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ELECTRICALLY CONDUCTING SECOND-GRADE FLUID FLOW IN A POROUS MEDIUM OVER A SHRINKING SHEET IN THE PRESENCE OF A CHEMICAL REACTION AND MAGNETIC FIELD

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ABSTRACT

An analysis is carried out to study the distribution of reactive species undergoing first-order chemical reaction in an incompressible homogeneous electrically conducting second-grade fluid over a shrinking surface in the presence of magnetic field applied normal to the plane of the flow. Using similarity variables, the boundary layer equations governing the flow and concentration field are reduced into a set of nonlinear ordinary differential equations which are then solved analytically for power-law surface concentration (PSC) as concentration boundary conditions. The analysis reveals that the velocity is getting more closure towards the wall for increasing magnetic, viscoelastic and porosity parameters. It is also found that the diffusion of reactive species is considerably reduced with increasing values of magnetic, reaction rate, viscoelastic and porosity parameters.

Keywords: *Shrinking sheet, Second-grade fluid, MHD, Porous medium, Chemically reactive species, Exact Solution*

1. INTRODUCTION

Boundary layer flow of viscoelastic fluid over a stretching sheet has huge applications in manufacturing industries like polymer sheet extrusion from a die, drawing of plastic films, glass fiber and paper production etc. Rajagopal et al. [1] independently examined the flow of a second order fluid over a stretching sheet and obtained similarity solutions of the boundary layer equations numerically. Later, this problem was extended by Rajagopal *et al.* [2] by introducing uniform free stream velocity in the problem formulation. Dandapat and Gupta [3] studied heat transfer of the problem considered by Rajagopal *et al.* [1]. Rollins and Vajravelu [4] solved the heat transfer problem in a second order fluid over a continuous stretching surface. Heat transfer in the viscoelastic fluid over a stretching sheet with different contexts were further studied by Lorence and Rao [5], Abel *et al.* [6] and others. Role of magnetic field in controlling momentum, heat and mass transfer in viscoelastic boundary layer flow over a stretching sheet were studied by Andersson [7], Char [8], Lawrence and Rao [9], Datti et al. [10] and others. The mass transfer along with chemical reaction in a viscoelastic boundary layer fluid flow is a hot research area now-a-days because of its huge engineering applications in polymer technology, metallurgy and chemical industries. Prasad *et al.* [11] studied the diffusion of chemically reactive species of a non-Newtonian fluid immersed in a porous medium over a stretching sheet. Recently, Cortell ([12], [13]) discussed the effects of magnetic field on the flow and mass transfer of a second grade fluid in porous medium over a stretching sheet with chemically reactive species and also explained the motion and mass transfer with chemically reactive species for two classes of viscoelastic fluid past a porous stretching sheet subjected to applied suction or blowing. The diffusion of first order chemical reaction on the viscoelastic fluid flow past an infinite vertical porous plate was studied by Damesh and Shannak [14].

On the other hand, the flow over a shrinking sheet is quite different from the stretching out case and it is a new field of research with wide prospects in various engineering applications. Recently, Miklavcic and Wang [15] obtained the existence and uniqueness of the solution for steady viscous hydrodynamic flow over a shrinking sheet with mass suction. Magnetohydrodynamic boundary layer flow of a second grade fluid over a shrinking sheet was studied by Hayat *et al.* [16]. They derived both exact and series solution using Homotopy Analysis Method (HAM). Sajid and Hayat [17] solved the problem of MHD viscous flow due to a shrinking sheet using HAM. Closed-form analytical solution for steady MHD flow over a porous shrinking sheet subjected to wall mass suction was obtained by Fang and Zhang [18]. Noor *et al.* [19] found a solution in the form of an infinite series of MHD viscous flow over a shrinking sheet by applying Adomian decomposition method (ADM). The boundary layer MHD flow and heat transfer over a permeable shrinking surface was studied by Midya [20]. Hayat *et al.* [21] analyzed the MHD flow and mass transfer of an upper-convected Maxwell fluid past a porous shrinking sheet with chemical reaction using the Homotopy Analysis Method (HAM). Effect of chemical reaction, heat and mass transfer on nonlinear boundary layer past a porous shrinking sheet in the presence of suction was studied numerically by Muhaimin *et al.* [22]. Midya [23] obtained an

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exact solution of the fluid flow and mass transfer over a shrinking sheet in the presence of chemical reaction and magnetic field. Recently, Van Gorder and Vajravelu [24] obtained multiple solutions for hydromagnetic flow of a second-grade fluid over a stretching or shrinking sheet with suction / injection. They derived the solution for fluid flow analytically. More recently, Midya [25] studied the diffusion of reactive species in a boundary layer flow of an incompressible homogeneous second order fluid over a linearly shrinking sheet in the presence of a transverse magnetic field.

In this work, we investigate the diffusion of chemically reactive species undergoing first-order chemical reaction in an electrically conducting second-grade fluid flowing in a porous medium over a linearly shrinking sheet in the presence of magnetic field. Using the similarity solution technique, the governing boundary layer partial differential equations are transformed into a set of nonlinear self-similar ordinary differential equations. After being substituted the analytical solution for fluid flow into the concentration equation, it is solved analytically for power-law surface concentration (PSC) as boundary conditions. Closed form exact solutions of the concentration equation are obtained in terms of Kummer's function. The effects of various parameters on the concentration distribution are analyzed and are presented graphically.

2. MATHEMATICAL FORMULATION

Consider the flow of an electrically conducting incompressible homogeneous fluid of second-grade over a flat plate coinciding with the plane y = 0. The flow is confined to y > 0. Two equal and opposite forces are applied opposite to the x -axis so that the wall is shrinked keeping the origin fixed. A magnetic induction B_0 is applied perpendicular to the shrinking surface. The shrinking sheet velocity is proportional to the distance i.e. $u_w = -ax$, (a > 0). Using boundary layer approximation and neglecting the induced magnetic field (by assuming the magnetic Reynolds number R_m for the flow to be very small i.e. $R_m << 1$ (see Midya et al. [26]), the equations for steady two-dimensional flow and the reactive concentration equation can be written in usual notation as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+\frac{\alpha_1}{\rho}\left(u\frac{\partial^3 u}{\partial x\partial y^2}+v\frac{\partial^3 u}{\partial y^3}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}-\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}\right)-\frac{\sigma B_0^2}{\rho}u-\frac{\nu}{k_p}u \tag{2}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 u}{\partial y^2}-r_1(C-C_\infty) \tag{3}$$

where u and v are the components of velocity respectively in the x and y directions, C is the concentration, C_∞ is the concentration far from the sheet, ρ is the fluid density (assumed constant), σ is the electrical conductivity of the fluid, ν (= μ/ρ) is the coefficient of fluid viscosity, D is the mass diffusion coefficient, α_1 (> 0) is the second-grade elastic parameter, k_p is the permeability of the porous medium, r_1 is the chemical reaction coefficient.

The boundary conditions for the velocity components and concentration are given by

$$u=-ax,\ \ v=0,\ \ C=C_w=C_\infty+Ax^p\quad at\ y=0 \tag{4}$$

and

$$u\longrightarrow 0,\ \ v\longrightarrow 0,\ \ C\longrightarrow C_\infty\quad at\ y\longrightarrow\infty \tag{5}$$

where C_w is the wall concentration.

3. SOLUTION OF THE PROBLEM

Equations (1) - (3) admit self-similar solutions of the form

$$u=axf'(\eta),\ \ v=-\sqrt{a\nu}f(\eta),\ \ \eta=y\sqrt{\frac{a}{\nu}},\ \ C=C_\infty+\theta(\eta)(C_w-C_\infty) \tag{6}$$

where f and θ are the dimensionless stream function and dimensionless temperature and η is the similarity variable. Substituting these Eqs. (2) and (3) become

$$\left(\frac{df}{d\eta}\right)^2-\frac{d^2f}{d\eta^2}=\frac{d^3f}{d\eta^3}+k\left(2\frac{df}{d\eta}\frac{d^3f}{d\eta^3}-\left(\frac{d^2f}{d\eta^2}\right)^2-f\frac{d^4f}{d\eta^4}\right)-\left(M^2+k'\right)\frac{df}{d\eta} \tag{7}$$

$$\frac{d^2\theta}{d\eta^2}+Scf\frac{d\theta}{d\eta}-Sc\left(\gamma+p\frac{df}{d\eta}\right)\theta=0 \tag{8}$$

where k' [=$\mu/(ak_p)$], M [=$\sqrt{\sigma B_0^2/(a\rho)}$], k [= $(a\alpha_1)/(\rho\nu)$ (> 0)], Sc [= ν/D] and γ [=r_1/a] represents respectively porosity parameter, magnetic parameter, visco-elastic parameter, Schmidt number and reaction rate parameter.

The boundary conditions are then reduces to

$$f'(0) = -1,\ \ f(0) = 0,\ \ and\ \ f'(\infty) = 0, \tag{9}$$

$$\theta(0) = 1,\ \ and\ \ \theta(\infty) = 0. \tag{10}$$

We now assume a solution to the nonlinear ordinary differential equation (7) in the form $f(\eta)=P+Qe^{-R\eta}$. This assumption is often motivated by the the result of Crane [27], where an exact solution was obtained for flow equation of Newtonian fluid flowing over a stretching sheet. Van Gorder and Vajravelu [24] also obtained an exact solution of the above form for hydromagnetic flow of a second-grade fluid over a stretching or shrinking sheet when porosity parameter $k' = 0$. In the present problem, we find that

$$f(\eta) = \frac{1}{\alpha}(e^{-\alpha\eta} - 1),\ \ \alpha = \sqrt{(M^2 + k' - 1)/(1 - k)} \tag{11}$$

is a solution of the equation (7) with associated boundary conditions (9). It is seen that this solution is valid for $M^2 + k' > 1$and $0< k < 1$ or $M^2+k' < 1$ and $k > 1$.

The non-dimensional horizontal velocity component is given by

$$f'(\eta) = -e^{-\alpha\eta} \tag{12}$$

The shear stress at the wall is denoted by τ_w and is defined as

$$\tau_w = \mu(\partial u/\partial y)_{y=0} = \mu a x\sqrt{\frac{a}{\nu}} f''(0) = \mu a x\sqrt{\frac{a}{\nu}}\ \alpha \tag{13}$$

The skin friction coefficient C_f at the wall is obtained as

$$C_f = \frac{\tau_w}{\left(\mu a x\sqrt{\frac{a}{\nu}}\right)} = f''(0) = \alpha \tag{14}$$

Now, substituting (11) into Eq. (8), we have

$$\frac{d^2\theta}{d\eta^2} + \frac{Sc}{\alpha}(e^{-\alpha\eta} - 1)\frac{d\theta}{d\eta} - Sc(\gamma - pe^{-\alpha\eta})\theta = 0 \tag{15}$$

Now, let us introduce a new variable $\xi = \frac{Sc}{\alpha^2}e^{-\alpha\eta}$ so that the above equation transforms to

$$\xi\frac{d^2\theta}{d\xi^2} + \left(1 + \frac{Sc}{\alpha^2} - \xi\right)\frac{d\theta}{d\xi} - \left(\frac{Sc\gamma}{\alpha^2\xi} - p\right)\theta = 0 \tag{16}$$

The boundary conditions (10) then become

$$\theta\left(\frac{Sc}{\alpha^2}\right) = 1,\ \ and\ \ \theta(0) = 0 \tag{17}$$

Now, transforming the above equation (16) into confluent hypergeometric equation, we can obtain the solution (see Abramowitz and Stegun [28]) given by

$$\theta(\xi) = (\alpha^2\xi/Sc)^{\beta}\,\Phi(\beta - p, 1 + b_0, \xi)/\Phi(\beta - p, 1 + b_0, Sc/\alpha^2), \tag{18}$$

where $\beta = (b_0 - a_0)/2$, $a_0 = Sc/\alpha^2$, $b_0 = \sqrt{a_0^2 + 4a_0\gamma}$ and $\boldsymbol{\Phi}(a', b', x)$ is the confluent hypergeometric function of the first kind or Kummer function.

Therefore,

$$\theta(\eta) = e^{-\alpha\beta\eta}\,\Phi(\beta - p, 1 + b_0, Sce^{-\alpha\eta}/\alpha^2)/\Phi(\beta - p, 1 + b_0, Sc/\alpha^2). \tag{19}$$

The dimensionless wall concentration gradient $\theta'(0)$ is obtained as

$$\theta'(0) = -\alpha\beta - \frac{Sc}{\alpha}\left(\frac{\beta - p}{1 + b_0}\right)\frac{\Phi(1 + \beta - p, 2 + b_0, Sc/\alpha^2)}{\Phi(\beta - p, 1 + b_0, Sc/\alpha^2)} \tag{20}$$

4. RESULTS AND DISCUSSION

In this section some examples will be shown and discussed for certain values of the controlling parameters.

The velocity distributions for various values of magnetic interaction parameter M is displayed in Figure 1(a) for fixed viscoelastic parameter k = 0.1 and porosity parameter $k^{/}$ = 0.1. It is seen that the velocity is going closer to the wall and boundary layer thickness becomes thinner for larger magnetic field parameter. The reason behind this is that increase in M results the increase in Lorentz force which in turn produce more resistance to the velocity field. As a result, the boundary layer thickness becomes thinner for larger magnetic field parameter.

Figure 1(b) presents the velocity curves for various values of viscoelastic parameter k for fixed magnetic field parameter M = 2 and viscoelastic parameter $k^{/}$ = 0.1. The figure reveals that velocity profiles are going closer to the wall and the boundary layer thickness becomes thinner for the increasing viscoelastic parameter k.

The effects of porosity parameter $k^{/}$ on the velocity field is shown in Figure 1(c) for fixed magnetic parameter M = 1.1 and k = 0.2. The figure reflects that velocity profiles are going closer to the wall and the same time, the boundary layer thickness is decreasing for increasing porosity parameter $k^{/}$. It is obvious that increase in porosity parameter leads to the increase in resistance in the velocity field and as a result the velocity goes closure to the wall.

Figure 2(a) demonstrates the effects of Schmidt number Sc on the concentration profile. It is seen that concentration is decreased with the enhanced values of Sc for M = 2, p = 0, k = 0.1, $k^{/}$ = 0.1, γ = 0.2. Physically it means that increase of Schmidt number Sc induces a decrease of molecular diffusivity D which in turn decrease in concentration boundary layer. Hence, concentration of the species is higher for small values of Sc and it is lower for higher values of Sc.

We shall now discuss the effect of reaction rate parameter σ on the diffusion of chemically reactive species and this is presented in Figure 2(b) for fixed values of k = 0.1, $k^{/}$ = 0.1, Sc = 0.6, M = 2 and p = 1. It is noticed from the graph that the thickness of concentration boundary layer decreases with increasing γ. This is due to the decelerating nature of reaction rate parameter. As a result it thins the boundary layer formed in the neighbourhood of the sheet.

Now, we shall draw our attention to the effects of solute distribution when the initial distribution of solute is varied over the sheet. The concentration profiles for different values of power-law exponent p are plotted in Figure 2(c) for M = 2, γ = 0.1, Sc = 0.6, k = 0.1, $k^{/}$ = 0.1. It is observed from the figure that the rate of species transfer is increased with the increase of distribution of concentration at the shrinking sheet. Concentration overshoot at the sheet is noticed for higher values of p.

Next, Figure 2(d) presents the concentration profiles for various values of M with fixed k = 0.1, $k^{/}$ = 0.1, Sc = 0.8, γ = 0.1, p = 1. The figure reveals that the value of concentration at a particular η is reduced with increasing values of magnetic field parameter M. Due to increase in the magnetic parameter M, the velocity boundary layer thickness becomes thinner and thinner and consequently decrease in concentration of reactive species in the shrinking sheet.

Now we concentrate in the diffusion of concentration boundary layer thickness for various values of viscoelastic parameter k and this is presented in Figure 2(e). Other parameters are kept fixed at M = 2, Sc = 0.5, γ = 0.05, p = 2, $k^{/}$ = 0.1. It is observed that concentration boundary layer thickness is decreased for enhanced values of viscoelastic parameter k.

The concentration boundary layer thickness for different values of the viscoelastic parameter $k^{/}$ is displayed in Figure 2(f) keeping all other parameters fixed at M = 2, Sc = 1.5, γ = 0.05, p = 3, k = 0.1. It is observed that concentration boundary layer thickness is decreased for increasing porosity parameter $k^{/}$.

The skin friction coefficient $f^{//}(0)$ and the concentration gradient on the wall is tabulated in the Tables I-VI. From the results of Tables I-III, it is found that the wall skin friction coefficient is increased and the wall concentration gradient is decreased for increasing values of M, k and $k^{/}$. From Tables IV - V, it is observed that wall concentration gradient is decreased with the increase of Sc and γ. Table VI indicates that concentration gradient at the wall is increased for increasing power-law index p.

5. CONCLUSION

The chemically reactive species distribution in an electrically conducting fluid of second grade over a shrinking sheet embedded in porous medium and subjected to transverse magnetic field is investigated in this work. The basic boundary layer equations of momentum and concentration field are converted into nonlinear ordinary differential equations by means of similarity transformations. The resulting nonlinear ordinary differential equations of momentum are solved exactly. The concentration equation is transformed to a confluent hypergeometric differential equation using a new variable and the solution is expressed in terms of Kummer's function. It is noticed that the rate of reactant transfer is decreased for increasing values of Schmidt number, viscoelastic, reaction rate, magnetic and porosity parameters. It is also seen that increase in power-law exponent leads to the increase in reactive species transport rate.

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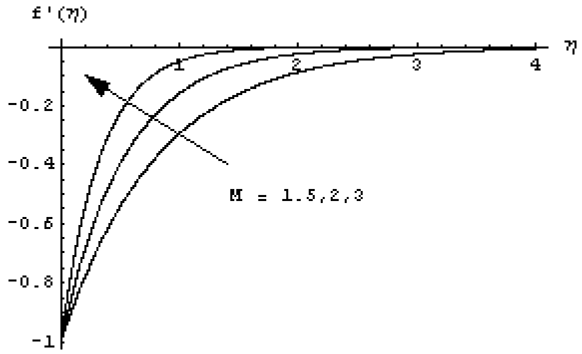

Figure 1(a): Velocity profiles $f^{/}(\eta)$ for various values of magnetic parameter M (M = 1.5,2,3) for k = 0.1 and $k^{/}$ = 0.1.

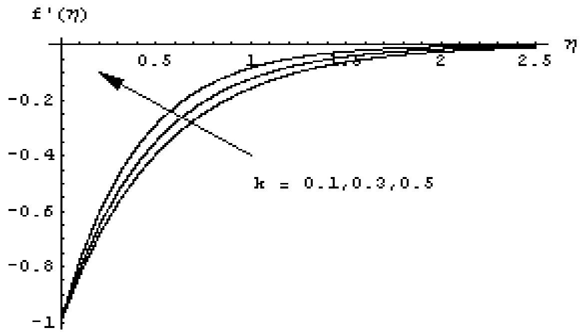

Figure 1(b): Velocity profiles $f^{/}(\eta)$ for various values of visco-elastic parameter k (k = 0.1,0.3,0.5) for M = 2 and $k^{/}$ = 0.1.

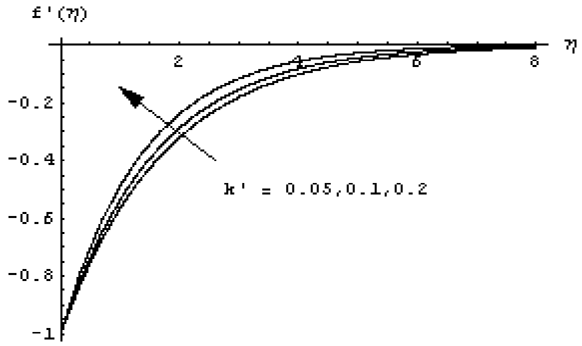

Figure 1(c): Velocity profiles $f^{/}(\eta)$ for various values of porosity parameter k ($k^{/}$ = 0.05, 0.1, 0.2) for M = 1.1 and k = 0.2.

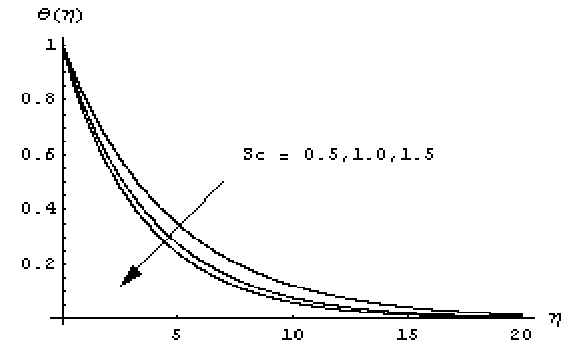

Figure 2(a): Variation of concentration for several values of Sc (Sc = 0.5, 1.0, 1.5), for M = 2, k = 0.1, $k^{/}$ = 0.1, γ = 0.2 and p = 0.

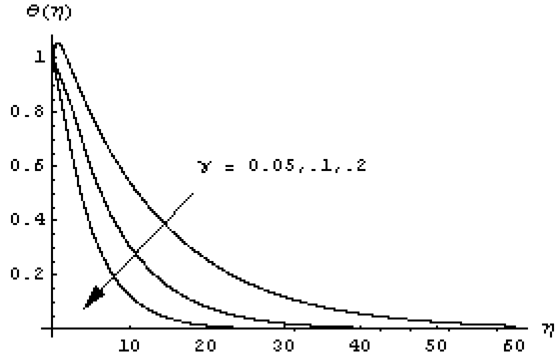

Figure 2(b): The concentration profiles for different values of γ (= 0.05, 0.1, 0.2), for M = 2, k = 0.1, $k^{/}$ = 0.1, Sc = 0.6 and p = 1.

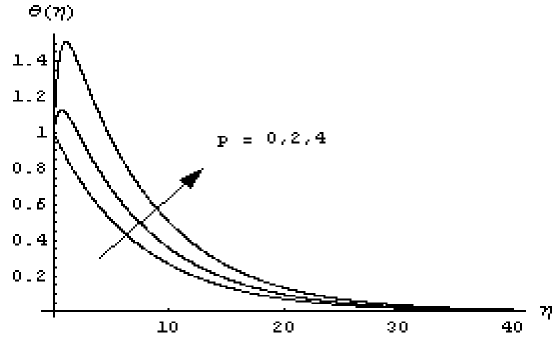

Figure 2(c): The concentration distribution for several values of p (p = 0,2,4) with M = 2, γ = 0.1 k = 0.1, $k^{/}$ = 0.1, Sc = 0.6.

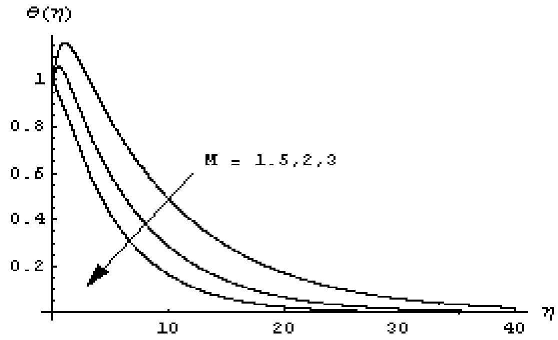

Figure 2(d): The concentration profiles for several values of M (M = 1.5, 2, 3) for γ = 0.1, k = 0.1, $k^{/}$ = 0.1, Sc = 0.8 and p = 1.

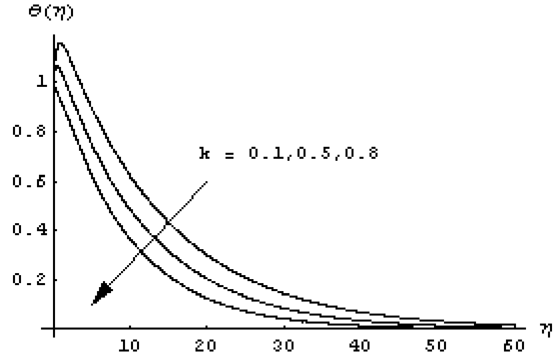

Figure 2(e): Variation of concentration for several values of k (k = 0.1, 0.5, 0.8) with M = 2, $k^{/}$ = 0.1, Sc = 0.5, γ = 0.05 and p = 2.

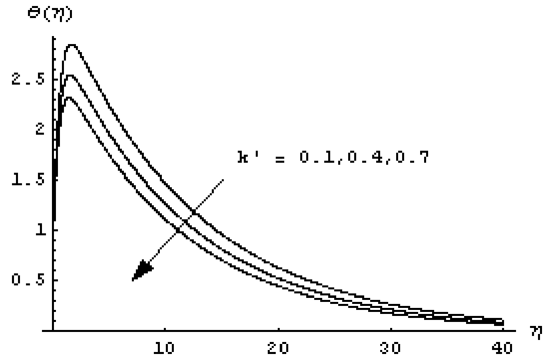

Figure 2(f): Variation of concentration for several values of $k^{/}$ ($k^{/}$ = 0.1, 0.4, 0.7) for M = 2, k = 0.1, Sc = 1.5, γ = 0.05 and p = 3.

Table I Effect of M on $f^{//}(0)$ and $\theta^{/}(0)$ with Sc=1, k=0.1, $k^{/}$=0.1, γ=0.2 and p=0

M	1.1	2	2.5	3
$f^{//}(0)$	0.58689	1.85592	2.43812	3
$\theta^{/}(0)$	-0.248512	-0.304831	-0.323599	-0.337964

Table II Effect of k on $f^{//}(0)$ and $\theta^{/}(0)$ with Sc=1, M=2, $k^{/}$=0.1, γ=0.2 and p=0

k	0.1	0.3	0.5	0.7
$f^{//}(0)$	1.85592	2.10442	2.48998	3.21455
$\theta^{/}(0)$	-0.304828	-0.31339	-0.325068	-0.342665

Table III Effect of $k^{/}$ on $f^{//}(0)$ and $\theta^{/}(0)$ with Sc=1, k=0.1, M=2, γ=0.2 and p=0

$k^{/}$	0.1	0.3	0.5	0.7
$f^{//}(0)$	1.85592	1.91485	1.97203	2.02759
$\theta^{/}(0)$	-0.304828	-0.306937	-0.308936	-0.310835

Table IV Effect of Sc on $\theta^{/}(0)$ with k = 0.1, $k^{/}$ = 0.1, M = 2, γ = 0.2 and p = 0

Sc	0.5	1.0	1.5	2.0	2.5
$\theta^{/}(0)$	-0.23245	-0.30483	-0.35719	-0.40019	-0.43756

Table V Effect of p on $\theta^{/}(0)$ with Sc = 1, $k^{/}$ = 0.1, M = 2, γ = 0.2 and k = 0.1

p	0	1	2	3	4
$\theta^{/}(0)$	-0.30483	0.1084	0.61116	1.25139	2.12071

Table VI Effect of γ on $\theta^{/}(0)$ with Sc = 1, $k^{/}$ = 0.1, M = 2, p = 0 and k = 0.1

γ	0.1	0.2	0.5	1	1.5
$\theta^{/}(0)$	-0.17885	-0.30483	-0.57247	-0.87961	-1.11465

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