

EFFECT OF HALL CURRENTS ON MHD FLOW OF VISCO-ELASTIC FLUID OF AN ARBITRARY INCLINED PLANE

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ABSTRACT

The effect of Hall currents on the flow of visco-elastic conducting fluid of Rivlin – Ericksen type flowing down an inclined plane in presence of a strong transverse magnetic field is investigated. This type of problem finds application in many technological and engineering fields such as rocket propulsion systems, space craft re-entry aerothermodynamics, cosmo flight aerodynamics, plasma physics, Glass production and furnace engineering. Velocity of the flow has been presented for various parameters. In this study velocity of fluid increases with the increase in m (Hall currents parameter), but it decreases with the increase in H (Hartmann number).

Key words: Rivlin-Ericksen fluid, Magnetic field, Hall Currents, An arbitrary inclined plane.

INTRODUCTION

Some hydromagnetic problems as investigated by Sengupta and Ghosh [1] and Sengupta and Bhattacharya [2] may be referred. Some fluids, sometimes exhibits various property of solids and viscous property of liquids are operation. Some problems associated with visco-elastic liquids have been considered by Sengupta and his research collaborators (Sengupta and Ghosh [3], Sengupta and Das [4], Sengupta and Kundu [5], Sengupta and Mukherjee [6]). Recently, Sultana and Ahmmed [7] have analysed MHD flow of visco-elastic fluid of an arbitrary inclined plane.

In the present paper we consider the problem Sultana and Ahmmed [7] with Hall currents. The purpose of this study is to investigate the effect of Hall currents on the flow of visco-elastic conducting fluid of Rivlin – Ericksen type flowing down an inclined plane in presence of a strong transverse magnetic field.

MATERIALS AND METHODS

Let us consider electrical conducting visco-elastic Rivlin-Ericksen type fluid between two parallel inclined planes, the lower plane is at rest and the upper plane is in motion. A strong transverse uniform magnetic field B_0 has been applied perpendicular to the time varying body force $F(t')$ taking the fluid initially at rest. The effect due to induced magnetic field and the perturbation of the magnetic field is neglected.

The equation of slow motion of a conducting visco-elastic Rivlin-Ericksen type fluid with Hall currents in two dimensional form become

$$\frac{\partial u'}{\partial t'} = g' \sin \alpha - \frac{1}{\rho} \frac{\partial p'}{\partial x'} + \bar{\nu} \frac{\partial^2 u'}{\partial y'^2} - \left(\frac{\sigma B_0^2}{\rho(1+m^2)} \right) u' \quad (1)$$

$$0 = g' \cos \alpha + \frac{1}{\rho} \frac{\partial p'}{\partial y'} \quad (2)$$

and the equation of continuity is

$$\frac{\partial u'}{\partial x'} = 0 \quad (3)$$

$$\text{where } \bar{\nu} = \frac{\mu(1+\lambda' \frac{\partial}{\partial t'})}{\rho} \quad (4)$$

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ρ is the density, g' is the acceleration due to gravity, σ is the electric conductivity, B_0 is the magnetic induction, m is Hall currents parameter, α is the inclination of the plane to the horizontal and h is the height between two parallel plates. So, $g' \sin \alpha - \frac{1}{\rho} \frac{\partial p'}{\partial x'}$ is a function of t' alone and therefore, we can write

$$g' \sin \alpha - \frac{1}{\rho} \frac{\partial p'}{\partial x'} = -F(t') \quad (5)$$

The value of F can be found out if the pressure is known at a given point $(x_0, 0)$. Now the equation (1) reduces to

$$\frac{\partial u'}{\partial t'} = -F(t') + \nu(1 + \lambda' \frac{\partial}{\partial t'}) \frac{\partial^2 u'}{\partial y'^2} - \left(\frac{\sigma B_0^2}{\rho(1 + m^2)} \right) u' \quad (6)$$

SOLUTION OF THE PROBLEM

We are now going to put the equation in a non-dimensional form by setting

$$u = \frac{u'}{U_0}, \quad p = \frac{p'}{\rho U_0^2}, \quad g = \frac{hg'}{U_0^2}, \quad t = \frac{t' U_0}{h}, \quad y = \frac{y'}{h}, \quad x = \frac{x'}{h}, \quad \lambda = \frac{\lambda' U_0}{h},$$

$$H = B_0 h \sqrt{\frac{\sigma}{\rho}} \quad (\text{Hartmann number})$$

$$R = \frac{h U_0}{\nu} \quad (\text{Reynolds number})$$

Thus the governing equation in non-dimensional form is

$$\frac{\partial u}{\partial t} = -F(t) + \frac{1}{R}(1 + \lambda \frac{\partial}{\partial t}) \frac{\partial^2 u}{\partial y^2} - \frac{Q}{R} u \quad (7)$$

where

$$Q = \frac{H^2}{1 + m^2}$$

Fluid motion due to transient body force

Firstly, we suppose that a transient force $F_0 e^{-\frac{\omega U_0 t}{h}}$ is applied to the fluid and the velocity u of the fluid is considered as

$$u = \frac{\bar{u}}{U_0} e^{-\frac{\omega U_0 t}{h}}$$

Putting $u = \frac{\bar{u}}{U_0} e^{-\frac{\omega U_0 t}{h}}$ and $F(t) = F_0 e^{-\frac{\omega U_0 t}{h}}$, the equation (7) takes the form

$$\frac{d^2 \bar{u}}{dy^2} + a_1^2 \bar{u} = \beta \quad (8)$$

where

$$a_1^2 = \frac{N}{\left(1 - \frac{\lambda \omega U_0}{h}\right)}$$

$$\beta = \frac{F_0 U_0 R}{\left(1 - \frac{\lambda \omega U_0}{h}\right)}$$

$$N = \frac{R\omega U_o}{h} - Q$$

The solution of the equation (8) is

$$u = \frac{1}{U_o} \left[A \cos a_1 y + B \sin a_1 y + \frac{RF_o U_o}{N} \right] e^{-\frac{\omega U_o t}{h}} \quad (9)$$

Applying boundary conditions

(i) $u = 0, y = 0$ for all t

(ii) $u = \frac{F_o U_o}{h} e^{-\frac{\omega U_o t}{h}}, y = 1, \forall t$

We get

$$A = \frac{-RF_o U_o}{N} \text{ And } B = \frac{F_o U_o}{\sin a_1} \left[\frac{U_o}{h} + \frac{R(\cos a_1 - 1)}{N} \right]$$

Therefore,

$$u = \frac{F_o}{N} \left[R(1 - \cos a_1 y) + \frac{U_o N \sin a_1 y}{h \sin a_1} + \frac{R(\cos a_1 - 1) \sin a_1 y}{\sin a_1} \right] e^{-\frac{\omega U_o t}{h}} \quad (10)$$

RESULTS AND DISCUSSION

Fluid Velocity Profiles are tabulated in Table-1 & 2 and plotted in Fig.-1 & 2 for $U_o = 0.2, F_o = 1, R = 15, \omega = 25, h = 0.5, y = 0.5, \lambda = 0.05$ and different values of H (Hartmann number) and m (Hall currents parameter).

It is observed from Fig.-1 & 2 that all velocity Graphs are decreasing sharply up to $t = 0.4$, then after velocity in each Graphs begins to decrease and tends to zero with the increase in t . It is also observed from Fig.-1 & 2 that velocity increases with the increase in m , but it decreases with the increase in H .

CONCLUSION

The velocity of fluid increases with the increase in m (Hall currents parameter).

PARTICULAR CASE

When m is equal to zero (Graph-1 of Fig.-2), this problem reduces to the problem of Sultana and Ahmmed [7].

Table-1: Values of velocity at $U_o = 0.2, F_o = 1, R = 15, \omega = 25, h = 0.5, y = 0.5, \lambda = 0.05, m = 0.2$ and different values of H .

t	Graph-1 (H = 6)	Graph-2 (H = 7)	Graph-3 (H = 8)
0	0.403385	0.231591	0.202174
0.2	0.054592	0.031342	0.027361
0.4	0.007388	0.004242	0.003703
0.6	0.001000	0.000574	0.000501
0.8	0.000135	0.000078	0.000068
1	0.000018	0.000011	0.000009

Table-2 : Values of velocity at $U_0 = 0.2$, $F_0 = 1$, $R = 15$, $\omega = 25$, $h = 0.5$, $y = 0.5$, $\lambda = 0.05$, $H = 6$ and different values of m .

t	Graph-1 (m = 0)	Graph-2 (m = 0.2)	Graph-3 (m = 0.5)
0	0.356583	0.403385	1.229827
0.2	0.048258	0.054592	0.166439
0.4	0.006531	0.007388	0.022525
0.6	0.000884	0.001000	0.003048
0.8	0.000120	0.000135	0.000413
1	0.000016	0.000018	0.000056

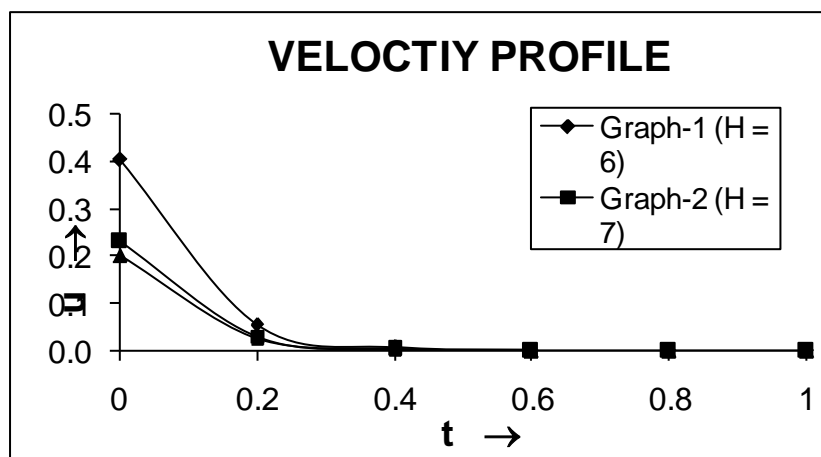


Fig.-1

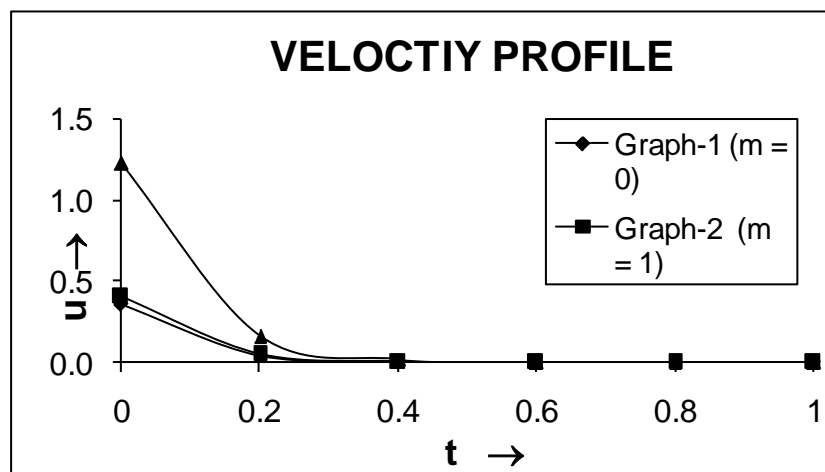


Fig.-2

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