - C_1 e^{At}}\right)$ where D is a constant

(b) Spatial Volume

$$V = (-g)^{1/2} = \left(-e^{3\lambda} r^4 \sin^2\theta\, e^{\mu}\right)^{1/2}$$

$$= -e^{\frac{3}{2}\lambda} r^2 \sin\theta\, e^{\frac{\mu}{2}}$$

where $\lambda = \frac{1}{D}\left[t - \frac{2}{A}\log\left(1 - C_1 e^{At}\right)\right]$

(c) Expansion Scalar

$$\theta = u^l_{;l} = \left(\frac{3+a}{2}\right)\dot{\lambda} = \left(\frac{3+a}{2}\right)\frac{1}{D}\left(\frac{1 + C_1 e^{At}}{1 - C_1 e^{At}}\right)$$

(d)
$$\frac{\sigma}{\theta} = \frac{\dot{\lambda}}{\left(\frac{3+a}{2}\right)\dot{\lambda}} = \frac{2}{3+a}$$

The model does not approaches to isotropy as $\sigma \neq 0$ and $\frac{\sigma}{\theta} = \text{constant}\ (\neq 0)$, for all t

Thus in this case the anisotropy exist through out the evolution.

when $t \to 0, \sigma = \text{constant}$

when $t \to \infty, \sigma = \text{constant}$

So the shape of the universe does not change during evolution. In case $C_1 = \frac{1}{2}$, $\lim t \to \infty, \sigma = 0$ and the model approaches to isotropy

(e) The deceleration parameter q is given by (Feinstein et al., 1995, [22])

$$q = -3\theta^2\left[\theta;_{\alpha}\upsilon^{\alpha} + \frac{1}{3}\theta^2\right]$$

$$= -3\left\{\frac{(3+a)^2}{4D^2}\frac{\left(1 + C_1 e^{At}\right)^4}{\left(1 - C_1 e^{At}\right)^6}\left[2C_1 A e^{4t} + \frac{\left(1 + C_1 e^{At}\right)^2}{3}\right]\right\}$$

for $A > 0,\ C > 0$ and $D > 0$, the Value of the deceleration parameters is negative, which indicates inflation in the model.

5. CONCLUSION

In this paper it is shown that the cosmological model for Takabayasi String does not exist in five dimensional string cosmological model with bulk viscous fluid in Lyra manifold, the equation of state for Takabayasi String viz. $\rho = (1+\omega)\lambda_s$ admits model only when $\omega = 0$. In case of constant bulk viscous coefficient, at the initial epoch $t = 0, \theta$ is finite and θ decreases when $t \to \infty$. The expansion in the models stops at infinite time. Thus there is finite expansion in the model.

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