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SEMIPRE GENERALIZED CLOSED SETS IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

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ABSTRACT

This paper is devoted to the study of intuitionistic fuzzy topological spaces. In this paper an intuitionistic fuzzy semipre generalized closed set introduced. Some of its properties are studied.

Keywords: *Intuitionistic fuzzy topology, intuitionistic fuzzy semipre generalized closed sets and intuitionistic fuzzy points.*

AMS Subject Classification (2000): *54A40, 03F55.*

1. INTRODUCTION

Fuzzy set(FS), proposed by Zadeh [13] in 1965, as a framework to encounter uncertainty, vagueness and partial truth, represents a degree of membership for each member of the universe of discourse to a subset of it. Later on, fuzzy topology was introduced by Chang [3] in 1967. By adding the degree of non-membership to FS, Atanassov [1] proposed intuitionistic fuzzy set (IFS) in 1983 which looks more accurately to uncertainty quantification and provides the opportunity to precisely model the problem based on the existing knowledge and observations. In the last few years various concepts in fuzzy were extended to intuitionistic fuzzy sets. In 1997, Coker [4] introduced the concept of intuitionistic fuzzy topological space. In 2005, R. K. Saraf [8] introduced the concept of fuzzy semipre generalized closed sets in fuzzy topological space. In this paper, we introduce intuitionistic fuzzy semipre generalized closed sets in intuitionistic fuzzy topological space. We established their properties and relationships with other classes of early defined forms.

2. PRELIMINARIES

Definition 2.1: [1] Let X be a non-empty fixed set. An intuitionistic fuzzy set (IFS in short) A in X is an object having the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$ where the functions $\mu_A: X \to [0,1]$ and $\nu_A: X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \le \mu_A(x) + \nu_A(x) \le 1$ for each $x \in X$. Denote by IFS(X), the set of all intuitionistic fuzzy sets in X.

Definition 2.2: [1] Let A and B be IFSs of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle / x \in X\}$. Then

(i) $A \subseteq B$ if and only if $\mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \ge \nu_B(x)$ for all $x \in X$,
(ii) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$,
(iii) $A^c = \{ \langle x, \nu_A(x), \mu_A(x)\rangle / x \in X \}$,
(iv) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle / x \in X\}$,
(v) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle / x \in X\}$.

For the sake of simplicity, we shall use the notation $A = \langle x, \mu_A, \nu_A\rangle$ instead of $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$.

Definition 2.3: [4] An intuitionistic fuzzy topology (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms:

(i) $0_\sim, 1_\sim \in \tau$,
(ii) $G_1 \cap G_2 \in \tau$, for any $G_1, G_2 \in \tau$,
(iii) $\cup G_i \in \tau$ for any family $\{G_i / i \in J\} \subseteq \tau$.

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In this case the pair (X, τ) is called an *intuitionistic fuzzy topological space* (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set(IFOS in short)in X. The complement A^c of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 2.4: [4] Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS in X. Then the intuitionistic fuzzy interior and an intuitionistic fuzzy closure are defined by
int(A) = ∪{ G / G is an IFOS in X and G ⊆ A },
cl(A) = ∩{ K / K is an IFCS in X and A ⊆ K }.

Definition 2.5: [5] An IFS A of an IFTS (X, τ) is an
(i) *intuitionistic fuzzy semiclosed set* (IFSCS in short) if int(cl(A)) ⊆ A,
(ii) *intuitionistic fuzzy semiopen set* (IFSOS in short) if A ⊆ cl(int(A)).

Definition 2.6: [5] An IFS A of an IFTS (X, τ) is an
(i) *intuitionistic fuzzy preclosed set* (IFPCS in short) if cl(int(A)) ⊆ A,
(ii) *intuitionistic fuzzy preopen set* (IFPOS in short) if A ⊆ int(cl(A)).

Note that every IFOS in (X, τ) is an IFPOS in X.

Definition 2.7: [5] An IFS A of an IFTS (X, τ) is an intuitionistic fuzzy semipre closed set (IFSPCS for short) if there exists an IFPCS B such that int(B) ⊆ A ⊆ B.

Definition 2.8: [5] An IFS A of an IFTS (X, τ) is an
(i) *intuitionistic fuzzy α-closed set* (IFαCS in short) if cl(int(cl(A))) ⊆ A,
(ii) *intuitionistic fuzzy α-open set* (IFαOS in short) if A ⊆ int(cl(int(A))),
(iii) *intuitionistic fuzzy regular closed set* (IFRCS in short) if A = cl(int(A)),
(iv) *intuitionistic fuzzy regular open set* (IFROS in short) if A = int(cl(A)),
(v) *intuitionistic fuzzy β-closed set* (IFβCS in short) if int(cl(int(A))) ⊆ A,
(vi) *intuitionistic fuzzy β-open set* (IFβOS in short) if A ⊆ cl(int(cl(A))).

Definition 2.9: [12] An IFS A of an IFTS (X, τ) is an
(i) *intuitionistic fuzzy semipre closed set* (IFSPCS for short) if there exists an IFPCS B such that int(B) ⊆ A ⊆ B,
(ii) *intuitionistic fuzzy semipre open set* (IFSPOS for short) if there exists an IFPOS B such that B ⊆ A ⊆ cl(B).

Definition 2.10: [11] An IFS A of an IFTS (X, τ) is called an intuitionistic fuzzy w-closed set (IFWCS in short) if cl(A) ⊆ U whenever A ⊆ U and U is an IFSOS in (X, τ). An IFS A of an IFTS (X, τ) is called an intuitionistic fuzzy w-open set (IFWOS in short) if A^c is an IFWCS in (X, τ).

Definition 2.11: [6] An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X, τ) is called an
(i) *intuitionistic fuzzy semi generalized closed set*(IFSGCS in short) if scl(A) ⊆ O whenever A ⊆ O and O is an IFOS in (X, τ),
(ii) *intuitionistic fuzzy semi generalized open set*(IFSGOS in short) if its complement A^c is an IFSGCS in (X, τ).

Definition 2.12: [7] An IFS A is an IFTS (X, τ) is said to be an intuitionistic fuzzy generalized semipre closed set (IFGSPCS) if spcl(A) ⊆ U whenever A ⊆ U and U is an IFSOS in (X, τ). An IFS A of an IFTS (X, τ) is called an intuitionistic fuzzy generalized semipre open set (IFGSPOS in short) if A^c is an IFGSPCS in (X, τ).

Definition 2.13: [5] Two IFSs A and B are said to be q-coincident (A_q B in short) if and only if there exists and element x ∈X such that $\mu_A(x) > \nu_B(x)$ or $\nu_A(x) < \mu_B(x)$.

Definition 2.14: [5] Two IFSs are said to be not q-coincident ($A\ {}_q{}^c\ B$ in short) if and only if A ⊆ B^c.

Definition 2.15: [9] Let α, β ∈ [0, 1] and α+ β ≤ 1. An intuitionistic fuzzy point (IFP for short) $x_{(\alpha,\beta)}$ of X is an IFS of X defined by

$$x_{(\alpha,\beta)}(y) = \begin{cases} (\alpha,\beta) & if \quad y = x \\ (0,1) & if \quad y \neq x \end{cases}$$

Definition 2.16: [7] An IFS A in (X, τ) is an IFQ-set if int(cl(A)) = cl(int(A)).

Result 2.17: [2] For an IFS A in an IFTS(X, τ), we have spcl(A) ⊇ A ∪ (int(cl(int(A)))).

Result 2.18: [10] Every IFOS (IFCS) in an IFTS (X, τ) is an IFSOS (IFSCS).

3. INTUITIONISTIC FUZZY SEMIPRE GENERALIZED CLOSED SET

In this section we have introduced intuitionistic fuzzy semipre generalized closed set and have studied some of its properties.

Definition 3.1: An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy semipre generalized closed set (IFSPGCS for short) if spcl(A) ⊆ U whenever A ⊆ U and U is an IFSOS in (X, τ). The family of all IFSPGCSs of an IFTS(X, τ) is denoted by IFSPGC (X).

For the sake of simplicity, we shall use the notation A= ⟨x, (μ, μ), (ν, ν) ⟩ instead of A= ⟨x,(a/μ_a, b/μ_b), (a/ν_a, b/ν_b)⟩ in all the examples used in this paper.

Example 3.2: Let X= {a, b} and G= ⟨x, (0.5, 0.4), (0.5, 0.6) ⟩. Then τ = {$0_\sim$, G, $1_\sim$} is an IFT on X and the IFS A= ⟨x, (0.4, 0.2), (0.6, 0.7) ⟩ is an IFSPGCS in (X, τ).

Theorem 3.3: Every IFCS in (X, τ) is an IFSPGCS in (X, τ) but not conversely.

Proof: Let A ⊆ U and U be an IFSOS in (X, τ). Then spcl(A) ⊆ cl(A) = A ⊆ U, by hypothesis. Hence A is an IFSPGCS in (X, τ).

Example 3.4: In Example 3.2., the IFS A= ⟨x, (0.4, 0.2), (0.6, 0.7) ⟩ is an IFSPGCS but not an IFCS in (X, τ).

Theorem 3.5: Every IFSPCS in (X, τ) is an IFSPGCS in (X, τ) but not conversely.

Proof: Let A be an IFSPCS and A ⊆ U, U be an IFSOS in (X, τ). Then since spcl(A)= A and A⊆ U, we have spcl(A)⊆ U. Hence A is an IFSPGCS in (X, τ).

Example 3.6: Let X= {a, b} and G_1= ⟨x, (0.7, 0.8), (0.3, 0.2) ⟩, G_2= ⟨x, (0.2, 0.1), (0.8, 0.9) ⟩, G_3= ⟨x, (0.5, 0.6), (0.5, 0.4) ⟩ and G_4= ⟨x, (0.6, 0.7), (0.4, 0.3) ⟩. Then τ = {$0_\sim$, G_1, G_2, G_3, G_4, $1_\sim$} is an IFT on X and the IFS A= ⟨x, (0.9, 0.6), (0.1, 0.4) ⟩ is an IFSPGCS but not an IFSPCS in (X, τ) in (X, τ).

Theorem 3.7: Every IFβCS in (X, τ) is an IFSPGCS in (X, τ) but not conversely.

Proof: Let A be an IFβCS and A ⊆ U, U be an IFSOS in (X, τ). By hypothesis βcl(A)= A and A ⊆ U, we have spcl(A)⊆ U. Hence A is an IFSPGCS in (X, τ).

Example 3.8: Let X= {a, b} and G_1= ⟨x, (0.7, 0.8), (0.3, 0.2)⟩, G_2= ⟨x, (0.2, 0.1), (0.8, 0.9)⟩, G_3= ⟨x, (0.5, 0.6), (0.5, 0.4)⟩ and G_4= ⟨x, (0.6, 0.7), (0.4, 0.3)⟩. Then τ = {$0_\sim$, G_1, G_2, G_3, G_4, $1_\sim$} is an IFT on X and the IFS A= ⟨x, (0.9, 0.6), (0.1, 0.4) ⟩ is an IFSPGCS but not an IFβCS in (X, τ) in (X, τ).

Theorem 3.9: Every IFSCS in (X, τ) is an IFSPGCS in (X, τ) but not conversely.

Proof: Let A be an IFSCS in (X, τ) and let A ⊆ U, U be an IFSOS in (X, τ). Since spcl(A) ⊆ scl(A) = A ⊆ U. Hence spcl(A) ⊆ U. Therefore A is an IFSPGCS in (X, τ).

Example 3.10: In Example 3.2., the IFS A= ⟨x, (0.4, 0.2), (0.6, 0.7) ⟩ is an IFSPGCS but not an IFSCS in (X, τ).

Theorem 3.11: Every IFPCS in (X, τ) is an IFSPGCS in (X, τ) but not conversely.

Proof: Let A be an IFPCS in (X, τ) and let A ⊆ U, U be an IFSOS in (X, τ). By hypothesis, pcl(A) = A. Since spcl(A) ⊆ pcl(A). Therefore spcl(A) ⊆ A ⊆ U. Hence is an IFSPGCS in (X, τ).

Example 3.12: Let X= {a, b} and G= ⟨x, (0.3, 0.2), (0.6, 0.6)⟩. Then τ = {$0_\sim$, G, $1_\sim$} is an IFT on X and the IFS A= ⟨x, (0.3, 0.2), (0.6, 0.6) ⟩ is an IFSPGCS in (X, τ) but not an IFPCS in (X, τ).

Theorem 3.13: Every IFSGCS in (X, τ) is an IFSPGCS in (X, τ) but not conversely.

Proof: Let A be an IFSGCS in (X, τ) and A ⊆ U, U is an IFSOS in (X, τ). Since spcl(A) ⊆ scl(A) and scl(A) ⊆ U, by hypothesis. Hence A is an IFSPGCS in (X, τ).

Example 3.14: Let X= {a, b} and let τ = {$0_\sim$, G_1, G_2, $1_\sim$} where G_1= ⟨x, (0.7, 0.8), (0.3, 0.2)⟩ and G_2= ⟨x, (0.6, 0.7), (0.4, 0.3)⟩. Then the IFS A= ⟨x, (0.6, 0.4), (0.4, 0.3)⟩ is an IFSPGCS in (X, τ) but not an IFSGCS in (X, τ).

Theorem 3.15: Every IFWCS in (X, τ) is an IFSPGCS in (X, τ) but not conversely.

Proof: Let A be an IFWCS in (X, τ) and $A \subseteq U$, U is an IFSOS in (X, τ). By hypothesis $cl(A) \subseteq U$. Since $spcl(A) \subseteq cl(A)$. Therefore $spcl(A) \subseteq U$. Hence A is an IFSPGCS in (X, τ).

Example 3.16: In Example 3.2., the IFS $A= \langle x, (0.4, 0.2), (0.6, 0.7) \rangle$ is an IFSPGCS but not an IFWCS in (X, τ).

Theorem 3.17: Every IFαCS in (X, τ) is an IFSPGCS in (X, τ) but not conversely.

Proof: Let A be an IFαCS in (X, τ) and let $A \subseteq U$, U is an IFSOS in (X, τ). Since every IFαCS is an IFPCS, by Theorem 3.7., A is an IFSPGCS in (X, τ).

Example 3.18: In Example 3.2., the IFS $A= \langle x, (0.4, 0.2), (0.6, 0.7) \rangle$ is an IFSPGCS but not an IFαCS in (X, τ).

Theorem 3.19: Every IFRCS in (X, τ) is an IFSPGCS in (X, τ) but not conversely.

Proof: Let A be an IFRCS in (X, τ). Since every IFRCS is an IFCS, by Theorem 3.3., A is an IFSPGCS in (X, τ).

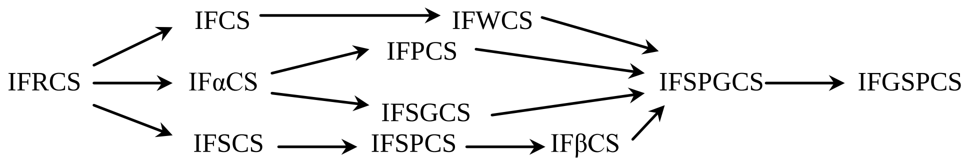
Example 3.20: In Example 3.4., the IFS $A= \langle x, (0.4, 0.2), (0.6, 0.7)\rangle$ is an IFSPGCS but not an IFRCS in (X, τ).

Theorem 3.21: Every IFSPGCS in (X, τ) is an IFGSPCS in (X, τ) but not conversely.

Proof: Let A be an IFSPGCS in (X, τ) and let $A\subseteq U$, U is an IFOS in (X, τ). Since every IFOS in (X, τ) is an IFSOS in (X, τ) and by hypothesis $spcl(A) \subseteq U$. Hence A is an IFGSPCS in (X, τ).

Example 3.22: Let $X= \{a, b\}$ and $G= \langle x, (0.3, 0.6), (0.7, 0.4) \rangle$**.** Then $\tau= \{0_\sim, G, 1_\sim\}$ is an IFT on X and the IFS $A= \langle x, (0.7, 0.6), (0.3, 0.4) \rangle$ is an IFGSPCS in (X, τ) but not an IFSPGCS in (X, τ).

In the following diagram, we have provided the relation between various types of intuitionistic fuzzy closedness.



In this diagram by "A $\longrightarrow$B" we mean A implies B but not conversely.

Remark 3.23: The union of any two IFSPGCSs in (X, τ) is not an IFSPGCS in (X, τ) in general as seen from the following example.

Example 3.24: Let $X= \{a, b\}$ and let $\tau= \{0_\sim, G_1, G_2, 1_\sim\}$ where $G_1= \langle x, (0.7, 0.8), (0.3, 0.2) \rangle$ and $G_2= \langle x, (0.6, 0.7), (0.4, 0.3) \rangle$**.** Then the IFSs $A= \langle x, (0.6, 0.4), (0.4, 0.3)\rangle$ and $B= \langle x, (0.4, 0.8), (0.4, 0.2)\rangle$are IFSPGCSs in (X, τ) but $A\cup B=\langle x, (0.6, 0.8), (0.4, 0.2)\rangle$is not an IFSPGCS in (X, τ). Let $U= \langle x, (0.6, 0.8), (0.4, 0.2)\rangle$ be an IFSOS in (X, τ).

Since $A\cup B \subseteq U$ but $spcl(A\cup B)= 1_\sim \nsubseteq U$.

Remark 3.25: The intersection of any two IFSPGCSs in (X, τ) is not an IFSPGCS in (X, τ) in general as seen from the following example.

Example 3.26: Let $X= \{a, b\}$ and let $\tau = \{0_\sim, G_1, G_2, G_3, G_4, 1_\sim\}$ where $G_1= \langle x, (0.7, 0.8), (0.3, 0.2)\rangle$, $G_2= \langle x, (0.2, 0.1), (0.8, 0.9)\rangle$, $G_3= \langle x, (0.5, 0.6), (0.5, 0.4)\rangle$ and $G_4= \langle x, (0.6, 0.7), (0.4, 0.3) \rangle$**.** Then the IFSs $A= \langle x, (0.9, 0.6), (0.1, 0.4)\rangle$ and $B= \langle x, (0.5, 0.9), (0.5, 0)\rangle$ are IFSPGCSs in (X, τ) but $A\cap B= \langle x, (0.5, 0.6), (0.5, 0.4)\rangle$ is not an IFSPGCS in (X, τ). Let $U= \langle x, (0.5, 0.6), (0.5, 0.4) \rangle$ be an IFSOS in (X, τ). Since $A\cap B \subseteq U$ but $spcl(A\cap B)= \langle x, (0.7, 0.8), (0.3, 0.2)\rangle \nsubseteq U$.

Theorem 3.27: Let (X, τ) be an IFTS. Then for every $A\in IFSPGC(X)$ and for every $B \in IFS(X)$, $A\subseteq B \subseteq spcl (A)$ implies $B \in IFSPGC(X)$.

Proof: Let $B \subseteq U$ and U be an IFSOS in (X, τ). Then since $A \subseteq B$, $A \subseteq U$. Since A is an IFSPGCS, it follows that $spcl(A) \subseteq U$. Now $B\subseteq spcl(A)$ implies $spcl(B) \subseteq spcl(spcl(A))= spcl(A)$. Thus, $spcl(B) \subseteq U$. This proves that $B \in IFSPGC(X)$.

Theorem 3.28: If A is an IFSOS and an IFSPGCS in (X, τ), then A is an IFSPCS in (X, τ).

Proof: Since A ⊆ A and A is an IFSOS in (X, τ), by hypothesis, spcl(A) ⊆ A. But since A ⊆ spcl(A). Therefore spcl(A)= A. Hence A is an IFSPCS in (X, τ).

Theorem 3.29: An IFS A of an IFTS (X, τ) is an IFSPGCS in (X, τ) if and only if $A\ {}_q{}^c\ F \Rightarrow spcl(A)\ {}_q{}^c\ F$ for every IFSCS F of X.

Proof: Necessity: Let F be an IFSCS in (X, τ) and $A\ {}_q{}^c\ F$. Then by Definition 2.14, $A \subseteq F^c$, where F^c is an IFSOS in (X, τ). Then $spcl(A) \subseteq F^c$, by hypothesis. Hence again by Definition 2.14, $spcl(A)\ {}_q{}^c\ F$.

Sufficiency: Let U be an IFSOS in (X, τ) such that A ⊆ U. Then U^c is an IFSCS in (X, τ) and $A \subseteq (U^c)^c$. By hypothesis, $A\ {}_q{}^c\ U^c \Rightarrow spcl(A)\ {}_q{}^c\ U^c$. Hence by Definition 2.14, $spcl(A) \subseteq (U^c)^c = U$. Therefore spcl(A) ⊆ U. Hence A is an IFSPGCS in (X, τ).

Theorem 3.30: Let A be an IFSPGCS in (X, τ) and c(α,β) be an IFP in X such that $c(\alpha,\beta)\ {}_q\ scl(A)$. Then $scl(c(\alpha,\beta))_q\ A$.

Proof: Let A be an IFSPGCS in (X, τ) and $c(\alpha,\beta)\ {}_q\ scl(A)$. If $scl(c(\alpha,\beta))\ {}_q{}^c\ A$, then by Definition 2.14, $A \subseteq (scl(c(\alpha,\beta)))^c$ where $(scl(c(\alpha,\beta)))^c$ is an IFSOS in (X, τ). Then by hypothesis, $scl(A) \subseteq (scl(c(\alpha,\beta)))^c \subseteq (c(\alpha,\beta))^c$. Therefore by Definition 2.14, $c(\alpha,\beta)_q{}^c\ scl(A)$, which is a contradiction to the hypothesis. Hence $scl(c(\alpha,\beta))\ {}_q\ A$.

Theorem 3.31: For an IFS A in (X, τ), the following conditions are equivalent:
(i) A is an IFOS and an IFSPGCS in (X, τ),
(ii) A is an IFROS in (X, τ).

Proof: (i)⇒(ii) Let A be an IFOS and an IFSPGCS in (X, τ). Since every IFOS in (X, τ) is an IFSOS in (X, τ). Then spcl(A) ⊆ A. This implies that int(cl(int(A))) ⊆ A. Since A is an IFOS, int(A)= A. Therefore int(cl(A)) ⊆ A. Since A is an IFOS, it is an IFPOS. Hence A ⊆ int(cl(A)). Therefore A= int(cl(A)). Hence A is an IFROS in (X, τ).

(ii)⇒(i) Let A be an IFROS in (X, τ). Therefore A= int(cl(A)). Since every IFROS is an IFOS, A is an IFOS and A ⊆ A. This implies that int(cl(A)) ⊆ A. Therefore A is an IFSCS. Hence by Theorem 3.9., A is an IFSPGCS in (X, τ).

Theorem 3.32: Let F ⊆ A ⊆ X where A is an IFSOS and an IFSPGCS in X. Then F is an IFSPGCS in A if and only if F is an IFSPGCS in X.

Proof: Necessity: Let U be an IFSOS in X and F ⊆ A. Also let F be an IFSPGCS in A. Then clearly F ⊆ A ⋂ U and A ⋂ U is an IFSOS in A. Hence the semipre closure of F in A, $spcl_A(F) \subseteq A \cap U$. By Theorem 3.28, A is an IFSPCS. Therefore spcl(A) = A and the semi-pre closure of F in X, $spcl(F) \subseteq spcl(F) \cap spcl(A) = spcl(F) \cap A = spcl_A(F) \subseteq A \cap U \subseteq U$. That is spcl (F) ⊆ U whenever F ⊆ U. Hence F is an IFSPGCS in X.

Sufficiency: Let V be an IFSOS in A such that F ⊆ V. Since A is an IFSOS in X, V is an IFSOS in X. Therefore spcl(F) ⊆ V, since F is an IFSPGCS in X. Thus $spcl_A(F) = spcl(F) \cap A \subseteq V \cap A \subseteq V$. Hence F is an IFSPGCS in A.

Theorem 3.33: For an IFOS A in (X, τ), the following conditions are equivalent:
(i) A is an IFCS in (X, τ),
(ii) A is an IFSPGCS and an IFQ-set in (X, τ).

Proof: (i)⇒(ii) Since A is an IFCS, it is an IFSPGCS in (X, τ). Now int(cl(A))= int(A) = A = cl(A) = cl(int(A)), by hypothesis. Hence A is an IFQ-set in (X, τ).

(ii)⇒(i) Since A is an IFOS and an IFSPGCS in (X, τ), by Theorem 3.31, A is an IROS in (X, τ). Therefore A= int(cl(A)) = cl(int(A)) = cl(A), by hypothesis. Hence A is an IFCS in (X, τ).

Theorem 3.34: Let (X, τ) be an IFTS. Then for every A ∈ IFSPC(X) and for every IFS B in X, int(A) ⊆ B ⊆ A ⇒ B ∈ IFSPGC(X).

Proof: Let A be an IFSPCS in X. Then by Definition 2.9, there exists an IFPCS, say C such that int(C) ⊆ A ⊆ C. By hypothesis, B ⊆ A. Therefore B ⊆ C. Since int(C) ⊆ A, int(C) ⊆ int(A) and int(C) ⊆ B. Thus int(C) ⊆ B ⊆ C and by Definition 2.9, B ∈ IFSPC(X). Hence by Theorem 3.5, B ∈ IFSPGC(X).

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