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UNSTEADY FLOW OF VISCO-ELASTIC [RIVLIN-ERICKSEN (1955) MODEL] FLUID THROUGH POROUS MEDIUM IN A LONG UNIFORM RECTANGULAR CHANNEL

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ABSTRACT

The flow of visco-elastic Rivlin-Ericksen (1955) type fluid through Porous medium in a long uniform straight tube of rectangular cross-section has been studied. The flow has been considered under the influence of time varying pressure gradient. Using integral transform technique, the exact solution for velocity of fluid has been obtained. A few particular cases of pressure gradient have been discussed in detail. Besides, the corresponding viscous flow problem has been derived as a limiting case when the relaxation time parameter tends to become zero. We have also derived the case when porous medium is withdrawn i.e. if $K = \infty$.

INTRODUCTION

The problem of viscous fluid motion subjected to uniform and periodic body force for a finite time has been investigated by Dutta (1958). The flow of visco-elastic fluid between two parallel plates under uniform, exponential or periodic pressure gradient has been investigated by Das (1991) and Pal and Sengupta (1986). Roy, Sen and Lahiri (1990) studied the problem of unsteady flow of Rivlin-Ericksen fluid through a rectangular duct under impulsive pressure gradient. Bagchi (1965) has considered a similar problem through rectangular channel and through two parallel plates with transient pressure gradient. Drake (1965) studied the flow of an incompressible viscous fluid along a rectangular channel due to a periodic pressure gradient. The flow of immicible visco-elastic Maxwell fluid through a rectangular channel was studied by Sengupta and Ray Mahapatra (1971). Sengupta and Banerjee (2005) studied the unsteady MHD flow of visco-elastic Rivlin-Ericksen and Walter fluid through a straight tube. Kumar, Singh and Sharma (2009) discussed the unsteady flow of Rivlin-Ericksen visco-elastic fluid through porous medium in a confocal elliptical duct.

In the present paper, the unsteady flow of Rivlin-Ericksen visco-elastic fluid through porous medium in a uniform long straight channel of rectangular cross-section under the influence of time dependent pressure gradient has been studied. Various particular cases have also been discussed in detail.

BASIC THEORY

The visco-elastic fluid of Rivlin-Ericksen type is consitituted by the rheological equations:

$$\tau_{ij} = -p\delta_{ij} + \tau'_{ij},$$

$$\tau'_{ij} = 2\mu\left(1 + \mu_1 \frac{\partial}{\partial t}\right)e_{ij},$$

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}),$$

where τ_{ij} is the stress tensor, τ'_{ij} the deviatoric stress tensor, e_{ij} the rate of strain tensor, μ_1 the kinematical co-efficient of visco-elasticity, μ the co-efficient of viscosity, p the hydrodynamic pressure, u_i represents the velocity components of the fluid and δ_{ij} is the kronecker delta.

Now, the fundamental Navier-Stokes equation of motion is

$$\frac{d\vec{q}}{dt} = -\frac{1}{\rho}\vec{\nabla}p + v^*\nabla^2\vec{q} + \vec{F},$$

where

$$\tau'_{ij} = 2\mu^* e_{ij},$$

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i.e., $\mu^* = \mu\left(1 + \mu_1 \frac{\partial}{\partial t}\right)$ and $v^* = \frac{\mu^*}{\rho}$,

Finally, we get the Navier-Stokes equation of motion for Rivlin-Ericksen fluid as

$$\frac{d\vec{q}}{dt} = -\frac{1}{\rho}\vec{\nabla}p + v\left(1 + \mu_1 \frac{\partial}{\partial t}\right)\nabla^2\vec{q} + \vec{F},$$

where $\vec{q}$ is the velocity, $\vec{F}$ is the body force and $v\left(= \frac{\mu}{\rho}\right)$ is the co-efficient of viscosity

FORMULATION OF THE PROBLEM

Flow of visco-elastic Rivlin-Ericksen fluid through porous medium in a long uniform rectangular channel is considered here. The boundary of the walls of the rectangular tube are considered to be the planes $x = \pm a,\ y = \pm b,$ the motion is under the influence of a time varying pressure gradient taking the fluid initially at rest.

The Navier-Stokes equation of motion for Rivlin-Ericksen fluid through porous medium is given by

$$\frac{\partial W}{\partial t} = -\frac{1}{\rho}\cdot\frac{\partial p}{\partial z} + v\left(1 + \mu_1 \frac{\partial}{\partial t}\right)\left(\frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial y^2}\right) - \frac{vW}{K}, \tag{1}$$

where $W(x, y, t)$ is the velocity of the fluid in *z*-direction, μ_1 the kinematical co-efficient of visco-elasticity, ρ the density of the fluid and *K* is the permeabilits of porous medium.

We introduce the following non-dimensional quantities:

$$x^* = \frac{x}{a}, y^* = \frac{y}{a}, z^* = \frac{z}{a}, t^* = \frac{v}{a^2}t,\ p^* = \frac{a^2}{\rho v^2}p$$

$$W^* = \frac{a}{v}W, \mu_1^* = \frac{v}{a^2}\mu_1, K^* = \frac{1}{a^2}K$$

in equation (1), we have after dropping the stars:

$$\frac{\partial W}{\partial z} = -\frac{\partial p}{\partial z} + \left(1 + \mu_1 \frac{\partial}{\partial t}\right)\left(\frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial y^2}\right) - MW, \tag{2}$$

where $M = \frac{1}{K}$

Due to symmetric consideration, the flow is considered in the region $z \geq 0,\ y \geq 0,$ with no slip boundary condition. The fluid is at rest, initially and the flow takes place under the action of time varying pressure-gradient. The initial and boundary conditions are:

$$W(x, y, 0) = 0 \tag{3}$$

$$\left.\begin{aligned} &W(1, y, t) = 0, \quad 0 \leq y \leq l, \qquad t > 0, \\ &W(x, l, t) = 0, \quad 0 \leq x \leq 1, \qquad t > 0, \end{aligned}\right\} \tag{4}$$

$$\left.\begin{aligned} &\frac{\partial W}{\partial x} = 0, \qquad x = 0, \\ &\frac{\partial W}{\partial y} = 0, \qquad y = 0, \end{aligned}\right\} \tag{5}$$

where $l = \frac{b}{a}$.

SOLUTION OF THE PROBLEM

The following finite Fourier cosine transforms are used to find out the solution of the problem:

$$W_c(m, y, t) = \int_0^1 W(x, y, t)\cos(p_m x)\, dx \tag{6}$$

$$W_{\bar{c}}(m, n, t) = \int_0^1 W_c(x, y, t)\cos(p_n y)\, dy, \tag{7}$$

where

$$p_m = (2m+1)\frac{\pi}{2}, \quad p_n = (2n+1)\frac{\pi}{2l}.$$

Consequently, we have the following inverse of finite Fourier cosine transforms defined as:

$$W(x, y, t) = 2\sum_{m=0}^{\infty} W_c(m, y, t)\cos(p_m x), \tag{8}$$

$$W_c(m, y, t) = \frac{2}{l}\sum_{n=0}^{\infty} W_{\bar{c}}(m, n, t)\cos(p_n y), \tag{9}$$

We apply transforms (6) and (7) to initial condition (3) and we have

$$W_{\bar{c}}(m, n, 0) = 0 \tag{10}$$

Also taking finite Fourier cosine transform to boundary conditions we get

$$\left.\begin{aligned} &W_c(m, l, t) = 0 \\ &\frac{\partial W_c}{\partial y}(m, 0, t) = 0 \end{aligned}\right\} \tag{11}$$

We apply (6) and (7) to the equation of motion (2) and using transformed initial and boundary conditions (10) and (11), we get

$$[1 + \mu_1(p_m + p_n^2)]\frac{\partial W_{\bar{c}}}{\partial t} + (M + p_m^2 + p_n^2)W_{\bar{c}} = \frac{(-1)^{m+n}}{p_m p_n}G(t),$$

where

$$W_{\bar{c}} = \int_0^1\int_0^l W(x, y, t)\cos(p_m x)\cos(p_n y)dx\, dy,$$

$$\frac{\partial p}{\partial z} = -G(t) \quad \text{(an arbitrary function of time).}$$

Now, considering $1 + \mu_1(p_m^2 + p_n^2) = \eta$ and $M + p_m^2 + p_n^2 = \xi$, the above equation takes the form,

$$\eta\frac{\partial W_{\bar{c}}}{\partial t} + \xi W_{\bar{c}} = \frac{(-1)^{m+n}G(t)}{p_m p_n} \tag{12}$$

Using Laplace integral transform defined by

$$Q_{\bar{c}}(s) = \int_0^{\infty} W_{\bar{c}}\, e^{-st}dt,$$

$$\bar{G}(s) = \int_0^{\infty} G(t)\, e^{-st}dt, \tag{13}$$

Using condition (11) in equation (12), we have

$$\eta s Q_{\bar{c}} + \xi Q_{\bar{c}} = \frac{(-1)^{m+n}\bar{G}(s)}{p_m p_n} \tag{14}$$

Now, solving equation (14) by Laplace inversion formula and using convolution theorem, we get,

$$W_{\bar{c}} = \frac{(-1)^{m+n}}{p_m p_n \eta} \int_0^t G(t-u) e^{-(\xi/\eta)u} du. \tag{15}$$

Finally applying inverse Fourier cosine formula as given in equations (8) and (9), the expression of velocity is obtained from (15) as

$$W(x,y,t) = \frac{4}{l} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{(-1)^{m+n}}{p_m p_n \eta} \left[\int_0^t G(t-u) e^{-ku} du \right] \times \cos(p_m x) \cos(p_n y), \tag{16}$$

where

$$k = \frac{\xi}{\eta},\ p_m = (2m+1)\frac{\pi}{2},\ p_n = (2n+1)\frac{\pi}{2l},$$

We discuss the nature of velocity for following different particular cases:

Case I: Flow under constant pressure gradient:

Let, $G(t) = P_0$ (a constant).

From equation (16), the velocity reduced to

$$W = \frac{4}{l} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \left[\frac{(-1)^{m+n} P_0}{p_m p_n \xi} (1 - e^{-kt}) \cos(p_m x) \cos(p_n y) \right]. \quad . \tag{17}$$

Case II: Flow under impulsive pressure gradient:

Let, $G(t) = F_0 \delta(t),$

where $\delta(t)$ is the unit impulse function defined as

$$\delta(t) = \begin{cases} 0, & t \neq 0 \\ 1, & t = 0 \end{cases}$$

So, from equation (16), the velocity reduced to

$$W = \frac{4}{l} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \left[\frac{(-1)^{m+n} F_0}{p_m p_n \eta} e^{-kt} \cos(p_m x) \cos(p_n y) \right]. \tag{18}$$

Case III: Flow under transient pressure gradient:

Let, $G(t) = F_1 e^{-\omega t}, (\omega > 0),$

where F_1 is a constant.

From equation (16), the velocity reduced to

$$W = \frac{4}{l} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \left[\frac{(-1)^{m+n} F_1 e^{-\omega t}}{p_m p_n (\xi - \eta)} \left\{ 1 - e^{-(k-\omega t)} \right\} \times \cos(p_m x) \cos(p_n y) \right] \tag{19}$$

Case IV: Flow under periodic pressure gradient:

Let, $G(t) = \mathrm{Re}(f_1 e^{i\omega t}),$

where f_1 is a constant,

From equation (16), the velocity reduces to

$$W = \frac{4}{l}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\left[\frac{(-1)^{m+n}F_1}{p_m p_n(\xi^2 + N^2\eta^2)}\left\{N\eta\sin\omega t + \xi(\cos\omega t - 1)\right\}\times\cos(p_m x)\cos(p_n y)\right]. \qquad (20)$$

Case V: When the fluid is purely viscous:

For purely viscous fluid the kinematical coefficient of visco-elasticity $\mu_1 = 0$, and we have

$$W = \frac{4}{l}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\left[\frac{(-1)^{m+n}}{p_m p_n}\int_0^t G(t-u)e^{-\xi u}du\right]\times\cos(p_m x)\cos(p_n y), \qquad (21)$$

as here

$$\xi = M + p_m^2 + p_n^2, \qquad \eta = 1,$$

so,

$$k = \frac{\xi}{\eta} = \xi.$$

Case VI: When porous medium is withdrawn i.e. $K = \infty$ or $M = 0$.

We get all results for fluid motion in the absence of porous medium.

The values of ξ and η are given by

$$\xi = p_m^2 + p_n^2 \quad \text{and} \quad \eta = 1 + \mu_1(p_m^2 + p_n^2) = 1 + \mu_1\xi \qquad (22)$$

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