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sα-closed sets in Bitopological spaces

Mrs. Veronica Vijayan* & K. Reena**

****Department of mathematics, Nirmala College for women, Coimbatore, India***
*****Department of mathematics, Sree Narayana Guru College, Coimbatore, India***

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ABASTRACT

In this paper, we introduce sα–closed sets in bitopological spaces. Properties of these sets are investigated and we introduce six new bitopological spaces namely, (i,j)-$T^{\sim}$, (i,j)-$T^{\sim s}$,(i,j)-${}^{\sim}T$, (i,j)-${}^{\sim s}T$, (i,j)-T^{α} ,(i,j)-$T^{\alpha\sim}$ spaces as applications. Further, we introduce and study (i,j)-sα–continuous and (i,j)-sα-irresolute maps.

***Keywords**: (i,j)-sα-closed sets, (i,j)-$T^{\sim}$, (i,j)-$T^{\sim s}$, (i,j)-${}^{\sim}T$, (i,j)- ${}^{\sim s}T$, (i,j)-T^{α} , (i,j)-$T^{\alpha\sim}$ spaces, (i,j)-sα –continuous and (i,j)-sα-irresolute maps.*

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1. INTRODUCTION

A triple (X ,τ_1, τ_2) where X is a nonempty set and τ_1 and τ_2 are topologies on X is called a bitopological space and Kelly initiated the study of such spaces. Levine introduced and studied semi-open sets and generalized closed sets in 1963 and 1970 respectively. S.P. Arya and T. Nour defined generalized semi-closed sets (briefly gs-closed sets) in 1990 for obtaining some characterizations of s-normal spaces. Njåstad and Abd El-Monsef et. al introduced α-sets (called as α-closed sets) and semi-preopen sets respectively. Semi-preopen sets are also known as β-sets. Maki et.al. introduced generalized α-closed sets (briefly gα-closed sets) and α-generalized closed sets (briefly αg-closed sets) in 1993 and 1994 respectively.

The purpose of this paper is to introduce the concepts of sα-closed sets, $T^{\sim}$ space, $T^{\sim s}$ space, ${}^{\sim}T$ space, ${}^{\sim s}T$ space, T^{α} space, $T^{\alpha\sim}$ space, sα-continuous and sα -irresolute maps for bitopological spaces and investigate some of their properties.

2. PREREQUISITES

Throughout this paper (X,τ_1, τ_2), (Y, σ_1, σ_2) and (Z, η_1, η_2) represent non-empty bitopological spaces on which no separation axioms are assumed unless otherwise mentioned. If A is a subset of X with topology τ then cl(A), int(A) and C(A) denote the closure of A, the interior of A and the complement of A in X respectively. We recall the following definitions, which will be used often throughout this paper.

Definition 2.1: A subset A of a space (X, τ) is called
(1) a preopen set if A $\subseteq$ int(cl(A)) and a preclosed set if cl(int(A)) $\subseteq$ A.
(2) a semi-open set if A $\subseteq$ cl(int(A)) and a semi-closed set if int(cl(A)) $\subseteq$ A.
(3) an α-open set if A$\subseteq$int(cl(int(A))) and a α-closed set if cl(int(cl(A)))$\subseteq$ A.
(4) a semi-preopen set (=β-open) if A $\subseteq$ cl(int(cl(A))) and a semi-preclosed set (=β-closed) if int(cl(int(A))) $\subseteq$ A.

The semi-closure (resp. α-closure) of a subset A of (X,τ) is denoted by scl(A) (resp. αcl(A) and spcl(A))and is the intersection of all semi-closed (resp. α-closed and semi-preclosed) sets containing **A.**

Definition 2.2: A subset A of a space (X,τ) is called
(1) a generalized closed (briefly g-closed) set[2][10] if cl(A) $\subseteq$ U whenever A $\subseteq$U and U is open in (X, τ).
(2) a generalized semi-closed (briefly gs-closed) set[3][3] if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X, τ).
(3) a generalized semi-preclosed (briefly gsp-closed) set[12][9] if spcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X, τ).
(4) an α-generalized closed (briefly αg-closed) set[8][12] if αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X, τ).
(5) a generalized α-closed (briefly gα-closed) set[7][13] if αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is α-open in (X, τ).

Corresponding author: K. Reena*
*****Department of mathematics, Sree Narayana Guru College, Coimbatore, India***

Definition 2.3: A function f: $(X, \tau) \to (Y, \sigma)$ is called
(1) *semi-continuous*[1] [11] if $f^{-1}(V)$ is semi-open in (X, τ) for every open set V of (Y, σ).
(2) *pre-continuous*[11] [14] if $f^{-1}(V)$ is pre-closed in (X, τ) for every closed set V of (Y, σ).
(3) *α-continuous*[12] [15] if $f^{-1}(V)$ is α-closed in (X, τ) for every closed set V of (Y, σ).
(4) *β-continuous*[5] [1] if $f^{-1}(V)$ is semi-preopen in (X, τ) for every open set V of (Y, σ).
(5) *g-continuous*[13] [4] if $f^{-1}(V)$ is g-closed in (X,τ) for every closed set V of (Y, σ).
(6) *gs-continuous*[14] [7] if $f^{-1}(V)$ is gs-closed in (X, τ) for every closed set V of (Y, σ).
(7) *αg-continuous*[2] [10] if $f^{-1}(V)$ is αg-closed in (X, τ) for every closed set V of (Y, σ).
(8) *gα-continuous*[7] [13] if $f^{-1}(V)$ is gα-closed in (X, τ) for every closed set V of (Y, σ).
(9) *gsp-continuous*[16] [9] if $f^{-1}(V)$ is gsp-closed in (X, τ) for every closed set V of (Y, σ).
(10) *αg-irresolute*[10] [6] if $f^{-1}(V)$ is αg-closed in (X,τ) for every αg-closed set V of (Y, σ).
(11) *pre-semi-open*[15] [5] if f (U) is semi-open in (Y,σ) for every semi-open set U in (X,τ).

Definition 2.4: A topological space (X, τ) is said to be
(1) a $T_{1/2}$ space if every g-closed set in it is closed.
(2) a T_b space if every gs-closed set in it is closed.
(3) an ${}_\alpha T_b$ space if every αg-closed set in it is closed.

Definition 2.5: A subset A of a bitopological space (X, τ_1, τ_2) is called:
(1) (i,j)-g–closed if τ_j–cl(A)$\subseteq$U whenever A$\subseteq$U and U is open in τ_i
(2) (i,j)-g*–closed if τ_j–cl(A)$\subseteq$U whenever A$\subseteq$U and U is g-open in τ_i
(3) (i,j)-rg–closed if τ_j–cl(A)$\subseteq$U whenever A$\subseteq$U and U is regular open in τ_i
(4) (i,j)-gpr–closed if τ_j–pcl(A)$\subseteq$U whenever A$\subseteq$U and U is regular open in τ_i

The family of all (i,j)-g–closed sets (resp. (i,j)-g*–closed, (i,j)-rg–closed, (i,j)-gpr–closed) subsets of a bitopological space (X,τ_1, τ_2) is denoted by D(i, j) (resp. D*(i, j), D_r(i, j),ξ(i, j)).

Definition 2.6: A subset A of a bitopological space (X ,τ_1, τ_2) is called:
(1) (i,j)-$T_{1/2}$ space if every (i,j)-g-closed sets is τ_j- closed.
(2) (i,j)-T_b space if every (i,j)-gs-closed set is τ_j - closed.
(3) (i,j)-${}_\alpha T_b$ space if every (i,j)-αg-closed set is τ_j - closed**.**

Definition 2.7: A function f : $(X,\tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called
(1) τ_j- *semi-continuous*[1]**[11]** if $f^{-1}(V)$ is semi-open in (X, τ_1, τ_2) for every open set V of (Y, σ_1, σ_2).
(2) τ_j- *α-continuous*[12]**[15]** if $f^{-1}(V)$ is α-closed in (X, τ_1, τ_2) for every closed set V of (Y, σ_1, σ_2).
(3) τ_j- σ_k – continuous if $f^{-1}(V) \in \tau_j$, for every $V \in \sigma_k$.
(4) (i,j)-*gs-continuous*[14]**[7]** if $f^{-1}(V)$ is gs-closed in $(X,\tau_1, \tau_2,)$ for every closed set V of (Y, σ_1, σ_2).
(5) (i,j)-*gsp-continuous*[14]**[7]** if $f^{-1}(V)$ is gsp-closed in $(X,\tau_1, \tau_2,)$ for every closed set V of (Y, σ_1, σ_2).

3. sα –closed sets in Bitopological spaces

In this section we introduce the concept of sα–closed sets in bitopological spaces and discuss the related properties**.**

Definition 3.1: A Subset A of a space (X,τ_i ,τ_j) is called a (i,j)-sα-closed set if τ_j -scl(A)$\subseteq$U whenever A$\subseteq$U and U is α-open in τ_i

Remark 3.2: By setting $\tau_i = \tau_j$ in Definition 3.1, a (i,j)-sα-closed set is a sα-closed set.

Theorem 3.3:
1. If A is τ_j – closed subset of (X,τ_i ,τ_j) then A is (i,j)-sα-closed.
2. If A is τ_j –semi closed subset of (X,τ_i ,τ_j) then A is (i,j)-sα-closed.
3. If A is τ_j –α closed subset of (X,τ_i ,τ_j) then A is (i,j)-sα-closed.
4. Every (i,j)-gα-closed set is (i,j)-sα-closed.
5. Every (i,j)-w-closed set is (i,j)-sα-closed.

Proof: Straight forward. Converse of the above need not be true as in the following examples.

Example 3.4: Let X={a, b, c} , τ_1 = {ϕ,X,{a, b}}, τ_2 = {ϕ,X,{a}} then {b} is (1,2)-sα –closed but not τ_2–closed.

Example 3.5: Let X={a, b, c} , τ_1 = {ϕ,X,{a ,b}}, τ_2 = {ϕ,X,{a}} then {a, c} is (1,2)-sα –closed but not τ_2 –semi closed **.**

Example 3.6: Let X={a, b, c} , τ_1 = {ϕ, X, {a}, {b, c}}, τ_2 = {ϕ,X,{a},{a, c}} then{a, b} is (1,2)-sα -closed but not τ_2-α -closed

Example 3.7: Let X={a, b, c} , τ_1 = {ϕ,X,{a}}, τ_2 = {ϕ,X,{a},{b}, {a, b}} then{a} is (1,2)-sα -closed but not (1,2) -gα -closed .

Example 3.8: Let X={a, b, c} , τ_1 = {ϕ,X,{a, b}}, τ_2 = {ϕ,X,{a}} then{b} is (1,2)-sα -closed but not (1,2) -w-closed.

Thus the class of (i,j)- sα -closed sets properly contains the classes of τ_j –closed sets, τ_j -α-closed sets,τ_j –semi-closed sets, (i,j)-gα-closed sets, (i,j)-w-closed sets.

Theorem 3.9: In a bitopological space (X,τ_i ,τ_j), every (i,j)-sα-closed set is :
1. (i,j)-gs-closed and
2. (i,j)-gsp-closed.

Proof: follows from the definitions.

The following examples show that the reverse implications of above proposition are not true.

Example 3.10: Let X={a, b, c}, τ_1 = {ϕ,X,{a},{a, c}}, τ_2 ={ϕ,X,{a},{b, c}} then {b} is (1,2)-gs-closed but not (1,2)-sα –closed

Example 3.11: Let X={a, b, c}, τ_1 ={ϕ, X, {a}}, τ_2 ={ϕ, X,{a},{b, c}} then {b} is (1,2)-gsp-closed but not (1,2) -sα -closed.

So the class of (i,j)- sα-closed sets is properly contained in the classes of (i,j)-gs-closed and (i,j)-gsp-closed sets .

The following examples shows that (i,j)-sα-closedness is independent from (i,j)- αg-closedness, (i,j)-rg-closedness, (i,j)-gp-closedness , (i,j)-gpr-closedness**.**

Example 3.12: Let X={a, b, c}, τ_1 ={ϕ,X,{a}}, τ_2 ={ϕ,X,{a},{b},{a, b}} then the set {a, b} is (1,2)- αg-closed set, (1,2)-rg-closed set, (1,2)-gp-closed set, (1,2)-gpr-closed set but not (1,2)-sα -closed.

Proposition 3.13: If A is (i,j)-sα-closed set such that A$\subseteq$B$\subseteq$$\tau_j$-Scl(A) then B is also (i,j)-sα-closed.

Proof: Follows

Proposition 3.14: If A is (i,j)-sα-closed then τ_j-Scl(A) – A contains no non-empty τ_i-α-closed set.

Proof: Let A be an (i,j)-sα-closed set and F be a non-empty τ_i -α-closed subset such that F$\subseteq$$\tau_j$-Scl(A) – A = τ_j-Scl(A)$\cap$$A^c$ $\therefore$ F$\subseteq$$\tau_j$-Scl(A) and F$\subseteq$$A^c$ Since F^c is τ_i -α-open and A is (i,j)-sα -closed we have, τ_j-Scl(A) $\subseteq$ F^c i.e F $\subseteq$ $(\tau_j\text{-Scl}(A))^c$ Hence F$\subseteq$ τ_j-Scl(A) $\cap$ $(\tau_j\text{-Scl}(A))^c$ = ϕ

$\therefore$ τ_j-Scl(A) – A contains no non-empty τ_i -α-closed set

Corollary 3.15: If A is (i,j)-sα-closed set in (X, τ_i, τ_j), then A is τ_j –semi-closed iff τ_j-Scl(A) – A is τ_i -α-closed.

Proof: Necessity: If A is τ_j-semi-closed then τ_j-Scl(A)=A i.e τ_j-Scl(A) – A = ϕ and hence τ_j-Scl(A) – A is τ_i -α-closed. [By prop.3.14]

Sufficiency: If τ_j-Scl(A)–A is τ_i -α-closed then by proposition 3.14 we have,τ_j-Scl(A) – A = ϕ [since A is (i,j)-sα -closed]
$\therefore$ τ_j-Scl(A) = A. Hence A is τ_j – semi-closed.

Proposition 3.16: For each element x of (X, τ_i, τ_j), {x} is τ_i -α-closed (or) $\{x\}^c$ is (i,j)-sα -closed.

Proof: If {x} is not τ_i -α-closed then the only τ_i -α-open set containing X-{x} is X Thus X-{x} is (i,j)-sα -closed. i.e $\{x\}^c$ is (i,j)-sα-closed. Hence Proved**.**

Proposition 3.17: If A is an τ_i -α-open and (i,j)-sα-closed set of (X, τ_i , τ_j) then A is τ_j –semi-closed.

Proof: Let A be τ_i-α-open and (i,j)-sα-closed. Since A is (i,j)-sα-closed, we have τ_j–scl(A) ⊆ U whenever A⊆U and U is τ_i -α-open ⇒ τ_j–scl(A) = A ⇒A is τ_j–semi-closed.

Remark 3.18: An (i,j)-sα-closed set need not be (j,i)-sα-closed.

Proof: Consider the Example Let X={a, b, c} , τ_1 = {ϕ,X,{c},{a, b}}, τ_2 = {ϕ,X,{a}} then {a, c} is (1,2)-sα –closed but not (2,1)-sα –closed

Definition 3.19: The family of all (i,j)-sα-closed set in (X, τ_i, τ_j) is defined as D^s(i,j)

Proposition 3.20: If A, B∈ D^s(i,j) then A∪B∈ D^s(i,j)

Proof: Follows.

Proposition 3.21: The intersection of (i,j)-sα-closed sets need not be (i,j)-sα-closed as seen from the following example.

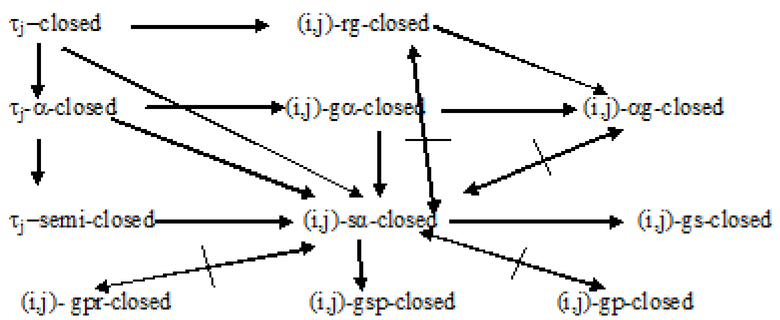
Example 3.22: Let X={a, b, c}, τ_1 ={ϕ,X,{a},{b, c}}, τ_2 ={ϕ,X,{a},{a, c}} then the sets {a, b}, {a, c} are (1,2)- sα-closed but {a} is not a (1,2)- sα-closed set.

Remark 3.23: The family of all τ_j–semi-closed set is denoted by F_j **.**

Theorem 3.24: In a bitopological space (X,τ_i ,τ_j), αo(X, τ_i)⊆ F_j. Iff every subset of X is an (i,j)-sα-closed set.

Proof: Follows.

The following figure shows the relationships of (i,j)-sα-closed sets with other sets



Where A ⟶ B represents A implies B and A ⟷ B represents A and B are independent.

4. Applications of (i,j)-sα-closed Set

In this chapter we introduce six new spaces namely (i,j)-T˜ space, (i,j)-T˜s space ,(i,j)-˜T space, (i,j)- ˜sT space, (i,j)-T^{α} space , (i,j)-$T^{\alpha\sim}$ space.

We now introduce a new space (i,j)-T˜ space.

Definition 4.1: A space (X, τ_i, τ_j) is called an (i,j)-T˜ space if every (i,j)-sα-closed set is τ_j–closed.

Proposition 4.2: Every (i,j)-T_b space is an (i,j)-T˜ space but not conversely.

Proof: follows

The converse of above proposition need not be true which is shown by the following example.

Example 4.3: Consider the example X={a,b,c},τ_1={ϕ,X,{a}},τ_2 ={ϕ,X,{a},{b,c}} then (X,τ_1 ,τ_2) is (1,2)-T˜ space but not (1,2)-T_b–space.

Characterization of (i,j)-$T^{\sim}$ space

Theorem 4.4: If (X, τ_i , τ_j) is an (i,j)-$T^{\sim}$ space, then every singleton of X is either τ_i -α-closed or τ_j -open

Proof: Let $x \in X$ and suppose that {x} is not τ_i -α-closed. Then X-{x} is (i,j)-sα-closed set since X is the only τ_i -α-open set containing X-{x}.So X-{x} is τ_j-closed.(i.e) {x} is τ_j -open

Remark 4.5: (X, τ_1) space is not generally $T^{\sim}$ space even if (X,τ_1 ,τ_2 is (1,2)-$T^{\sim}$ space shown in the following example.

Example 4.6: Consider the example X={a, b, c},τ_1={ϕ,X,{a}},τ_2 ={ϕ,X,{a},{b, c}} then (X,τ_1 ,τ_2) is (1,2)-$T^{\sim}$ space but (X,τ_1) is not $T^{\sim}$ –space.

We now introduce a new space (i,j)-$T^{\sim s}$

Definition 4.7: A space (X, τ_i , τ_j) is called (i,j)-$T^{\sim s}$ space if every (i,j)-sα-closed set is τ_j –semi closed.

Proposition 4.8: Every (i,j)-T_b space is an (i,j)-$T^{\sim s}$ space but not conversely.

Proof: follows.

Example 4.9: Let X={a,b,c}, τ_1 ={ϕ,X,{a}}, τ_2 ={ϕ,X,{a},{b},{a,b}} then (X, τ_1, τ_2) is (1,2)-$T^{\sim s}$ space but not (1,2)-T_b space**.**

Proposition 4.10: Every (i,j)-$T_{1/2}$ space is an (i,j)-$T^{\sim s}$ space but not conversely.

Proof: follows.

Example 4.11: Let X={a, b, c}, τ_1 ={ϕ,X,{a}}, τ_2 ={ϕ,X,{a},{b, c}} then (X, τ_1, τ_2) is (1,2)-$T^{\sim s}$ space but not (1,2)-$T_{1/2}$ space**.**

Proposition 4.12: Every (i,j)-$T^{\sim}$ space is (i,j)-$T^{\sim s}$ space but not conversely.

Proof: Follows

Example 4.13: Let X={a, b, c}, τ_1 ={ϕ,X,{a}}, τ_2 ={ϕ,X,{a},{b}, {a, b}}. Then (X, τ_1, τ_2) is (1, 2)-$T^{\sim s}$ space but not (1,2)-$T^{\sim}$ space

Characterization of (i,j)-$T^{\sim s}$ space

Theorem 4.14: For a space (X,τ_i ,τ_j) the following are equivalent.
1. (X, τ_i , τ_j) is a (i,j)-$T^{\sim s}$ space
2. Every singleton of X is either τ_i -α-closed or τ_j –semi open.

Proof: To Prove (1)⇒(2) Let $x \in X$ and suppose that {x} is not τ_i -α-closed. Then X-{x} is (i,j)-sα-closed set since X is the only τ_i -α-open set containing X-{x}. Therefore X-{x} is τ_j – semi-closed.(i.e) {x} is τ_j - semi-open

To Prove (2)⇒(1) Let A be a (i,j)-sα-closed set of (X, τ_i , τ_j).Clearly $A \subseteq \tau_j$ – scl(A)**.**Let $x \in X$. by (2) {x} is either τ_i -α-closed or τ_j - semi-open

Case (i): Suppose {x} is τ_i -α-closed. If $x \notin A$, then τ_j – scl (A)-A contains the τ_i -α-closed set {x} and A is (i,j)-sα-closed set. Hence we arrive at a contradiction. Thus $x \in A$.

Case (ii): Suppose that {x} is τ_j - semi-open. Since $x \in \tau_j$ – scl(A), then $\{x\} \cap A \neq \phi$. So $x \in A$. Thus in any case $x \in A$. So τ_j – scl(A)$\subseteq$A $\therefore$ A=τ_j – scl(A) (or) equivalently A is τ_j - semi-closed. Thus (X, τ_i , τ_j) is an (i,j)- $T^{\sim s}$ space.

Definition 4.15: A space (X, τ_i , τ_j) is called strongly pairwise $T^{\sim s}$ space if it is both (1,2)- $T^{\sim s}$ and (2,1)- $T^{\sim s}$

Proposition 4.16: If (X, τ_1 , τ_2) is strongly pairwise T_b space then it is strongly pairwise $T^{\sim s}$ space but not conversely.

Proof: follows

Example 4.17: Let X={a, b, c}, τ_1 ={ϕ,X,{a},{a, b}}, τ_2 ={ϕ,X,{a}} then (X, τ_1, τ_2) is strongly pairwise $T^{\sim s}$ space but not strongly pairwise T_b space**.**

We introduce another new space (i,j)-$^{\sim}T$ space

Definition 4.18: A space (X, τ_i , τ_j) is called (i,j)- $^{\sim}T$ space if every (i,j)-sα-closed set is τ_j –α-closed

Proposition 4.19: Every (i,j)-T_b space is (i,j)-$^{\sim}T$ space but not conversely.

Proof: Let (X,τ_i ,τ_j) be a (i,j)-T_b space and A be a (i,j)-sα-closed set of (X,τ_i ,τ_j). Since(X, τ_i , τ_j) is a (i,j)-T_b space, A is τ_j –closed. Since every τ_j –closed set is τ_j –α-closed set. Implies A is τ_j –α-closed∴ (X, τ_i , τ_j) is a (i,j)- $^{\sim}T$ space.

Example 4.20: Let X={a, b, c}, τ_1 ={ϕ,X,{a},{b},{a, b}}, τ_2 ={ϕ,X} then (X, τ_1 , τ_2) is (1,2)- $^{\sim}T$ space but not (1,2)-T_b space.

Proposition 4.21: Every (i,j)- $^{\sim}T$ space is (i,j)-$T^{\sim s}$ space but not conversely.

Proof: follows

Example 4.22: Let X={a, b, c}, τ_1 ={ϕ,X,{a}}, τ_2 ={ϕ,X,{a},{b},{a, b}} then (X, τ_1 , τ_2) is (1,2)- $T^{\sim s}$ space but not (1,2)- $^{\sim}T$ space.

Proposition 4.23: Every (i,j)-$T^{\sim}$ space is (i,j)-$^{\sim}T$ space but not conversely.

Proof: follows.

Example 4.24: Let X={a, b, c}, τ_1 ={ϕ,X,{a},{b},{a, b}}, τ_2 ={ϕ,X} then (X, τ_1 , τ_2) is (1,2)- $^{\sim}T$ space but not (1,2)- $T^{\sim}$ space.

Theorem 4.25: If (X, τ_i , τ_j) is a (i,j)-$^{\sim}T$ space, then every singleton of X is either τ_i –α-closed or τ_j –α-open.

Proof: Suppose that (X, τ_i , τ_j) is a (i,j)- $^{\sim}T$ space. Suppose that {x} is not τ_i –α-closed for some x∈X. Then X-{x} is not τ_i –α-open. Then X is the only τ_i –α-open set containing X-{x. So X-{x} is a (i,j)-sα-closed. Since (X, τ_i , τ_j) is a (i,j)- $^{\sim}T$ space, X-{x} is τ_j –α-closed or equivalently {x} is τ_j –α-open.

We now introduce a new space (i,j)-$^{\sim s}T$ space

Definition 4.26: A space (X,τ_i ,τ_j) is called a (i,j)-$^{\sim s}T$ space if every (i,j)-gs-closed set is (i,j)-sα-closed.

Proposition 4.27: Every (i,j)-$T_{1/2}$ space is a (i,j)-$^{\sim s}T$ space but not conversely.

Proof: Let (X, τ_i , τ_j) be a (i,j)-$T_{1/2}$ space .Let A be a (i,j)-gs-closed set. Since (X, τ_i , τ_j) is (i,j)-$T_{1/2}$ space, A is τ_j –semi-closed. Therefore A is (i,j)-sα-closed. Hence (X, τ_i , τ_j) is a (i,j)- $^{\sim s}T$ space. Hence proved.

Example 4.28: Let X={a, b, c}, τ_1 ={ϕ,X,{a, c},{c}}, τ_2 ={ϕ,X,{a}} then (X,τ_1 ,τ_2) is (1,2)- $^{\sim s}T$ space but not (1,2)-$T_{1/2}$ space**.**

Proposition 4.29: Every (i,j)-T_b space is (i,j)-$^{\sim s}T$ space but not conversely.

Proof: follows

Example 4.30: Let X={a, b, c}, τ_1 ={ϕ,X,{c},{a, b}}, τ_2 ={ϕ,X,{a}} then (X, τ_1 , τ_2) is (1,2)- $^{\sim s}T$ space but not (1,2)-T_b space.

Theorem 4.31: A space (X, τ_i, τ_j) is a (i,j)-$T_{1/2}$ –space if and only if (X, τ_i, τ_j) is (i,j)- $^{\sim s}T$ and (i,j)- $T^{\sim s}$ space.

Proof: follows**.**

Theorem 4.32: A space (X, τ_i, τ_j) is a (i,j)-T_b –space if and only if (X, τ_i, τ_j) is (i,j)-$^{\sim s}T$ and (i,j)- $T^{\sim}$ space.

Proof: follows**.**

We now introduce a new space (i,j)-T^{α} space

Definition 4.33: A space (X, τ_i, τ_j) is called (i,j)-T^{α} space if every (i,j)-sα-closed set is (i,j)-gα-closed.

Proposition 4.34: Every (i,j)- $T^{\sim}$ space is (i,j)- T^{α} space but not conversely.

Proof: follows

Example 4.35: Let X={a, b, c}, τ_1 ={ϕ,X,{a}}, τ_2 ={ϕ,X,{c},{a, b}} then (X, τ_1, τ_2) is (1,2)- T^{α} space but not (1,2)- $T^{\sim}$ space.

Proposition 4.36: Every (i,j)- $^{\sim}T$ space is (i,j)- T^{α} space but not conversely.

Proof: follows

Example 4.37: Let X={a, b, c}, τ_1 ={ϕ,X,{a, b}}, τ_2 ={ϕ,X,{c},{a, b}} then (X, τ_1, τ_2) is (1,2)- T^{α} space but not (1,2)- $^{\sim}T$ space.

We now introduce a new space (i,j)-$T^{\alpha\sim}$ space

Definition 4.38: A space (X, τ_i, τ_j) is called (i,j)-$T^{\alpha\sim}$ space if every (i,j)-sα-closed set is (i,j)-w-closed.

Proposition 4.39: Every (i,j)-T_b space is (i,j)- $T^{\alpha\sim}$ space but not conversely.

Proof: follows

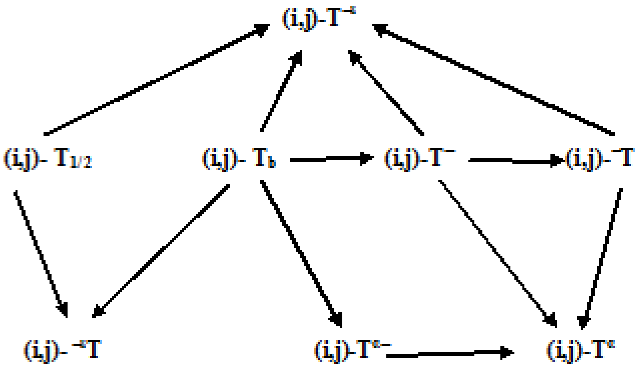
Example 4.40: Let X={a, b, c}, τ_1 ={ϕ,X,{a, b}}, τ_2 ={ϕ,X,{a},{b, c}} then (X, τ_1, τ_2) is (1,2)- $T^{\alpha\sim}$ space but not (1,2)- T_b space.

Proposition 4.41: Every (i,j)-$T^{\alpha\sim}$ space is (i,j)- T^{α} space but not conversely.

Proof: follows

Example 4.42: Let X={a, b, c}, τ_1 ={ϕ,X,{a, b}}, τ_2 ={ϕ,X,{a}} then (X, τ_1, τ_2) is (1,2)- T^{α} space but not (1,2)- $T^{\alpha\sim}$ space.

The following diagram shows the inter relationships between the separation axioms discussed in this section.



where A $\longrightarrow$ B represents A implies B but B need not imply A .

5. sα–continuous maps in bitopological spaces

We introduce the following definition.

Definition 5.1 : A function f : $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called (i ,j)- sα –continuous if $f^{-1}(V)$ is (i ,j)-sα– closed set of (X, τ_1, τ_2) for every closed set V of (Y, σ_1, σ_2).

Proposition 5.2: If f : $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is τ_j -σ_k–continuous then it is (i ,j)- sα –continuous but not conversely.

Proof: follows from the definitions.

Example 5.3: Let X = {a, b, c}, τ_1 = {ϕ, X,{a, b}}, τ_2 ={ϕ, X, {a}} and Y = {p, q}, σ_1 = {ϕ, Y, {p}}, σ_2 = {ϕ, Y, {q}}. Define a map f: (X, τ_1, τ_2) $\to$ (Y, σ_1, σ_2) by f(a) = q, f(b) = f(c) = p. then f is (1 ,2)- sα –continuous but not τ_1 -σ_2 –continuous.

Proposition 5.4: If f : (X, τ_1, τ_2) $\to$ (Y, σ_1, σ_2) is (i ,j)- sα –continuous, then it is (i ,j)- gs –continuous and (i ,j)- gsp –continous but not conversely.

Proof: follows from the definitions.

The converses are not true which is shown by the following examples.

Example 5.5: Let X = {a, b, c}, τ_1 = {ϕ, X,{a}}, τ_2 ={ϕ, X, {a},{b, c}} and Y = {p, q}, σ_1 = {ϕ, Y, {p}}, σ_2 = {ϕ, Y, {q}}. Define a map f : (X, τ_1, τ_2) $\to$ (Y, σ_1, σ_2) by f(a) = f(c) =q, f(b) = p. then f is (1 ,2)- gs –continuous but not (1 ,2)- sα –continuous.

Example 5.6: Let X = {a, b, c}, τ_1 = {ϕ, X,{a}}, τ_2 ={ϕ, X, {a},{b, c}} and Y = {p, q}, σ_1 = {ϕ, Y, {p}}, σ_2 = {ϕ, Y, {q}}. Define a map f: (X, τ_1, τ_2) $\to$ (Y, σ_1, σ_2) by f(a) = f(c) = q, f(b) = p. then f is (1 ,2)- gsp –continuous but not (1 ,2)- sα –continuous.

Remark 5.7: (i ,j)- g–continuous and (i ,j)-sα-continuous are independent which are shown by the following example. Let X = {a, b, c}, τ_1 = {ϕ, X,{a}}, τ_2 ={ϕ, X, {a},{b, c}} and Y = {p, q}, σ_1 = {ϕ, Y, {p}}, σ_2 = {ϕ, Y, {q}}. Define a map f : (X, τ_1, τ_2) $\to$ (Y, σ_1, σ_2) by f(a) = f(c) = q, f(b) = p. then f is (1, 2)- g –continuous but not (1 ,2)- sα –continuous.

Theorem 5.8 : Let f : (X, τ_1, τ_2) $\to$ (Y, σ_1, σ_2) be a map.
1. If (X, τ_1, τ_2) is an (i,j)-$T_{1/2}$ space then f is (i ,j)- sα –continuous if it is (i,j)- g –continuous.
2. If (X, τ_1, τ_2) is an (i,j)-$T^{\sim}$ space then f is τ_j -σ_k–continuous. Iff it is (i ,j)- sα –continuous.

Proof:
1. Let V be a σ_k - closed set Since f is (i ,j)-g–continuous, f^{-1} (V) is (i ,j)-g –closed But (X, τ_1, τ_2) is an (i,j)-$T_{1/2}$ space we have every (i ,j)- g –closed set is τ_j -closed.

$\therefore f^{-1}$ (V) is (i ,j)-sα–closed. Hence f is (i, j)-sα–continuous.

2. Obviously, f is (i,j)-sα-continuous. Conversely, suppose that f is (i ,j)-sα- continuous. Let V be a σ_k - closed set.

Since f is (i ,j)-sα–continuous we have f^{-1} (V) is (i ,j)-sα–closed. But (X, τ_1, τ_2) is an (i,j)-$T^{\sim}$ space we have f^{-1} (V) is τ_j – closed $\therefore$ f is τ_j -σ_k –continuous Hence proved.

Theorem 5.9: Every τ_j -σ_k –semi-continuous map is (i, j)-sα–continuous but not conversely.

Proof: obvious.

The following example supports that the converse of the above theorem is not true in general.

Example 5.10: Let X = {a, b, c}, τ_1 = {ϕ, X,{a, b}}, τ_2 ={ϕ, X, {a}} and Y = {a, b, c}, σ_1 = {ϕ, Y, {a}}, σ_2 = {ϕ, Y, {b,c}}. Define a map f: (X, τ_1, τ_2) $\to$ (Y, σ_1, σ_2) by f(a) = b, f(b) = a, f(c) = c. then f is (1 ,2)-sα–continuous but not τ_1 -σ_2 –semi-continuous.

Theorem 5.11: Every τ_j -σ_k -α-continuous map is (i ,j)-sα-continuous. But not conversely.

Proof: follows from definitions.

The converse of the above theorem is not true which is shown by the following example.

Example 5.12: Let X = {a, b, c}, τ_1 = {ϕ, X,{a, b}}, τ_2 ={ϕ, X, {a}} and Y = {a, b, c}, σ_1 = {ϕ, Y, {a}}, σ_2 = {ϕ, Y, {b, c}}. Define a map f: (X, τ_1, τ_2) $\to$ (Y, σ_1, σ_2) by f(a) = b, f(b) = a, f(c) = c. then f is (1 ,2)-sα –continuous but not τ_1 -σ_2 -α –continuous.

Thus the class of (i ,j)-sα-continuous maps properly contains the class of τ_j -σ_k -continuous maps, the class of τ_j -σ_k -α-continuous maps, the class of τ_j -σ_k -semi-continuous maps. And also the class of (i ,j)-sα-continuous maps is properly contained in the class of (i ,j)- gs- continuous maps and hence in the class of (i ,j)- gsp-continuous maps.

Theorem 5.13: Let $f : (X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ be a (i, j)-sα-continuous map. If (X, τ_1, τ_2), the domain of f is an (i,j)-$T^{\sim s}$ space, then f is τ_j -σ_k–semi-continuous.

Proof: follows

Theorem 5.14: Let f: $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a (i, j)-sα-continuous map. If (X, τ_1, τ_2), the domain of f is (i,j)-$^{\sim}T$ space, then f is τ_j -σ_k–α-continuous.

Proof: follows

Theorem 5.15: Let f: $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a (i ,j)-sα-continuous map. If (X, τ_1, τ_2), the domain of f is (i,j)-$T^{\sim}$ space, then f is τ_j -σ_k-continuous.

Proof: follows

Theorem 5.16: Let f: $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a (i, j) - gs-continuous map. If (X, τ_1, τ_2), the domain of f is (i,j)-$^{\sim s}T$ space, then f is (i ,j)-sα –continuous.

Proof: follows

We introduce the following definition

Definition 5.17: A function f : $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called (i ,j)-sα –irresolute map if $f^{-1}(V)$ is (i ,j)-sα – closed set of (X, τ_1, τ_2) for every (i ,j)- sα – closed set of (Y, σ_1, σ_2).

Theorem 5.18: Every (i, j)-sα –irresolute map is (i, j)-sα–continuous but not conversely.

Proof: Let f is (i, j)-sα –irresolute Let V be σ_k –closed set. Then $f^{-1}(V)$ is (i, j) - sα –closed, since f is (i ,j)-sα –irresolute. hence f is (i ,j)-sα –continuous. Hence proved.

The converse of the above theorem is not true which is shown by the following example. Consider example 5.10

From example 5.10 we have, f is (1, 2)-sα –continuous.

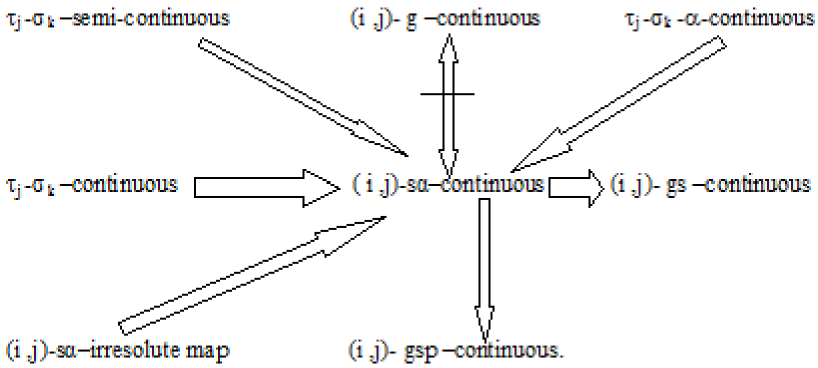
But f is not (1,2)-sα –irresolute because{b} is (1 ,2)-sα –closed and $f^{-1}(\{b\}) = \{a\}$ and {a} is not (1 ,2)- sα –closed. Hence proved.

Theorem 5.19: Let f: $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ and g: $(Y, \sigma_1, \sigma_2) \to (Z, \eta_1, \eta_2)$ be any two functions. Then

1. gof is (i,j)-sα –continuous, if g is σ_j -η_k - continuous and f is (i ,j)-sα–continuous.
2. gof is (i,j)-sα – irresolute, if g is (i,j)-sα – irresolute and f is (i ,j)-sα – irresolute.
3. gof is (i,j)-sα –continuous, if g is (i,j)-sα–continuous and f is (i,j)-sα-irresolute.

Proof: follows

Remark 5.20: The following diagram summarizes the above discussions.



where A $\longrightarrow$ B (resp. A $\nleftrightarrow$ B) represents A implies B but B need not imply A (resp. A and B are independent).

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