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COMMON FIXED POINT THEOREM FOR THREE MAPS BY ALTERING DISTANCES BETWEEN THE POINTS

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ABSTRACT

In this paper, we prove a common fixed point theorem for three maps under generalized weakly contractive condition without appeal to continuity. Our results extend and generalized the results of Choudhury et al. [2] and others.

Keywords: *Fixed point, Control Function, Weakly Compatible mappings, Weak Contraction.*

2000 Mathematics Subject Classifications: *47H10, 54H25.*

1. INTRODUCTION

The study of fixed points of mappings satisfying certain contractive conditions has been at the centre of rigorous research activity. In 1977, Rhoades [9] showed that there are several possible types of extended forms of contraction pairs. In 1986, Jungck [4] introduced the notion of compatible mappings which are more general than commuting and weakly commuting mappings. In 1998, Jungck and Rhoades [5] introduced the concept of weakly compatible and showed that compatible maps are weakly compatible but not conversely. In [3], R. Chugh and S. Kumar proved a fixed point theorem for weakly compatible maps without appeal to continuity.

Khan *et al.* [6] introduced the altering distance and used it solving for fixed point problem in metric spaces. Recently many authors for example [7], [11] and [12] used the altering distance function and obtained some fixed point theorem. Further, the concept of weak contraction was introduced in 1997 by Alber *et al.* [1] in Hilbert spaces and subsequently extended to metric spaces by Rhoades [10]. Recently, O. Popescu [8] proved fixed point problem involving weak contraction and mapping satisfying weak contractive type inequalities.

The main purpose of this paper is to present fixed points results for three maps satisfying a generalized weak contraction condition by using the concept of weakly compatible maps in a complete metric space. Our results extend and generalized the results of Choudhury *et al.* [2] and others.

2. PRELIMINARIES

Definition 2.1 ([10]): A mapping $T : X \to X$, where (X,d) is a metric space, is said to be weakly contractive if for $x, y \in X$

$$d(Tx,Ty) \le d(x,y) - \varphi(d(x,y)),$$

where $\varphi:[0,\infty) \to [0,\infty)$ is a continuous non-decreasing function such that $\varphi(t) = 0$ if and only if $t = 0$. If one takes $\varphi(t) = (1-k)t$, where $0 < k < 1$, a weak contraction reduces to a Banach contraction.

Definition 2.2 ([6]): A function $\psi :[0,\infty) \to [0,\infty)$ is called altering distance function if the following properties are satisfied,

(i) ψ is monotone increasing and continuous.

(ii) $\psi(t) = 0$ if and only if $t = 0$.

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Definition 2.3 ([2]): Let (X,d) be a metric space, T a self-mapping of X. We shall call T a generalized weakly contractive mapping if for all $x, y \in X$,

$$\psi\left(d\left(Tx,Ty\right)\right) \le \psi\left(m\left(x,y\right)\right) - \varphi\left(\max\left\{d\left(x,y\right), d\left(y,Ty\right)\right\}\right),, \tag{2.1}$$

where

$$m(x,y) = \max\{d(x,y), d(x,Tx), d(y,Ty), \frac{1}{2}[d(x,Ty)+d(y,Tx)]\},$$

ψ is an altering distance function and $\varphi:[0,\infty)\to[0,\infty)$ is continuous function with $\varphi(t)=0$ if and only if $t=0$.

A generalized weakly contractive mapping is more general than that satisfying,

$$d(Tx,Ty) \le km(x,y), \text{ for some constant } 0 \le k < 1, \tag{2.2}$$

and is included in those mappings which satisfy

$$d(Tx,Ty) < m(x,y). \tag{2.3}$$

Using the numbering scheme in [8], (2.2) and (2.3) are (21) and (22) respectively.

Definition 2.4 ([4]): Let S and T be mapping from a metric space (X,d) into itself. The mapping S and T are said to be compatible if

$$\lim_{n\to\infty} d(STx_n, TSx_n) = 0,$$

whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = t$ for some $t \in X$.

Definition 2.5 ([5]): Let S and T be mapping from a metric space (X,d) into itself. The mapping S and T are said to be weakly compatible if they commute at their coincidence points, that is, if $Tu = Su$ for some $u \in X$, then $TSu = STu$.

3. RESULTS

Now we state our main results

Theorem 3.1: Let (X,d) be a complete metric space. Let $S, f, g : X \to X$ be self- mappings such that for all, $x, y \in X$,

$$S(X) \subset f(X) \cap g(X), \tag{3.1}$$

$$\begin{aligned}\psi(d(Sx,Sy)) \le \psi(\max\{d(fx,gy), d(Sx,fx), d(Sy,gy), \frac{1}{2}[d(Sx,gy)+d(Sy,fx)]\}) \\ -\varphi(\max\{d(fx,gy), d(Sx,fx), d(Sy,gy)\}),\end{aligned} \tag{3.2}$$

where $\varphi:[0,\infty)\to[0,\infty)$ is a continuous function with $\varphi(t)=0$ if and only if $t=0$ and $\psi:[0,\infty)\to[0,\infty)$ is an altering distance function. Then S, f and S, g have a coincidence point. Further if (S, f) and (S, g) are weakly compatible pairs, then S, f and g have a unique common fixed point.

Proof: Let $x_0 \in X$ be arbitrary. We define a sequence $\{y_n\}$ such that

$$y_{2n} = Sx_{2n} = fx_{2n+1}$$
$$y_{2n+1} = Sx_{2n+1} = gx_{2n+2}$$

for all $n \in N$.

If there exist a positive integer $2n$ such that $y_{2n} = y_{2n+1}$, then y_{2n} is a coincidence of S and f. A similar conclusion holds if $y_{2n+1} = y_{2n+2}$, for some n, then S and g have a coincidence point. Therefore, we may assume that $y_n \neq y_{n+1}$, for all $n \geq 0$.

Applying contractive condition (3.2) we obtain that

$$\begin{aligned}\psi(d(y_{2n+1}, y_{2n+2})) &= \psi(d(Sx_{2n+1}, Sx_{2n+2}))\\ &\leq \psi(\max\{d(fx_{2n+1}, gx_{2n+2}), d(Sx_{2n+1}, fx_{2n+1}), d(Sx_{2n+2}, gx_{2n+2}),\\ &\quad \frac{1}{2}[d(Sx_{2n+1}, gx_{2n+2}) + d(Sx_{2n+2}, fx_{2n+1})]\})\\ &\quad -\varphi(\max\{d(fx_{2n+1}, gx_{2n+2}), d(Sx_{2n+1}, fx_{2n+1}), d(Sx_{2n+2}, gx_{2n+2})\})\end{aligned}$$

$$\begin{aligned}\psi(d(y_{2n+1}, y_{2n+2})) &\leq \psi(\max\{d(y_{2n}, y_{2n+1}), d(y_{2n+1}, y_{2n}), d(y_{2n+2}, y_{2n+1}),\\ &\quad \frac{1}{2}[d(y_{2n+1}, y_{2n+1}) + d(y_{2n+2}, y_{2n})]\})\\ &\quad -\varphi(\max\{d(y_{2n}, y_{2n+1}), d(y_{2n+1}, y_{2n}), d(y_{2n+2}, y_{2n+1})\})\end{aligned}$$

$$\begin{aligned}\psi(d(y_{2n+1}, y_{2n+2})) &\leq \psi(\max\{d(y_{2n}, y_{2n+1}), d(y_{2n+1}, y_{2n+2}), \frac{1}{2}[d(y_{2n}, y_{2n+1}) + d(y_{2n+1}, y_{2n+2})]\})\\ &\quad -\varphi(\max\{d(y_{2n}, y_{2n+1}), d(y_{2n+2}, y_{2n+1})\}).\end{aligned}$$

Since $\frac{1}{2}[d(y_{2n}, y_{2n+1}) + d(y_{2n+1}, y_{2n+2})] \leq \max\left\{d(y_{2n}, y_{2n+1}), d(y_{2n+1}, y_{2n+2})\right\}$ then it follows that

$$\begin{aligned}\psi(d(y_{2n+1}, y_{2n+2})) &\leq \psi(\max\{d(y_{2n}, y_{2n+1}), d(y_{2n+1}, y_{2n+2})\})\\ &\quad -\varphi(\max\{d(y_{2n}, y_{2n+1}), d(y_{2n+1}, y_{2n+2})\}).\end{aligned} \tag{3.3}$$

Suppose that $d(y_{2n}, y_{2n+1}) \leq d(y_{2n+1}, y_{2n+2})$ for some positive integer n.

Then from (3.3), we have

$$\psi(d(y_{2n+1}, y_{2n+2})) \leq \psi(d(y_{2n+1}, y_{2n+2})) - \varphi(d(y_{2n+1}, y_{2n+2})), \tag{3.4}$$

that is, $\varphi(d(y_{2n+1}, y_{2n+2})) \leq 0$ which implies that $d(y_{2n+1}, y_{2n+2}) = 0$, or that $y_{2n+1} = y_{2n+2}$, contradicting our assumption that $y_n \neq y_{n+1}$, for each n.

Therefore $d(y_{2n+1}, y_{2n+2}) < d(y_{2n}, y_{2n+1})$ for all $n \geq 0$.

Now

$$\begin{aligned}\psi\left(d(y_{2n+2}, y_{2n+3})\right) &= \psi\left(d(Sx_{2n+3}, Sx_{2n+2})\right)\\ &\leq \psi(\max\{d(fx_{2n+3}, gx_{2n+2}), d(Sx_{2n+3}, fx_{2n+3}), d(Sx_{2n+2}, gx_{2n+2}),\\ &\quad \frac{1}{2}[d(Sx_{2n+3}, gx_{2n+2}) + d(Sx_{2n+2}, fx_{2n+3})]\})\\ &\quad -\varphi(\max\{d(fx_{2n+3}, gx_{2n+2}), d(Sx_{2n+3}, fx_{2n+3}), d(Sx_{2n+2}, gx_{2n+2})\})\end{aligned}$$

$$\psi(d(y_{2n+2},y_{2n+3}))\le\psi(\max\{d(y_{2n+2},y_{2n+1}),d(y_{2n+3},y_{2n+2}),d(y_{2n+2},y_{2n+1}),\\ \frac{1}{2}[d(y_{2n+3},y_{2n+1})+d(y_{2n+2},y_{2n+2})]\})\\ -\varphi\left(\max\left\{d\left(y_{2n+2},y_{2n+1}\right),d\left(y_{2n+3},y_{2n+2}\right),d\left(y_{2n+2},y_{2n+1}\right)\right\}\right)$$

$$\psi\left(d\left(y_{2n+2},y_{2n+3}\right)\right)\le\psi(\max\{d(y_{2n+2},y_{2n+1}),d(y_{2n+3},y_{2n+2}),d(y_{2n+2},y_{2n+1}),\\ \frac{1}{2}[d(y_{2n+1},y_{2n+2})+d(y_{2n+2},y_{2n+3})])\\ -\varphi(\max\{d(y_{2n+2},y_{2n+1}),d(y_{2n+3},y_{2n+2}),d(y_{2n+2},y_{2n+1})\}).$$

Since $\frac{1}{2}[d(y_{2n+1},y_{2n+2})+d(y_{2n+2},y_{2n+3})]\le\max\{d(y_{2n+1},y_{2n+2}),d(y_{2n+2},y_{2n+3})\}$ then it follows that

$$\psi(d(y_{2n+2},y_{2n+3}))\le\psi(\max\{d(y_{2n+1},y_{2n+2}),d(y_{2n+2},y_{2n+3})\})\\ -\varphi(\max\{d(y_{2n+1},y_{2n+2}),d(y_{2n+2},y_{2n+3})\}).$$

Suppose that $d(y_{2n+1},y_{2n+2})\le d(y_{2n+2},y_{2n+3})$ for some positive integer n.

Then from (3.3), we have

$$\psi(d(y_{2n+2},y_{2n+3}))\le\psi(d(y_{2n+2},y_{2n+3}))-\varphi(d(y_{2n+2},y_{2n+3})), \tag{3.5}$$

that is, $\varphi(d(y_{2n+2},y_{2n+3}))\le 0$ which implies that $d(y_{2n+2},y_{2n+3})=0$ or that $y_{2n+2}=y_{2n+3}$, contradicting our assumption that $y_n\ne y_{n+1}$, for each n.

Therefore $d(y_{2n+2},y_{2n+3})<d(y_{2n+1},y_{2n+2})$ for all $n\ge 0$.

Thus $\{d(y_n,y_{n+1})\}$ is a monotone decreasing sequence of non-negative real numbers. Hence there exist an $r\ge 0$ such that

$$\lim_{n\to\infty}d(y_n,y_{n+1})=r. \tag{3.6}$$

In view of (3.3), for all $n\ge 0$

$$\psi(d(y_{2n+1},y_{2n+2}))\le\psi(d(y_{2n},y_{2n+1}))-\varphi(d(y_{2n},y_{2n+1})).$$

Taking the limit as $n\to\infty$ in the above inequality and using the continuities of φ and ψ we have

$$\psi(r)\le\psi(r)-\varphi(r),$$

Which is a contradiction unless $r=0$.

Hence we have

$$\lim_{n\to\infty}d(y_n,y_{n+1})=0. \tag{3.7}$$

Now we shall show that $\{y_n\}$ is a Cauchy sequence. It is sufficient to show that $\{y_{2n}\}$ is a Cauchy sequence. Suppose that $\{y_{2n}\}$ is not a Cauchy sequence.

Then there exist an $\varepsilon \ge 0$ such that for each even integer $2(k)$ there exist an even integer, $2m(k) > 2n(k) > 2(k)$ such that

$$d(y_{2n(k)}, y_{2m(k)}) \ge \varepsilon, \tag{3.8}$$

for every integer $2(k)$. Let $2m(k)$ be the least even integer exceeding $2n(k)$ satisfying (3.8) such that

$$d(y_{2n(k)}, y_{2m(k)-2}) < \varepsilon.$$

Using the triangle inequality, we have

$$\varepsilon \le d(y_{2n(k)}, y_{2m(k)}) \le d(y_{2n(k)}, y_{2m(k)-2}) + d(y_{2m(k)-2}, y_{2m(k)-1}) + d(y_{2m(k)-1}, y_{2m(k)}),$$

that is, $\varepsilon \le d(y_{2n(k)}, y_{2m(k)}) \le \varepsilon + d(y_{2m(k)-2}, y_{2m(k)-1}) + d(y_{2m(k)-1}, y_{2m(k)})$.

Letting $k \to \infty$ in the above inequality and using (3.7), we have

$$\lim_{k\to\infty} d(y_{2n(k)}, y_{2m(k)}) = \varepsilon. \tag{3.9}$$

Again

$$d(y_{2n(k)}, y_{2m(k)}) \le d(y_{2n(k)}, y_{2n(k)+1}) + d(y_{2n(k)+1}, y_{2m(k)+1}) + d(y_{2m(k)+1}, y_{2m(k)})$$

and

$$d(y_{2n(k)+1}, y_{2m(k)+1}) \le d(y_{2n(k)+1}, y_{2n(k)}) + d(y_{2n(k)}, y_{2m(k)}) + d(y_{2m(k)}, y_{2m(k)+1}).$$

Letting $k \to \infty$ in the above inequality and using (3.7) and (3.9), we have

$$\lim_{k\to\infty} d(y_{2n(k)+1}, y_{2m(k)+1}) = \varepsilon. \tag{3.10}$$

Again

$$d(y_{2n(k)}, y_{2m(k)+2}) \le d(y_{2n(k)}, y_{2n(k)+1}) + d(y_{2n(k)+1}, y_{2m(k)+1}) + d(y_{2m(k)+1}, y_{2m(k)+2}).$$

Letting $k \to \infty$ in the above inequality and using (3.7) and (3.10), we have

$$\lim_{k\to\infty} d(y_{2n(k)}, y_{2m(k)+2}) = \varepsilon. \tag{3.11}$$

Further

$$d(y_{2n(k)}, y_{2m(k)+1}) \le d(y_{2n(k)}, y_{2n(k)+1}) + d(y_{2n(k)+1}, y_{2m(k)+1}).$$

Letting $k \to \infty$ in the above inequality and using (3.7) and (3.10), we have

$$\lim_{k\to\infty} d(y_{2n(k)}, y_{2m(k)+1}) = \varepsilon. \tag{3.12}$$

For $x = y_{2n(k)}$ and $y = y_{2m(k)+1}$, we have from (3.2),

$$\begin{aligned}\psi(d(y_{2n(k)+1}, y_{2m(k)+2})) &= \psi(d(Sx_{2n(k)}, Sx_{2m(k)+1}))\\ &\le \psi(\max\{d(fx_{2n(k)}, gx_{2m(k)+1}), d(Sx_{2n(k)}, fx_{2n(k)}), d(Sx_{2m(k)+1}, gx_{2m(k)+1}),\\ &\quad \frac{1}{2}[d(Sx_{2n(k)}, gx_{2m(k)+1}) + d(Sx_{2m(k)+1}, fx_{2n(k)})]\})\\ &\quad -\varphi(\max\{d(fx_{2n(k)}, gx_{2m(k)+1}), d(Sx_{2n(k)}, fx_{2n(k)}), d(Sx_{2m(k)+1}, gx_{2m(k)+1})\})\end{aligned}$$

$$\psi(d(y_{2n(k)+1}, y_{2m(k)+2})) \le \psi(\max\{d(y_{2n(k)}, y_{2m(k)+1}), d(y_{2n(k)+1}, y_{2n(k)}), d(_{2m(k)+2}, y_{2m(k)+1}), \frac{1}{2}[d(y_{2n(k)+1}, y_{2m(k)+1}) + d(y_{2m(k)+2}, y_{2n(k)})]\}) - \varphi\left(\max\left\{d(y_{2n(k)}, y_{2m(k)+1}), d(y_{2n(k)+1}, y_{2n(k)}), d(y_{2m(k)+2}, y_{2m(k)+1})\right\}\right).$$

Letting $k \to \infty$ in the above inequality and using (3.7), (3.9-3.12) and using the continuities of φ and ψ, we have

$$\psi(\varepsilon) \le \psi(\varepsilon) - \varphi(\varepsilon),$$

which is a contradiction by virtue of a property of φ.

Therefore, $\{y_{2n}\}$ is a Cauchy sequence. In view of (3.7), $\{y_n\}$ is also a Cauchy sequence in X.

Since X is complete then there exist a point z in X such that

$$\lim_{n\to\infty} y_{2n} = \lim_{n\to\infty} Sx_{2n} = \lim_{n\to\infty} fx_{2n+1} = z \text{ and}$$
$$\lim_{n\to\infty} y_{2n+1} = \lim_{n\to\infty} Sx_{2n+1} = \lim_{n\to\infty} gx_{2n+2} = z.$$

Since $S(X) \subset f(X) \cap g(X)$, then there exist a points u and $v \in X$ such that $fu = z$ and $gv = z$.

We shall prove that $fu = Su$ and $gv = Sv$.

For this firstly we have,

$$\psi(d(Su, fx_{2n+1})) = \psi(d(Su, Sx_{2n}) \le \psi(\max\{d(fu, gx_{2n}), d(Su, fu), d(Sx_{2n}, gx_{2n}), \frac{1}{2}[d(Su, gx_{2n}) + d(Sx_{2n}, fu)]\}) - \varphi(\max\{d(fu, gx_{2n}), d(Su, fu), d(Sx_{2n}, gx_{2n})\}).$$

Taking limit $n \to \infty$ we have

$$\psi(d(Su, z)) \le \psi(\max\{d(z,z), d(Su,z), d(z,z), \frac{1}{2}[d(Su,z) + d(z,z)]\}) - \varphi(\max\{d(z,z), d(S\ u z), d(z,z)\})$$

$$\psi(d(Su,z)) \le \psi(\max\{0, d(Su,z), 0, \frac{1}{2}[d(Su,z) + 0]\}) - \varphi(\max\{0, d(Su,z), 0\})$$

$$\psi(d(Su,z)) \le \psi(d(Su,z)) - \varphi(d(Su,z)).$$

Which implies that $\varphi(d(Su,z)) = 0$. Hence $d(Su,z) = 0$, that is, $z = Su$.

Therefore $z = fu = Su$.

Now

$$\psi(d(gx_{2n+2},Sv))=\psi(d(Sx_{2n+1},Sv))$$

$$\le\psi(\max\{d(fx_{2n+1},gv),d(Sx_{2n+1},fx_{2n+1}),d(Sv,gv),$$

$$\frac{1}{2}[d(Sx_{2n+1},gv)+d(Sv,fx_{2n+1})]\})$$

$$-\varphi(\max(d(fx_{2n+1},gv),d(Sx_{2n+1},fx_{2n+1}),d(Sv,gv)\}).$$

Taking limit $n\to\infty$ we have

$$\psi(d(z,Sv))\le\psi(\max\{d(z,z),d(z,z),d(Sv,z),\frac{1}{2}[d(z,z)+d(Sv,z)]\})$$

$$-\varphi(\max\{d(z,z),d(z,z),d(Sv,z)\})$$

$$\psi(d(z,Sv))\le\psi(\max\{0,0,d(Sv,z),\frac{1}{2}[0+d(Sv,z)]\})-\varphi(\max\{0,0,d(Sv,z)\})$$

$$\psi(d(z,Sv))\le\psi(d(z,Sv))-\varphi(d(z,Sv)).$$

Which implies that $\varphi(d(z,Sv))=0.$ Hence $d(z,Sv)=0,$ that is, $z=Sv.$

Therefore, $z=gv=Sv.$ Thus $z=fu=Su=Sv=gv.$

Since pair of maps S and f are weakly compatible then $Sfu=fSu,$ that is, $Sz=fz.$

Now we show that z is a fixed point of f.

$$\psi(d(fz,z))=\psi(d(Sz,Sv))$$

$$\le\psi(\max\{d(fz,gv),d(Sz,fz),d(Sv,gv),\frac{1}{2}[d(Sz,gv)+d(Sv,fz)]\})$$

$$-\varphi(\max\{d(fz,gv),d(Sz,fz),d(Sv,gv)\})$$

$$=\psi(\max\{d(fz,z),0,0,\frac{1}{2}[d(fz,z)+d(z,fz)\})-\varphi(\max\{d(fz,z),0,0\})$$

$$\psi(d(fz,z))\le\psi(d(fz,z))-\varphi(d(fz,z)).$$

Which implies that $\varphi(d(fz,z))=0.$ Hence $d(fz,z)=0,$ that is, $fz=z.$

Therefore $z=fz=Sz.$

Similarly the pair of maps S and g are weakly compatible, then $Sgv=gSv,$ that is, $Sz=gz.$

Now we show that z is a fixed point of g.

$$\psi(d(z,gz))=\psi(d(Su,Sz))$$

$$\le\psi(\max\{d(fu,gz),d(Su,fu),d(Sz,gz),\frac{1}{2}[d(Su,gz)+d(Sz,fu)]\})$$

$$-\varphi(\max\{d(fu,gz),d(Su,fu),d(Sz,gz)\})$$

$$=\psi(\max\{d(z,gz),0,0,\frac{1}{2}[d(z,gz)+d(z,gz)]\})-\varphi(\max\{d(z,gz),0,0\})$$

$$\psi(d(z,gz)) \le \psi(d(z,gz)) - \varphi(d(z,gz)).$$

Which implies that $\varphi(d(z,gz)) = 0.$ Hence $d(z,gz) = 0,$ that is, $z = gz.$

Therefore $z = gz = Sz.$

Thus $z = Sz = fz = gz,$ and z is a common fixed point of S, f and $g.$

Finally in order to prove the uniqueness of z, suppose that z and w, $z \ne w$ are common fixed points of S, f and $g.$

Then by (3.2), we obtain,

$$\begin{aligned}\psi(d(z,w)) &= \psi(d(Sz,Sw)) \\ &\le \psi(\max\{d(fz,gw), d(Sz,fz), d(Sw,gw), \tfrac{1}{2}[d(Sz,gw)+d(Sw,fz)]\}), \\ &\quad -\varphi(\max\{d(fz,gw), d(Sz,fz), d(Sw,gw)\})\end{aligned}$$

$$\psi(d(z,w)) \le \psi(\max\{d(z,w),0,0,\tfrac{1}{2}[d(z,w)+d(z,w)]\}) - \varphi(\max\{d(z,w),0,0\})$$

$$\psi(d(z,w)) \le \psi(d(z,w)) - \varphi(d(z,w)).$$

Which implies that $\varphi(d(z,w)) = 0$. Hence $d(z,w) = 0,$ that is, $z = w.$

Corollary 3.1: Let (X,d) be a complete metric space. Let $S, f : X \to X$ be self-mappings such that for all $x, y \in X$,

$$S(X) \subset f(X), \tag{3.13}$$

$$\begin{aligned}\psi(d(Sx,Sy)) &\le \psi(\max\{d(fx,fy), d(Sx,fx), d(Sy,fy), \tfrac{1}{2}[d(Sx,fy)+d(Sy,fx)]\}) \\ &\quad -\varphi(\max\{d(fx,fy), d(Sx,fx), d(Sy,fy)\}),\end{aligned} \tag{3.14}$$

where $\varphi:[0,\infty) \to [0,\infty)$ is a continuous function with $\varphi(t) = 0$ if and only if $t = 0$ and $\psi:[0,\infty) \to [0,\infty)$ is an altering distance function. Then S and f have a coincidence point. Further if (S, f) is a weakly compatible pair, then S and f have a unique common fixed point.

Proof: By taking $f = g$ in theorem 3.1, we get the proof.

Remark 3.1: If we take f as an identity map in Corollary 3.1, then we obtain Theorem 3.1 of [2].

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