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AN SQP ALGORITHM OF LINEAR INEQUALITY CONSTRAINTS

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ABSTRACT

In this paper, a kind of optimization problems with linear inequality constraints are discussed, and a sequential quadratic programming feasible descent algorithm for solving the problems is presented. In order to show the algorithm is well defined, we obtained a feasible descent direction by using generalized projection technique. The high-order revised direction d^k which avoids Maratos effect is computed. Under some suitable conditions, the global and superlinear convergence can be obtained.

Keywords: *linear inequality constraints; SQP method; Global convergence; superlinear convergence.*

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1. INTRODUCTION

Consider the following linear inequality constrained optimization problem:

$$\begin{aligned} &\min f(x) \\ &s.t. g_j(x) = a_j^T x_j + b_j \le 0. j \in I = \{1.2, \ldots, m\} \end{aligned} \tag{1}$$

where f, g_j, $(j \in I)$ are continuous and differentiable functions and g_j $(j \in I)$ is the linear function .

Sequential quadratic programming (SQP) algorithms are widely acknowledged to be among the most successful algorithms for solving two or three nonlinear optimization problems. Because of its superlinear convergence rate, it is a topic of many active researches [1-10]. It is well known that standard SQP method for (1) generates a decent direction by solving the quadratic programming sub-problem:

$$\begin{aligned} &\min \nabla f(x)^T d + \frac{1}{2} d^T H d \\ &s.t. g_j(x) + \nabla g_j(x)^T d \le 0, j \in I \end{aligned} \tag{2}$$

where H is the approximate Hessian matrix of Lagrange function associated with (2). However, the traditional algorithms make it necessary to solve relatively complex and highly computational cost QP problems per single iteration, or let the Hessian matrix of the quadratic programming sub-problem be positive definite or uniformly positive definite (such as[11]). In order to simplify the structure of the algorithm, weaken hypothesis conditions, reduce the computational cost, and quicken the convergence rate, a lot of authors present many different types algorithms. For example [10] proposed a new SQP method for solving inequality constrained optimization, which is not necessary that the Hessian matrix is positive definite.

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Based on the idea of [10], in this paper, a kind of optimization problems with linear inequality constraints are discussed, and an improved sequential quadratic programming feasible descent algorithm for solving the problems is presented. In order to show our algorithm is well defined, like [8], we obtained a feasible descent direction by using generalized projection technique. Our line search techniques like as [5]. The high-order revised direction d^k which avoids Maratos effect is computed. Under some suitable conditions, the global and super linear convergence can be obtained.

2. ALGORITHM AND ITS THEORETICAL BASIS

For the sake of simplicity, we denote

Let $X=\{x\in R^n \mid g_j\left(x\right)\le 0,\, j\in I\}.I(x)=\{j\in I\mid g_j\left(x\right)=0\}$ (3)

We make the following general assumptions and let them hold throughout the paper :

H1: the feasible set $X\neq\phi,\ f(x)$ are two-times continuously differentiable;

H2: $\forall x\in X$, the vectors $\{a_j,\, j\in I(x)\}$ is linearly independent.

Definite matrix $H_k=H(x^k)$, Denote: $N(x)=(\nabla g_j(x^k).j\in I)=\{a_j,\, j\in I\}$

$$\left.\begin{aligned}
D(x)&=diag\left(D_j\left(x\right),\, j\in I\right)=diag(g_j^2(x^k),\, j\in I)\\
G(x^k)&=(g_j\left(x^k\right),\, j\in I),\quad A_k=A(x^k)=(\nabla g_j,\, j\in I)\\
B_k=B(x^k)&=(A_k^TH_k^{-1}A_k)^{-1}A_k^TH_k^{-1},\quad P_k=P(x^k)=H_k^{-1}(E_n-A_kB_k)\\
\pi^k&=\pi(x^k)=-B_k\nabla f(x^k)\\
V^k&=V(x^k)=(V_j^k,\, j\in I),\quad V_j^k=\begin{cases}-g_j(x^k), & \pi_j^k>0\\ \pi_j^k, & \pi_j^k\le 0\end{cases}\\
d_1^k&=-B_k^T(\|d_0^k\|^T\ e+G(x^k+d_0^k)),(e=(1,\ldots,1)\in R^{|J(x)|})\\
d^k&=d_0^k+d_1^k,\quad \tilde{d}_0^k=\tilde{d}\left(x^k\right)=-P_k\nabla f\left(x^k\right)+B_k^TV^k\\
d_2^k&=\frac{-\rho_k}{1+2|e^T\pi^k|}B_k^Te,\quad \rho_k=-\left(\nabla f\left(x^k\right)\right)^Td_0^k,\quad q^k=\rho_k\left(\tilde{d}_0^k+d_2^k\right)
\end{aligned}\right\}\qquad (4)$$

Lemma: 1 Let H2 hold, then the $D_j\ge 0,(\forall j\in I)$ and $D_j\ge 0,(\forall j\in I\setminus I(x))$ satisfy the diagonal $D=diag(D_j,\, j\in I),$,

$(N(x)^TN(x)+D)$ is a positive definite matrix so $(N(x)^TN(x)+D(x))$ is also positive definite.

And from the above definition and a small appropriate parameters $\sigma>0$, defined as a constraint set

$J(x)=\{j\in I\mid -\sigma|\pi_j(x)|\le g_j(x)\le 0\}$ (5)

Consider the following equality constraints Quadratic Programming

$$\begin{aligned}
&\min\nabla f(x)^Td+\frac{1}{2}d^THd\\
&s.t.g_j\left(x\right)+\nabla g_j\left(x\right)^Td=-\min\{0.\pi_j(x)\},\, j\in J(x)
\end{aligned}\qquad (6)$$

Lemma: 2 Suppose that $(d_0(x),\tilde{u}(x))$ is a K-T point pairs of (6) , if $d_0(x)=0,$ then, x is the K-T point of problem (1).

Proof: Let

$$u(x)=\left(u_j(x),\, j\in I\right), u_j(x)=\begin{cases}\tilde{u}_j(x), & j\in J(x)\\ 0, & j\in I\setminus J(x)\end{cases}$$

We have $d_0(x)=0$.

From (6), it is easy obtain that

$$\begin{aligned}&\nabla f(x)+\nabla g_{J(x)}(x)\tilde{u}(x)=0,\\ &g_j(x)+\min\{0,\pi_j(x)\}=0,\ j\in J(x)\end{aligned} \tag{7}$$

Where $\nabla g_{J(x)}(x)=\left(\nabla g_j(x),\, j\in J(x)\right)$

So $\quad g_j(x)=0, \pi_j(x)\ge 0,\ j\in J(x), D_j(x)u_j(x)=0,\ j\in I$

and $\quad \left(N(x)^T N(x)+D(x)\right)u(x)=N(x)^T\nabla g_{J(x)}(x)\tilde{u}(x)=-N(x)^T\nabla f(x)$

That $\quad u(x)=-\left(N(x)^T N(x)+D(x)\right)^{-1}N(x)^T\nabla f(x)=\pi(x)$

So from (7)

$$\nabla f(x)+\nabla g_{J(x)}(x)\tilde{u}(x)=0,$$

$$g_j(x)=0, \tilde{u}_j(x)\ge 0,\ j\in J(x)$$

this shows that x is the K-T point of problem (1).

Lemma: 3 For (4), if $\tilde{d}_0^k(x)=0$, then x is the K-T point of problem (1), if $\tilde{d}_0^k(x)>0$, then

$$\nabla f(x)^T q(x)\le -\frac{1}{2}\rho_k^2(x)<0,\ \nabla g_j(x)^T q^k(x)<0$$

Now, the algorithm for the solution of the problem (1) can be stated as follows.

ALGORITHM

Step 1: Given a starting point $x^1\in X$, and an initial symmetric positive definite matrix $H_1\in R^{n\times n}$,

Parameters $\xi,\nu\in(0,1),\alpha\in(0,\frac{1}{2}),\tau\in(2,3),\delta>2,k=1$

Step 2: From (4) compute

$$N_k=N(x^k), D_k=D(x^k), B_k=B(x^k), \pi_k=\pi(x^k), J_k=J(x^k);$$

Step 3: for x^k, H_k, solving equality constrained QP (6). let $(d_0^k, \tilde{u}^k)$ as a K-T point. If $d_0^k=0$, Stop, if (6) doesn't exist solution, go to Step 5

Step 4: Compute

$$d_1^k=-B_k^T(\| d_0^k\|^T e+G(x^k+d_0^k)), d^k=d_0^k+d_1^k \tag{8}$$

where $e = (1, \cdots, 1)^T \in R^{|I|}$

$$G(x^k + d_0^k) = (G_j(x^k + d_0^k), j \in I), G_j(x^k + d_0^k) = \begin{cases} g_j(x^k + d_0^k), j \in J_k \\ 0, \qquad j \in I \setminus J_k \end{cases}$$

If

$$\nabla f(x^k)^T d_0^k \le \min\left\{-\xi \| d_0^k \|^\delta, -\xi \| d^k \|^\delta\right\} \tag{9}$$

$$\| H_k d_0^k \| \le \xi \| d_0^k \|^{\frac{1}{2}}, \min\left\{\tilde{u}_j^k, j \in J_k\right\} \ge -\xi \| d_0^k \| \tag{10}$$

$$f(x^k + d^k) \le f(x^k) + \alpha \nabla f(x^k)^T d_0^k, \tag{11}$$

$$g_j(x^k + d^k) \le 0, j \in I, \tag{12}$$

Set $t_k = 1$, go to Step 6

Step 5: From (4) compute ρ_k, if $\rho_k = 0$, stopped; Otherwise compute $q^k = q\left(x^k\right)$.

Let $d^k = q^k$ Compute t_k, the first number $t \in \left\{1, \frac{1}{2}, \frac{1}{4}, \cdots\right\}$ satisfying

$$f(x^k + tq^k) \le f(x^k) + vt \nabla f(x^k)^T q^k, \tag{13}$$

$$g_j(x^k + tq^k) \le 0, j \in I \tag{14}$$

Step 6: Set $x^{k+1} = x^k + t_k d^k$, we can obtain H_{k+1} by updating the positive definite matrix H_k . Let k=k+1, then go back to step2

3. GLOBAL CONVERGENCE OF ALGORITHM

The algorithm is the global convergent, any accumulation x^* generated by $\left\{x^k\right\}$ is the K-T point of (1). Firstly, from Lemma 3, that the algorithm is convergent.

Theorem: 1 The algorithm either stops at the K-T point x^* of the problem(1) in finite number of steps, or generates an infinite sequence $\left\{x^k\right\}$ any accumulation point x^* of which is a K-T point of the problem (1).

Proof. The first half of the theorem, from Lemma 3, clearly hold, after the infinite point $\left\{x^k\right\}$ which algorithm generates, where x^* any accumulation point is. For $x^k \to x^*, J_k \equiv J, k \in K$ Firstly from (9), (11),(13) and Lemma 3 ,we know that $\left\{f\left(x^k\right)\right\}$ is monotonous decreasing, and when $k \in K, k \to \infty$ we have $f\left(x^k\right) \to f\left(x^*\right)$. Therefore, $f\left(x^k\right) \to f\left(x^*\right)$ $k \to \infty$

Together with (9) and (11), we get

$$0=\lim_{k\in K}(f(x^{k+1})-f(x^k))\le\lim_{k\in K}\alpha\nabla f(x^k)^T d_0^k\le\lim_{k\in K}(-\alpha\xi\|d_0^k\|^\delta)\le 0.$$

i.e. $d_0^k\to 0, k\in K$

So from (10) such that $H_k d_0^k\to 0, k\in K$. And from (6), we have

$$\nabla f(x^k)+H_k d_0^k+\nabla g_J(x^k)\tilde{u}^k=0 \tag{15}$$

$$g_j\left(x^K\right)+\min\{0,\pi_j^k\}+\nabla g_j\left(x^k\right)^T d_0^k=0. j\in J$$

So $g_j(x*)=0, j\in J$ Therefore, $J\subseteq I(x*)$

From H2, we know that $(\nabla g_J(x*)^T\nabla g_J(x*))$ is nonsingular. So for $k\in K$ and k is large enough, we get that $(\nabla g_J(x^k)^T\nabla g_J(x^k))$ is non-singular and

$$(\nabla g_J(x^k)^T\nabla g_J(x^k))^{-1}\to(\nabla g_J(x^*)^T\nabla g_J(x*))^{-1}, k\in K, k\to\infty$$

From (15)

$$\tilde{u}^k=-(\nabla g_J(x^k)^T\nabla g_J(x^k))^{-1}\nabla g_J(x^k)^T(\nabla f(x^k)+H_k d_0^k)$$

$$\to-(\nabla g_J(x^*)^T\nabla g_J(x^*))^{-1}\nabla g_J(x^*)^T(\nabla f(x^*)=\lambda^*, k\in K$$

So from (10) and (15), it holds that

$$\nabla f(x^*)+\nabla g_J(x^*)\lambda^*=0$$

$$g_j(x^*)=0,\lambda_j^*\ge 0, j\in J$$

We obtain that x^* is the K-T point of (1).

4. SUPER-LINEAR CONVERGENCE OF ALGORITHM

In this section, we discusses the convergent rate of the algorithm and prove that the sequence $\left\{x^k\right\}$ generated by the algorithm is one-step superlinearly convergence x^*.

Therefore, we add the following assumptions:

H3: $\left\{x^k\right\}$ are bounded and exists an accumulation x^*, such that x^* and their counterparts on the K-T multiplier $u^*=(u_j^*, j\in I)$ made the second-order sufficient condition and strict complementarity condition hold.

Lemma: 4 If H1~H3 hold, then $\lim_{k\to\infty}\|x^{k+1}-x^k\|=0$. Thereby, the algorithm is strong convergent. i.e., $\lim_{k\to\infty}x^k=x^*$

Lemma: 5 Let H1-H4 hold, for k is large enough d_0^k, $\tilde{u}^k=\pi^k+\left(A_k^T H_k^{-1}A_k\right)^{-1}G\left(x^k\right)$ is the only solution of quadratic programming

$$\min \nabla f(x)^T d + \frac{1}{2} d^T H d$$

$$s.t. g_j(x) + \nabla g_j(x)^T d = 0, \ j \in J(x)$$

and $J_k \equiv I(x^*) \overset{\Delta}{=} I_*, \lim_{k\to\infty} d_0^k = 0, \lim_{k\to\infty} \pi^k = (u_j^*, j \in I_*)$

Proof: Supposes that (d, u) is a K-T point pairs and from (4), it hold that

$$d = -P_k \nabla f(x^k) - B_k^T G(x^k), \ u = \pi^k + (A_k^T H_k^{-1} A_k)^{-1} G(x^k) = \tilde{u}^k$$

In addition, for k is large enough. From $\lim_{k\to\infty} \pi^k = (u_j^*, j \in I_*)$ and the stringent complementary conditions, we obtained $\pi_j^k > 0, j \in I_*$.

Therefore, from V^k definition of (4), we have

$$d_0^k = -P_k \nabla f(x^k) - B_k^T G(x^k) = d$$

Therefore, the claim holds.

Now, we show that $J_k \equiv I_*$. For $\forall j \in I_*$. From H3, it holds that $u_j^* > 0$. Therefore, from $x^k \to x^*, \pi^k \to u^*, k \to \infty$ and $g_j(j \in I)$ continuously, and for k sufficiently large it is easy to get

$$\pi_j^k > 0, 0 \geq g_j(x^k) \to g_j(x^*) = 0, \ j \in I_*$$

According to the definition J_k, for k is large enough $j \in J_k$. i.e. $I_* \subseteq J_k$

Below prove $J_k \subseteq I_*$, otherwise, there exists a j_0 and a infinite set K, such that

$$j_0 \in J_k \setminus I_*, g_{j_0}(x^*) < 0, g_{j_0}(x^k) \geq -\sigma |\pi_{j_0}^k|, \forall k \in K$$

Let $k \to \infty, k \in K$, then

$$0 > g_{j_0}(x^*) \geq -\sigma u_{j_0}^*, u_{j_0}^* > 0$$

This conflicts with the complementary conditions, so $J_k \equiv I_*$.

For $x^k \to x^*, H_k \to H_*, J_k \equiv I_*$, then, $d_0^k \to d_0^*, \pi^k \to \pi^*, k \to \infty$

As x^* is the K-T of (1), from

$$P_k A_k = H_k^{-1}(I_n - A_k B_k) A_k = H_k^{-1} A_k - H_k^{-1} A_k A_k^{-1} H_k A_k^{-T} A_k^T H_k^{-1} A_k = 0$$

$$P_k H_k P_k = P_k H_k H_k^{-1}(I_n - A_k B_k) = P_k, \quad B_k A_k = A_k^{-1} H_k A_k^{-T} A_k^T H_k^{-1} A_k = I_n$$

and the definition $\tilde{d}_0^k$ we have $\tilde{d}_0^k = 0$.

Then

$$0 = A_k^T \tilde{d}{}_0^k = V^k, \quad P_k \nabla f(x) = 0,$$

Therefore, from (4), and H_k positive definition

$$d_0^* = 0,\ \pi_j^k \ge 0, \quad \pi_j^k g_j(x) = 0, \quad j \in I(x)$$
$$\nabla f(x) + A_k \pi^k = 0$$

It shows the claim holds.

Lemma: 6

1. For k is large enough, there exist a constant $b, \eta > 0$, such that

$$\sum_{j\in I_*} \tilde{u}_j^k g_j(x^k) \le \eta z_k, \quad z_k \triangleq \left(\sum_{j\in I_*} g_j^2\left(x^k\right)\right)^{\frac{1}{2}}$$
$$-\left(d_0^k\right)^T H_k d_0^k \le -b\| d_0^k \|^2 + o\left(z_k\right) \tag{16}$$
$$\nabla f(x^k)^T d_0^k \le -b\| d_0^k \|^2$$

2. From (8) compute direction d_1^k, d^k, it hold that

$$\| d^k \| \sim \| d_0^k \|, \quad \| d_1^k \| = o(\| d_0^k \|^2) \tag{17}$$

Lemma: 7 For k is large enough, the algorithm steps 4 can satisfy the inequalities, i.e. $x^{k+1} = x^k + d^k, t_k \equiv 1$

Proof: Firstly from Lemma 5, Lemma 6 and strict complementary conditions ,we know that (10) holds, which from (16) (17), and $\delta > 2$ (9) holds, for (12),when $j \in I \setminus I_*$, from the $g_j\left(x^*\right) < 0, d_0^k \to 0$ and g_j continuous ,(12) hold;

When $j \in I_*$, by the Taylor expansion

$$\begin{aligned} g_j\left(x^k + d^k\right) &= g_j\left(x^k + d_0^k + d_1^k\right) = g_j\left(x^k + d_0^k\right) + \nabla g_j\left(x^k + d_0^k\right)^T d_1^k + O\left(\left\|d_1^k\right\|^2\right) \\ &= g_j\left(x^k + d_0^k\right) + \nabla g_j\left(x^k\right)^T d_1^k + O\left(\left\|d_0^k\right\|^3\right) \end{aligned}$$

from (4), (8), that

$$N_k^T d_1^k = -\left\|d_0^k\right\|^T e - G\left(x^k + d_0^k\right) + D_k\left(N_k^T N_k + D_k\right)^{-1}\left(\left\|d_0^k\right\|^T e + G\left(x^k + d_0^k\right)\right)$$

NOTE:

$$\left\|G\left(x^k + d_0^k\right)\right\| = o\left(\left\|d_0^k\right\|^2\right), \left(D_k\right)_{jj} = g_j^2\left(x^k\right) = \left(-\nabla g_j\left(x^k\right)^T d_0^k\right)^2 = O\left(\left\|d_0^k\right\|^2\right), j \in I_*$$

Therefore

$$\nabla g_j\left(x^k\right)^T d_1^k = -\left\|d_0^k\right\|^\tau - g_j\left(x^k + d_0^k\right) + O\left(\left\|d_0^k\right\|^4\right), j \in I_*$$

So

$$g_j\left(x^k + d^k\right) = -\left\|d_0^k\right\|^\tau + O\left(\left\|d_0^k\right\|^3\right), j \in I_* \tag{18}$$

Because $\tau \in (2,3)$, for $j \in I_*$, (12) holds. For (11), let

$$s = f\left(x^k + d^k\right) - f\left(x^k\right) - \alpha \nabla f\left(x^k\right)^T d_0^k$$
$$= \nabla f\left(x^k\right)^T d^k + \frac{1}{2}\left(d_0^k\right)^T \nabla^2 f\left(x^k\right) d_0^k - \alpha \nabla f\left(x^k\right)^T d_0^k + o\left(\left\|d_0^k\right\|^2\right)$$

and

$$\nabla f\left(x^k\right)^T d_0^k = -\left(d_0^k\right)^T H_k d_0^k - \sum_{j\in I_*} \tilde{u}_j^k \nabla g_j\left(x^k\right)^T d_0^k,$$

$$\nabla f\left(x^k\right)^T d^k = -\left(d_0^k\right)^T H_k d_0^k - \sum_{j\in I_*} \tilde{u}_j^k \nabla g_j\left(x^k\right)^T d^k + o\left(\left\|d_0^k\right\|^2\right),$$

$$g_j\left(x^k\right) + \nabla g_j\left(x^k\right)^T d_0^k = 0, j \in I_*$$

$$g_j\left(x^k + d^k\right) = g_j\left(x^k\right) + \nabla g_j\left(x^k\right)^T d^k + \frac{1}{2}\left(d_0^k\right)^T \nabla^2 g_j\left(x^k\right) d_0^k + o\left(\left\|d_0^k\right\|^2\right) = o\left(\left\|d_0^k\right\|^2\right), j \in I_*$$

Furthermore, from (18) ,then

$$-\sum_{j\in I_*} \tilde{u}_j^k \nabla g_j\left(x^k\right)^T d^k = \sum_{j\in I_*} \tilde{u}_j^k g_j\left(x^k\right) + \frac{1}{2}\left(d_0^k\right)^T \left(\sum_{j\in I_*} \tilde{u}_j^k \nabla^2 g_j\left(x^k\right)\right) d_0^k + o\left(\left\|d_0^k\right\|^2\right)$$

So

$$s = (\alpha - 1)\left(d_0^k\right)^T H_k d_0^k + \frac{1}{2}\left(d_0^k\right)^T \nabla_{xx}^2 \tilde{L}\left(x^k, \tilde{u}^k\right) d_0^k + \sum_{j\in I_*} (1-\alpha) \tilde{u}_j^k g_j\left(x^k\right) + o\left(\left\|d_0^k\right\|^2\right)$$
$$\le \left(\alpha - \frac{1}{2}\right)\left(d_0^k\right)^T H_k d_0^k + \frac{1}{2}\left(d_0^k\right)^T (\nabla_{xx}^2 \tilde{L}\left(x^k, \tilde{u}^k\right) - H_k) d_0^k - (1-\alpha)\eta z_k + o\left(\left\|d_0^k\right\|^2\right)$$
$$\le b\left(\alpha - \frac{1}{2}\right)\left\|d_0^k\right\|^2 + o\left(z_k\right) + \frac{1}{2}\left(\left(d_0^k\right)^T \tilde{P}_* + y_k^T\right)(\nabla_{xx}^2 \tilde{L}\left(x^k, \tilde{u}^k\right) - H_k) d_0^k - (1-\alpha)\eta z_k + o\left(\left\|d_0^k\right\|^2\right)$$
$$= -b\left(\frac{1}{2} - \alpha\right)\left\|d_0^k\right\|^2 + o\left(\left\|d_0^k\right\|^2\right) - (1-\alpha)\eta z_k + o\left(z_k\right) \le 0$$

For k large enough, $s \le 0$. i.e. $t_k = 1$ holds. .

According to Lemma 5 and under strict conditions of complementarity, when k is large enough, quadratic programming (6) right disturbance disappeared, that is, only the following quadratic programming

$$QP_k \quad \begin{array}{l} \min \nabla f\left(x^k\right)^T d + \frac{1}{2} d^T H_k d \\ s.t. g_j\left(x^k\right) + \nabla g_j\left(x^k\right)^T d = 0, j \in I_* \end{array}$$

Thus from Lemma 7 and the theorem 5.2 in [3] and the theorem 12.3.3 in [10], we get the following Convergent theorem.

Theorem 2 : Under the assumption, the algorithm is super-linear convergence, i.e., $\| x^{k+1} - x^* \| = o(\| x^k - x^* \|)$

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