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ON SOME TOPOLOGIES INDUCED BY BI$^+$ OPEN SETS IN SIMPLE EXTENSION IDEAL TOPOLOGICAL SPACES

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ABSTRACT

This paper aims at extending the idea of bI open sets in Simple Extension ideal topological spaces. Here we introduce the new concept of Ω_b^{+*} and $\mho_b^{+*}$ sets via bI$^+$ open and bI$^+$ closed sets in simple extension ideal topological spaces. Furthur Ω_b^{+*} and $\mho_b^{+*}$ functions are defined and their results are discussed.

1. INTRODUCTION

A new class of generalized open sets called b- open sets in topological spaces was defined by Andrijevic [8]. This type of sets was discussed by El Atik [13] under the name of γ open sets. The class of all b open sets generates the same topology as the class of all pre-open sets. In 1986, Maki [24] introduced the concept of generalized Λ sets and defined the associated closure operators by using the work of Levine [22] and Dunhem [12]. Caldas and Dontchev [9] introduced $\Lambda_s$sets, $\vee_s$ sets, g V_s sets and gΛ_s sets. Ganster and et al. [14] introduced the notion of pre Λ sets and pre $\vee$ sets and obtained new topologies via these sets. M.E. Abd El-Monsef *et al.* [3] defined bΛ sets and b$\vee$ sets on a topological space and proved that it forms a topology. In 1963 Levine [23] introduced the concept of a simple extension of a topology τ as τ (B) = $\{(B\cap O) \cup O' / B\notin\tau\}$. The concept of I open sets in ideal topological spaces were introduced by Jankovic and Hamlett [18], [19]. Further Abd El-Monsef *et al.* [2] investigated I open sets and I continuous functions. Dontchev [11] introduced the notion of pre I open sets and obtained a decomposition of I continuity. The notion of semi I open sets to obtain decomposition of continuity was introduced by Hatir and Noiri [16], [17]. In addition to this, Casksu Guler and Aslim [10] have introduced the concept of bI sets and bI continuous functions and futher research was done by Metin Akdag [28] on these sets. Nirmala and I. Arockiarani [30] have introduced the concept of bI open sets in the light of simple expansion topology. Using the above defined bI $^+$sets in simple extended ideal topological space (SEITS), we introduce the notion of Ω_b^{+*} sets and $\mho_b^{+*}$ sets in SEITS and study their properties. We also introduce Ω_b^{+*} functions and $\mho_b^{+*}$ functions and investigate some of its properties.

2. PRELIMINARIES

All through the paper the space X is a SEITS in which no separation axioms are assumed unless and otherwise stated. For any subset A of X, the interior of A is the same as the interior in usual topology and the closure of A is newly defined as a combination of the local function [30] in ideal topology and simple extension. In SEITS the new local function [30] is defined as $A^{+*}=\{x\in X/ U\cap A\notin I$ for each neighbourhood U of x in $\tau^+\}$ and $cl^{+*}(A) = A\cup A^{+*}$. Also $\tau^{+*} = \{V/ cl^{+*}(X\backslash V) = X\backslash V\}$, where $\tau^+\subseteq \tau^{+*}$.

Definition 2.1: A subset A of a topological space X is said to be
(1) α- open [29] if $A\subseteq$ int(cl(int(A))),
(2) semi-open [22] if $A\subseteq$ cl(int(A)),
(3) preopen [25] if $A\subseteq$ int(cl (A)),
(4) β-open [26] if $A\subseteq$ cl(int(cl(A))),
(5) b-open [8][13] if $A\subseteq$ int(cl(A)) $\cup$ cl(int(A)).

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The class of all semi-open (resp. pre-open, α-open) sets in X are denoted by SO(X,τ) (resp. PO(X,τ), αO(X,τ))

Definition 2.2: A subset A of X in SEITS (X, τ^+, I) is said to be
(1) αI^+ open [30] if $A \subseteq int(cl^{+*}(int(A)))$,
(2) semiI^+ open [30] if $A \subseteq cl^{+*}(int(A))$,
(3) pre I^+ open [30] if $A \subseteq int(cl^{+*}(A))$,
(4) βI^+ open [30] if $A \subseteq cl^{+*}(int(cl^{+*}(A)))$,
(5) bI^+open [30] if $A \subseteq int(cl^{+*}(A)) \cup cl^{+*}(int(A))$.

The class of all semiI^+ open (resp. preI^+-open, αI^+ open) sets in X are denoted by $SI^+O(X, \tau^+, I)$(resp. $PI^+O(X, \tau^+, I)$, $\alpha I^+O(X, \tau^+, I)$)

The complements of these sets are called semiI^+ closed (resp. preI^+-closed, αI^+ closed) sets in X and are denoted by $SI^+C(X, \tau^+, I)$ (resp. $PI^+C(X, \tau^+, I)$, $\alpha I^+C(X, \tau^+, I)$)

Definition 2.3: A topological space (X,τ) is said to be resolvable [15] if there is a subset A of X such that A and (X-A) are both dense in X.

3. Ωb^{+*} SETS

In this section we introduce the new idea of Ω_b^{+*} (resp. $\mho_b^{+*}$) sets via the concept of bI open sets under simple extension topology.

Definition 3.1: Let (X, τ^+, I) be a simple extension ideal topological space (SEITS) and A a subset of X. We define $\Omega_b^{+*}(A)$ and $\mho_b^{+*}(A)$ as follows,

a) $\Omega_b^{+*}(A) = \cap\{G: A \subseteq G, G \in BI^+O(X, \tau^+, I)\}$,

b) $\mho_b^{+*}(A) = \cup\{F: F \subseteq A, F \in BI^+C(X, \tau^+, I)\}$.

The class of all bI $^+$open (resp. bI $^+$closed) sets of a SEITS (X, τ^+, I) is denoted by $BI^+O(X, \tau^+, I)$ (resp. $BI^+C(X, \tau^+, I)$).

Example 3.2: Let X = {a, b, c},τ ={X,ϕ ,{a},{a, b}}, I={ϕ,{b}} and B={b}. Then τ^+ (B) ={X,ϕ, {a},{b},{a, b}}, τ^{+*}={ X,ϕ ,{a},{b},{a, b},{a, c}}.

Here $BI^+O(X, \tau^+, I)$ ={X , ϕ,{a}, {a, b},{a, c}}and $BI^+C(X, \tau^+, I)$={X ,ϕ,{b},{c},{b, c}}.

Here $\Omega_b^{+*}(a) = \{a\}$; $\Omega_b^{+*}(b) = \{a, b\}$; $\Omega_b^{+*}(c) = \{a, c\}$; $\Omega_b^{+*}(a, b) = \{a, b\}$; $\Omega_b^{+*}(a, c) = \{a, c\}$; $\Omega_b^{+*}(b, c) = X$. Also $\mho_b^{+*}(a) = \{\phi\}$; $\mho_b^{+*}(b) = \{b\}$; $\mho_b^{+*}(c) = \{c\}$; $\mho_b^{+*}(a, b) = \{b\}$; $\mho_b^{+*}(a, c) = \{c\}$; $\mho_b^{+*}(b, c) = \{b, c\}$.

Lemma 3.3: For subsets A, B and Ai (i $\in$I) of a space (X, τ^+, I), the following properties hold:
i) $A \subseteq \Omega_b^{+*}(A)$,
ii) If $A \subseteq B$, then $\Omega_b^{+*}(A) \subseteq \Omega_b^{+*}(B)$,
iii) $\Omega_b^{+*}(\Omega_b^{+*}(A)) = \Omega_b^{+*}(A)$,
iv) If $A \in BI^+O(X,\tau^+,I)$, then $A = \Omega_b^{+*}(A)$,
v) $\Omega_b^{+*}(\cup\{A_i : i \in I\}) = \cup\{\Omega_b^{+*}(A_i) : i \in I\}$,
vi) $\Omega_b^{+*}(\cap\{A_i \ i \in I\}) \subseteq \cap\{\Omega_b^{+*}(A_i) : i \in I\}$,
vii) $\Omega_b^{+*}(X \setminus A) = X \setminus \mho_b^{+*}(A)$.

Proof: (i), (ii), (iv), (vi), (vii): These are immediate consequences of the Definition 3.1(a).
(iii): We know from Definition 3.1(a)$\Omega_b^{+*}(A) \subseteq \Omega_b^{+*}(\Omega_b^{+*}(A))$. Now we prove the converse inclusion$\Omega_b^{+*}(\Omega_b^{+*}(A)) \subseteq \Omega_b^{+*}(A)$. Let us consider $x \notin \Omega_b^{+*}(A)$, then there exists a $G \in BI^+O(X, \tau^+, I)$ such that $A \subseteq G$, and $x \notin G$. By (ii) and (iv), $\Omega_b^{+*}(A) \subseteq \Omega_b^{+*}(G) = G$. Since $\Omega_b^{+*}(\Omega_b^{+*}(A)) = \cap\{G: \Omega_b^{+*}(A) \subseteq G, G \in BI^+O(X, \tau^+, I)\}$, consequently we have $x \notin \Omega_b^{+*}(\Omega_b^{+*}(A))$. Therefore, we have $\Omega_b^{+*}(\Omega_b^{+*}(A)) \subseteq \Omega_b^{+*}(A)$ and hence $\Omega_b^{+*}(\Omega_b^{+*}(A)) = \Omega_b^{+*}(A)$.

(v): Let $A = \cup\{A_i: i \in I\}$. Since $A_i \subseteq A$, by (ii) we have $\Omega_b^{+*}(A_i) \subseteq \Omega_b^{+*}(A)$ and hence $\cup\{\Omega_b^{+*}(A_i): i \in I\} \subseteq \Omega_b^{+*}(A)$.

Conversely, if $x \notin \cup \{\Omega_b^{+*}(A_i): i \in I\}$, then for each $i \in I$, there exists $G_i \in BI^+O(X, \tau^+, I)$ such that $A_i \subseteq G_i$, and $x \notin G_i$. If $G = \cup \{G_i: i \in I\}$, then $G \in BI^+O(X, \tau^+, I)$ such that $A \subseteq G$ and $x \notin G$. Hence $x \notin \Omega_b^{+*}(A)$ and hence (v) holds.

By using Lemma 3.3 (vii), we can easily verify the next result.

Lemma 3.4: Let (X, τ^+, I) be a SEITS. Let A, B and $\{A_i: i\in I\}$ be subsets of X. Then the following properties hold:

i) $\mho_b^{+*}(A) \subseteq A$,

ii) If $A \subseteq B$, then $\mho_b^{+*}(A) \subseteq \mho_b^{+*}(B)$,

iii) $\mho_b^{+*}(\mho_b^{+*}(A)) = \mho_b^{+*}(A)$,

iv) If $A \in BI^+C(X, \tau^+, I)$, then $A = \mho_b^{+*}(A)$,

v) $\mho_b^{+*}\{\cap \{A_i: i \in I\}\} = \cap\{\mho_b^{+*}(A_i): i \in I\}$,

vi) $\cup\{\mho_b^{+*}(A_i): i \in I\} \subseteq \mho_b^{+*}\{\cup \{A_i: i \in I\}\}$.

Remark 3.5: In general, for any subsets $A, B \in (X, \tau^+, I)$, $\Omega_b^{+*}(A\cap B) \neq \Omega_b^{+*}(A) \cap \Omega_b^{+*}(B)$ and $\mho_b^{+*}(A\cup B) \neq \mho_b^{+*}(A) \cup \mho_b^{+*}(B)$ as noted in the following example.

Example 3.6: Let $X=\{a, b, c\}$, $\tau =\{X,\phi ,\{a\},\{a, b\}\}$, $I=\{\phi,\{b\}\}$ and $B=\{b\}$, then $\tau^+(B)=\{X, \phi, \{a\},\{b\},\{a, b\}\}$. Let $A=\{a\}$ and $B=\{c\}$, here $\Omega_b^{+*}(A\cap B) \neq \Omega_b^{+*}(A) \cap \Omega_b^{+*}(B)$. When $A= \{a\}$; $B = \{b, c\}$, then $\mho_b^{+*}(A\cup B) \neq \mho_b^{+*}(A) \cup \mho_b^{+*}(B)$.

Definition 3.7: A subset A of a SEITS (X, τ^+, I) is called an Ω_b^{+*} set (resp. $\mho_b^{+*}$ set) if $A=\Omega_b^{+*}(A)$ [resp. $A = \mho_b^{+*}(A)$]. The family of all Ω_b^{+*} sets (resp. $\mho_b^{+*}$ sets) is denoted as Ω_b^{+*} [resp. $\mho_b^{+*}$].

Example 3.8: Let $X = \{a, b, c\}$, $\tau =\{X, \phi,\{a\}, \{a, b\}\}$, $I=\{\phi,\{b\}\}$ and $B=\{b\}$. Then $\tau^+(B)=\{X, \phi ,\{a\}, \{b\}, \{a, b\}\}$, $\Omega_b^{+*} = \{ X, \phi ,\{a\},\{a, c\},\{a, b\}\}$ and $\mho_b^{+*} = \{X, \phi, \{b\},\{c\},\{b, c\}\}$.

Proposition 3.9: In a SEITS (X, τ^+, I), Ω_b^{+*} (resp. $\mho_b^{+*}$) is a topology for X.

Proof: It is obvious from Definition 3.1 that X and ϕ are Ω_b^{+*} sets. Let $A_i \in \Omega_b^{+*}$ for each $i\in I$. By Lemma 3.3 ,$\Omega_b^{+*}(\cap_{i\in I} A_i) \subseteq \cap_{i\in I}\Omega_b^{+*}(A_i) = \cap_{i\in I} A_i$ and hence $\Omega_b^{+*}(\cap_{i\in I} A_i) = \cap_{i\in I} A_i$. Therefore $\cap_{i\in I} A_i \in \Omega_b^{+*}$. Let $\{A_i: i\in I\}$ be a family of Ω_b^{+*} sets in (X, τ^+, I) .Then by Lemma 3.3, $\cup_{i\in I} A_i = \cup_{i\in I} \Omega_b^{+*}(A_i) = \Omega_b^{+*}(\cup_{i\in I} A_i)$.

This implies that the union of Ω_b^{+*} sets is also an Ω_b^{+*} set.

Hence the family of Ω_b^{+*} sets forms a topology for X.

Proposition 3.10: In a space (X, Ω_b^{+*}) the following statements are verified.

1) If every subset A of X is nowhere dense in (X,τ), then $\Omega_b^{+*} = \Omega_s^{+*}$, where $\Omega_s^{+*}(A) =\{A\subset X: \Omega_s^{+*}(A) = A\}$ and $\Omega_s^{+*}(A) = \cap\{G:A \subseteq G, G \in SI^+O(X ,\tau^+, I)\}$.

2) If (X, τ^+, I) is an indiscrete space, then each Ω_b^{+*} set is a $preI^+\,\Omega$ set but not a $semiI^+\,\Omega$ set.

Proof: 1) Since every subset A is nowhere dense in (X, τ), we have $Int(cl^{+*}(A))= \phi$ for all A. Then $BI^+O(X, \tau^+, I) = SI^+O(X, \tau^+, I)$ and hence $\Omega_b^{+*}(A)= \Omega_s^{+*}(A)$ for every A of X .Hence $\Omega_b^{+*} = \Omega_s^{+*}$.
2) This is obvious, since each bI^+ open set in indiscrete space is a $preI^+$ open set but not a $semiI^+$ open set.

Definition 3.11: A space (X, τ^+, I) is called a $b^{+*}T_1$ space if for each pair of distinct points x and y of X, there exist two bI^+ open sets U and V such that $x \in U$, $y\notin U$ and $y \in V$, $x\notin V$.

Theorem 3.12: For a space (X, τ^+, I), the following properties are equivalent:
1) (X, τ^+, I) is $b^{+*}T_1$;
2) For each $x \in X$, $\{x\}$ is bI^+ closed;
3) For each $x \in X$, $\{x\}$ is an Ω_b^{+*} set;
4) For each subset A of X, A is an Ω_b^{+*} set.

Proof: (1) ⇨(2): Let y be any point of X-{x}.There exists a bI^+ open set V_y such that $x \notin V_y$ and $y \in V_y$.

Hence X-{x} = $\cup$ {V_y ; y ∊ X-{x}} and hence X-{x} is bI^+ open.

Therefore, {x} is bI^+ closed.

(2) ⇨(3): Let x be any point of X and y ∊ X-{x}. By (2), X-{y} is bI^+ open and x ∊ X-{y}.By Lemma 3.3, $\Omega_b^{+*}(\{x\}) \subset$ X-{y} and hence $\Omega_b^{+*}(\{x\}) = \{x\}$.Therefore,{x} is an Ω_b^{+*} set.

(3) ⇨(4): Let A be any subset of X. By (3) and Lemma 3.3,$\Omega_b^{+*}(A) = \Omega_b^{+*}(\cup\{x/x\in A\}) = \cup\{ \Omega_b^{+*}\{x\}/x\in A\} = \cup\{ x / x \in A\}$= A. Therefore, A is an Ω_b^{+*}set.

(4) ⇨(1): Let x and y be any distinct points.Then $y \notin \Omega_b^{+*}(\{x\}) = \{x\}$ and there exists a bI^+ open set U_x such that $y \notin U_x$ and $x \in U_x$ .Similarly $x \notin \Omega_b^{+*}(\{y\})$ and there exists a bI^+ open set U_y such that $y \in U_y$ and $x \notin U_y$ .This shows that (X,τ^+,I) is $b^{+*}T_1$.

Proposition 3.13: A SEITS (X, τ^+, I) is $b^{+*}T_1$ if and only if (X, Ω_b^{+*}) is a discrete space.

Proof: Let (X, τ^+, I) be $b^{+*}T_1$ and $x \in X$. Then, by Theorem 3.12, {x} is an Ω_b^{+*} set and {x} is open in (X, Ω_b^{+*}). Therefore (X, Ω_b^{+*}) is a discrete space. Conversely, suppose that (X, Ω_b^{+*}) is a discrete space. For any point $x \in X$, {x} is an Ω_b^{+*} set. By Theorem 3.12, (X, τ^+, I) is $b^{+*}T_1$.

Definition 3.14: The space (X, τ^+, I) is said to be resolvable in SEITS if it is the union of two disjoint dense subsets.

Proposition 3.15: If (X, τ^+, I) is resolvable in SEITS, then (X, Ω_b^{+*}) and $(X, \mho_b^{+*})$ are discrete.

Proof: We shall show that (X, τ^+, I) is $b^{+*}T_1$. Consider (X, τ^+, I) to be resolvable in SEITS

i.e.: X = D U E, where D and E are disjoint dense subsets of (X, τ^+, I).

Let $x \in X$, say $x \in D$ then X \{x}= E ∪ [D \{x}] is dense in (X, τ^+, I). Hence X-{x} is a $preI^+$ open and hence {x}is $preI^+$- closed. Since {x} is bI^+ closed, by Theorem 3.12 (X, τ^+, I) is $b^{+*}T_1$. By proposition 3.13, (X, Ω_b^{+*}) and $(X, \mho_b^{+*})$ are discrete.

Proposition 3.16: If (X, Ω_b^{+*}) is connected, then (X, τ^+, I) is bI^+ connected ie) X cannot be represented as a disjoint union of non empty bI^+ open subsets of (X , τ^+, I)

Proof: Since every bI^+-open set is an Ω_b^{+*} set, the proof is obvious.

4. $L\Omega b^{+*}$ - CLOSED SETS

Definition 4.1: A subset A of a SEITS (X, τ^+, I) is said to be $L\Omega b^{+*}$ - closed if $A= L \cap F$, where L is an Ω_b^{+*}- set and F is a closed set in (X , τ^{+*}) .

Remark 4.2: Every Ω_b^{+*} -set and every closed set in (X, τ^{+*}) are $L\Omega b^{+*}$ -closed.

Proposition 4.3: For a subset A of a SEITS (X, τ^+, I), the following properties are equivalent:
(1) A is $L\Omega b^{+*}$ -closed,
(2) $A = L \cap cl^{+*}(A)$, where L is an Ω_b^{+*} set,
(3) $A = \Omega_b^{+*}(A) \cap cl^{+*}(A)$.

Proof: (1) → (2): Let A be $L\Omega b^{+*}$ - closed. Then $A = L \cap F$, where L is an Ω_b^{+*} set and F is closed in $(X, \tau^{+}*)$. Since $A \subseteq F$, we have $cl^{+*}(A) \subseteq cl^{+*}(F)= F$. Therefore $A \subseteq L \cap cl^{+*}(A) \subseteq L \cap F = A$ and hence $A = L \cap cl^{+*}(A)$.

(2) → (3): Let $A = L \cap cl^{+*}(A)$, where L is an Ω_b^{+*} set. Since $A \subseteq L$, we have $\Omega_b^{+*}(A) \subseteq \Omega_b^{+*}(L) = L$. And hence
$A \subseteq \Omega_b^{+*}(A) \cap cl^{+*}(A) \subseteq L \cap cl^{+*}(A) =A$.

Thus we have obtained $A = \Omega_b^{+*}(A) \cap cl^{+*}(A)$.

(3) → (1): Since $\Omega_b^{+*}(A)$ is an Ω_b^{+*}set, the proof is obvious.

5. Ω_b^{+*} and $\mho_b^{+*}$ MAPPINGS

Definition 5.1: Let (X, τ^+, I) and (Y, σ^+, J) be SEITS. A map $f: (X, \tau^+, I) \rightarrow (Y, \sigma^+, J)$ is said to be

(i) Ω_b^{+*} map if $f(U) \in BI^+C(Y, \sigma^+, J)$ for all $U \in \Omega_b^{+*}$,

(ii) $\mho_b^{+*}$ map if $f(U) \in BI^+O(Y, \sigma^+, J)$ for all $U \in \mho_b^{+*}$.

Theorem 5.2: For a map $f: (X, \tau^+, I) \longrightarrow (Y, \sigma^+, J)$, the following are equivalent:
(i) f is Ω_b^{+*} map,
(ii) For each $A \subseteq Y$ and each $F \in \mho_b^{+*}$ with $f^{-1}(A) \subseteq F$, there exists $G \in BI^+O(Y, \sigma^+, J)$ such that $A \subseteq G$ and $f^{-1}(G) \subseteq F$.

Proof: (i)⇒(ii): For each $A \subseteq Y$ and each $F \in \mho_b^{+*}$ with $f^{-1}(A) \subseteq F$, let $G = Y - f(X-F)$.

Since f is Ω_b^{+*} map, $f(X-F) \in BI^+C(Y, \sigma^+, J)$ and hence $G \in BI^+O(Y, \sigma^+, J)$.

Since $f^{-1}(A) \subseteq F$, we have $X-F \subseteq X-f^{-1}(A) = f^{-1}(Y-A)$ and $f(X-F) \subseteq Y-A$.

Taking complements we have $A \subseteq Y- f(X-F) = G$.

Moreover $f^{-1}(G) = f^{-1}(Y- f(X-F)) = f^{-1}(Y) - f^{-1}(f(X-F)) \subset X-(X-F) = F$.

(ii)⇒(i): Let $A \in \Omega_b^{+*}$, $y \in Y\backslash f(A)$ and let $F=X\backslash A$. Since $F \in \mho_b^{+*}$ and $f^{-1}(y) \subset F$, by (ii) there exists $O_y \in BI^+O(Y, \sigma^+, J)$ with $y \in O_y$ and $f^{-1}(O_y) \subseteq F$. Since $F = X-A$, $y \in O_y \subseteq Y\backslash f(A)$. Hence $Y\backslash f(A) = \cup\{O_y : y \in Y\backslash f(A)\}$.

Thus $f(A) \in BI^+C(Y, \sigma^+, J)$.Therefore f is Ω_b^{+*} map.

Theorem 5.3: For a map $f: (X, \tau^+, I) \longrightarrow (Y, \sigma^+, J)$, the following are equivalent:
(i) f is $\mho_b^{+*}$ map,
(ii) For each $A \subseteq Y$ and each $F \in \Omega_b^{+*}$ with $f^{-1}(A) \subseteq F$, there exists $G \in BI^+C(Y, \sigma^+, J)$ with $A \subseteq G$ with $f^{-1}(G) \subseteq F$.

Proof: The proof is similar to the proof of Theorem 5.2.

Theorem 5.4: If $f: (X, \tau^+, I) \longrightarrow (Y, \sigma^+, J)$ is a surjective Ω_b^{+*} map and (X, τ^+, I) is $b^{+*}T_1$, then (Y, σ^+, J) is $b^{+*}T_{1.}$

Proof: Let y be any point of Y. Since f is surjective, there exists $x \in X$ such that $f(x) = y$. Since (X, τ^+, I) is $b^{+*}T_1$, by Theorem 3.12, $\{x\}$ is an Ω_b^{+*}set and hence $f(\{x\})$ is bI^+ closed. Therefore, $\{y\}$ is bI^+ closed and hence by Theorem 3.12 (Y, σ^+, J) is $b^{+*}T_{1.}$

REFERENCES

[1] M.E Abd El-Monsef, S. N El-Deeb and R.A. Mahmoud, "β-open sets and β-continuous mapping", Bull. Fac. Assuit Univ., 12(1983),77-90.

[2] M.E Abd El-Monsef, E. F Lashien and A. A. Nasef, "On I open sets and I continuous functions", Kyungpook Math. J., 32 (1992), 21-30.

[3] M.E. Abd El-Monsef, A.A El- Atik and M.M. El-Sharkasy, "Some topologies induced by b - open sets", Kyunpook Math. J., 45(2005), 539-547.

[4] P.Alexandroff, "Discrete Raume", Mat.Sb. 2(1937), 501-508

[5] D.Andrijević, "Some properties of the topology of α-sets", Mat. Vesnik, 36(1984), 1- 10.

[6] D. Andrijević, "Semi-preopen sets", Mat. Vesnik, 38(1986), 24-32.

[7] D. Andrijević, "On the topology generated by pre-open sets", Mat. Vesnik, 39(1987), 367- 376.

[8] D. Andrijević, "On b-open sets", Mat. Vesnik, 48, (1996), 59-64.

[9] M. Caldas and J. Dontchev, "$G\Lambda_s$- sets and $G\Lambda_s$ -sets", Mem. Fac. Sci. Kochi Univ. Math., 21(2000), 21-30.

[10] A. Caksu Guler and G. Aslim, “bI open sets and decomposition of continuity via idealization”, Proc. Inst. Math. and Mech., Nat. Acad Sci. Azerbaijan., 22(2005), 27-32.

[11] J.Dontchev, “On pre -I- open sets and a decomposition of I- continuity”, Banyan Math. J., 2 (1996).

[12] W.A Dunham, “New closure operator for non T_1 topologies”, Kyunpook Math. J., 22(1982), 55-60.

[13] A.A El-Atik, “A study of some type of mappings on topological spaces”, M. Sc thesis, Tanta University, Tanta, Eygpt, 1997.

[14] M. Ganster, S. Jafari and T. Noiri, “On pre-Λ-sets and pre-∨- sets”, Acta. Math. Hungar., 95(4)(2002),337-343.

[15] M. Ganster, “Preopen sets and presolvable spaces”, Kyunpook Math. J., 27(1987), 135-143.

[16] E. Hatir and T. Noiri, “On decompositions of continuity via idealization”, Acta Math. Hungar, 96 (2002), 341-349.

[17] E. Hatir and T. Noiri, “On Semi-I-open sets and semi-I-continuous functions,” Acta Math. Hungar.107(2005), 345-353.

[18] D. Jankovic and T. R Hamlett, “New topologies from old via ideals”, Amer. Math. Monthly, 97(1990), 295-310.

[19] D. Jankovic and T. R Hamlett, “Compatible extensions of ideals”, Boll. Un. Mat. Ital. B. Serie VII, .6(1992), 453 - 465.

[20] E. Khalimsky, R. Kopperman and P.R Meyer, “Computer graphics and connected topologies on finite ordered sets”, TopologyAppl.,36(1)(1990),1-17.

[21] N. Levine, “Generalized closed sets in topology” Rend. Circ. Mat. Palermo (2), 19(1970), 89-96.

[22] N. Levine, “Semi-open sets and semi-continuity in topological spaces” Amer. Math. Monthly, 70 (1963), 36-41.

[23] N. Levine **, “**Simple extension of topology”, Amer. Math. Monthly,71(1964), 22-105.

[24] H. Maki, “Generalized Λ-sets and the associated closure operator”, The Special Issue in commemoration of Prof. Kazusada Ikeda’s retirement, 1.Oct.(1986),139-146.

[25] A. S. Mashhour, M. E. Abd El Monsef and S. N. El-Deeb, “On precontinuous and weak precontinuous mappings”, Proc. Math. Phys. Soc. Eygpt, 53(1982), 47-53.

[26] A. S. Mashhour I.A Hasanein and S. N. El-Deeb, “α-continuous and α-open mappings”, Acta Math. Hungar., 41 (1983), 213-218.

[27] A. S. Mashhour, M. E. Abd El Monsef, I.A Hasanein and T. Noiri, “Strongly compact spaces”, Delta. J. Sci., 8 (1984), 30-46.

[28] Metin Adkag, “On bI open sets and bI continuous functions”, Internat. J. Math. Math. Sci., Vol 2007, Article ID 75721, 13 pages.

[29] O. Njastad , “On some classes of nearly open sets”, Pacific J.Math.,15(1965), 961-970.

[30] F. Nirmala Irudayam and Sr. I. Arockiarani, “A note on the weaker form of bI set and its generalization in SEITS”, International Journal of Computer Application, Issue 2, Vol.4 (Aug 2012), 42-54.

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