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ROTATION EFFECT ON STEADY LAMINAR FREE CONVECTIVE FLOW OF VISCOUS FLUID ALONG A MOVING POROUS HOT VERTICAL PLATE

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ABSTRACT

The purpose of the present problem is to study the Rotation effect on steady laminar free convective flow of an electrically conducting viscous fluid through porous medium along a moving porous hot vertical plate in the presence of heat source with mass transfer. The governing equations of motions are solved by a regular perturbation technique. The velocity of fluid, skin friction, temperature and concentration are discussed with the help of tables and graphs. The primary velocity of fluid increases with the increase in G_r (Grashof Number) and K (Porosity parameter), but it decreases with the increase in M (Hartman number) and Ω (Rotation parameter). The secondary velocity of fluid decreases with the increase in G_r, K and Ω, but it increases with the increase in M.

Keywords: *An electrically conducting fluid, MHD Flow, Rotation Heat transfer, Mass transfer, Heat Source, Porous medium.*

INTRODUCTION

The problem of free convection flow of an electrically conducting fluid past a vertical plate under the influence of a magnetic field attracted many scientists, in view of its application in Aerodynamics, Astrophysics, Geophysics and Engineering.

Laminar natural convection and heat transfer in fluids flow with and without heat source in channels with constant wall temperature was discussed by OSTRACH [5]. An analysis of laminar free convective flow and heat transfer on a flat plate parallel to the direction of governing body force was studied by OSTRACH [6]. Combined natural and forced convection laminar flow and heat transfer in fluid, with and without source channels, with linearly varying wall temperature was discussed by OSTRACH [7]. SASTRI [11] dealt heat transfer in the flow over a flat plate with suction and constant heat source. Also SASTRI [12] studied a problem of heat transfer in the presence of temperature dependent heat source in the flow over a flat plate with suction. Forced and natural flows were discussed by SCHLITCHTING [13], ECKERT and DRAKE [3] and BANSAL [1]. Free convection effects on the stokes problem for an infinite vertical plate has been studied by SOUNDALGEKAR [15]. POP and SOUNDALGEKAR [8] investigated free convection flow past an accelerated vertical infinite plate. RAPTIS et al [10] studied effects of free convection currents on the flow of an electrically conducting fluid of an accelerated vertical infinite plate with variable suction. SHARMA [14] investigated free convection effect on the flow past an infinite vertical, porous plate with constant suction and heat flux. KUMAR and VARSHNEY [4] studied steady laminar free convection flow of an electrically conducting fluid along a porous hot vertical plate in the presence of heat source with mass transfer. Recently, VARSHNEY and SINGH [16] discussed effect of porous medium on steady laminar free convective flow of an electrically conducting fluid in the presence of heat source with mass transfer. PRASAD et al [9] have studied steady laminar free convective flow of an electrically conducting fluid along a moving porous hot vertical plate in the presence of heat source through porous medium with mass transfer.

Present study is an extension of the work PRASAD et al [9] with rotation. The aim of present study is to investigate the effect of rotation on the velocity of the fluid.

FORMULATION OF THE PROBLEM

Consider the steady free convective flow with mass transfer of an electrically conducting viscous fluid past an infinite vertical porous plate at $z^* = 0$. Let the fluid and the plate be in a state of rigid rotation with constant angular velocity Ω about z^*-axis, taken normal to the plate. A constant transverse magnetic field B_o is acting parallel to the axis of rotation. Taking the magnetic Reynolds number to be small, the induced magnetic field is neglected in comparison to the applied magnetic field B_o. Since the length of the plate is large, therefore, all the physical variables depend on z^*

only. In the present problem, magnetic field B_o is assumed to be constant throughout the motion. We further assume that the electric field is equal to zero. The governing equations of continuity, momentum, energy and diffusion for a free convective flow of an electrically conducting fluid along a hot, non conducting porous vertical plate in the presence of heat source and rotation are given as :

$$\frac{dV^*}{dz^*} = 0$$

i.e.

$$V^* = -v_o \text{ (constant)} \tag{1}$$

$$w^*\frac{du^*}{dz^*} - 2\Omega^* v^* = \nu\frac{d^2u^*}{dz^{*2}} + g\beta(T^* - T_\infty) + g\beta'(C^* - C_\infty) - \frac{\sigma B_o^2}{\rho}u^* - \frac{\nu}{K^*}u^* \tag{2}$$

$$w^*\frac{dv^*}{dz^*} - 2\Omega^* u^* = \nu\frac{d^2v^*}{dz^{*2}} - \frac{\sigma B_o^2}{\rho}v^* - \frac{\nu}{K^*}v^* \tag{3}$$

$$\frac{dp^*}{dz^*} = 0$$

i.e.

$$p^* = \text{constant} \tag{4}$$

$$v^*\frac{dT^*}{dz^*} = \frac{k}{\rho C_P}\frac{d^2T^*}{dz^{*2}} + \frac{S^*}{\rho C_P}(T^* - T_\infty) \tag{5}$$

$$v^*\frac{dC^*}{dz^*} = D\frac{d^2C^*}{dz^{*2}} \tag{6}$$

where ρ is the density, g is the acceleration due to gravity, β is the coefficient of volume expansion, β' is the coefficient of concentration expansion, ν is the Kinematic viscosity, T_∞ is the temperature of the fluid in the free stream, σ is the electric conductivity, B_o is the magnetic induction, D is the chemical molecular diffusivity, k is the thermal conductivity, S* is the coefficient of heat source, C_∞ is the concentration at infinity, C_p is the specific heat at constant pressure, v_o is the suction velocity, Ω^* is the angular velocity.

The boundary conditions at the wall and in the free stream are:

$$\begin{aligned} &u^* = u_w, \quad v^* = 0, \quad T^* = T_w, \quad C^* = C_w \quad \text{at} \quad z^* = 0 \\ &u^* \to 0, \quad v^* \to 0, \quad T^* \to T_\infty, \quad C^* \to C_\infty \quad \text{at} \quad z^* \to \infty \end{aligned} \tag{7}$$

On introducing the following non dimensional quantities

$$\left.\begin{aligned} &u = \frac{u^*}{v_o}, \qquad v = \frac{v^*}{v_o}, \qquad z = \frac{z^* v_o}{\nu}, \\ &\theta = \frac{T^* - T_\infty}{T_w^* - T_\infty}, \qquad \phi = \frac{C^* - C_\infty}{C_w^* - C_\infty} \end{aligned}\right\}$$

In equations (2), (3), (5) and (6), we get

$$\frac{d^2q}{dz^2} + \frac{dq}{dz} - M_1 q = -G_r\theta - G_m\phi \tag{8}$$

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$$\frac{d^2\theta}{dz^2}+P_r\frac{d\theta}{dz}+S\theta=0 \tag{9}$$

$$\frac{d^2\phi}{dz^2}+S_c\frac{d\phi}{dz}=0 \tag{10}$$

and corresponding boundary conditions are

$$\begin{aligned} &q = Q,\quad \theta = 1,\quad \phi = 1 \qquad \text{at } z = 0 \\ &q \to 0,\ \theta \to 0,\ \phi \to 0 \qquad \text{as } z \to \infty \end{aligned} \tag{11}$$

where

$G_r = \dfrac{\nu g\beta(T_w - T_\infty)}{v_o^3}$ (Grashof number)

$G_m = \dfrac{\nu g\beta'(C_w - C_\infty)}{v_o^3}$ (Modified Grashof number)

$M = \dfrac{\sigma B_0^2\nu}{\rho v_o^2}$ (Hartmann number)

$K = \dfrac{K^* v_o^2}{\nu^2}$ (Porosity parameter)

$S = \dfrac{\nu^2 S^*}{k v_o^2}$ (Heat source parameter)

$Q = \dfrac{u_w}{v_o}$ (Velocity ratio parameter)

$P_r = \dfrac{\rho\nu C_p}{k}$ (Prandtl number)

$S_c = \dfrac{\nu}{D}$ (Schmidt number)

$\Omega = \dfrac{\nu\Omega^*}{v_o^2}$ (Rotation parameter)

$q = u + iv$, $M_1 = M_2 + 2iE$, $M_2 = M + 1/K$

On solving equations (8) to (10) which are ordinary linear differential equation in u, θ, ϕ with boundary conditions (11). We get the value of u, θ, ϕ as

$$q = [(Q + G_r A_3 + G_m A_4)e^{-A_1 y} - G_r A_3 e^{-A_2 y} - G_m A_4 e^{-S_c y}] \tag{12}$$

$$\theta = e^{-A_2 y} \tag{13}$$

$$\phi = e^{-S_c y} \tag{14}$$

where

$$A_1 = \frac{1+\left\{1+4M_1\right\}^{1/2}}{2}$$

$$A_2 = \frac{P_r + \{P_r^2 - 4S\}^{1/2}}{2}$$

$$A_3 = \frac{1}{A_2^2 - A_2 - M_1}$$

$$A_4 = \frac{1}{S_c^2 - S_c - M_1}$$

The primary velocity u (real part of q) and secondary velocity v (imaginary part of q) from equation (12) are given as

$$u = [(Q + G_r K_1 + G_m K_3)\cos(bz/2) + (G_r K_2 + G_m K_4)\sin(bz/2)]e^{-(1+a)z/2} - G_r K_1 e^{-A_2 z} - G_m K_3 e^{-S_c z} \quad (15)$$

$$v = [(G_r K_2 + G_m K_4)\cos(bz/2) - (Q + G_r K_1 + G_m K_3)\sin(bz/2)]e^{-(1+a)z/2} - G_r K_2 e^{-A_2 z} - G_c K_4 e^{-S_c z} \quad (16)$$

where

$$a = \left[\frac{\{(1+4M_2)^2 + 64\Omega^2\}^{1/2} + \left\{1+4M_2\right\}}{2}\right]^{1/2}$$

$$b = \left[\frac{\{(1+4M_2)^2 + 64\Omega^2\}^{1/2} - \left\{1+4M_2\right\}}{2}\right]^{1/2}$$

$$K_1 = \frac{A_2^2 - A_2 - M_2}{(A_2^2 - A_2 - M_2)^2 + 4\Omega^2} \qquad K_2 = \frac{2\Omega}{(A_2^2 - A_2 - M_2)^2 + 4\Omega^2}$$

$$K_3 = \frac{S_c^2 - S_c - M_2}{(S_c^2 - S_c - M_2)^2 + 4\Omega^2} \qquad K_4 = \frac{2\Omega}{(S_c^2 - S_c - M_2)^2 + 4\Omega^2}$$

Skin Friction Coefficient at the plate is given by :

$$\tau_x = \left(\frac{\partial u}{\partial z}\right)_{z=0} = -(Q + G_r K_1 + G_m K_3)\{(1+a)/2\} + (G_r K_2 + G_m K_4)(b/2) + G_r K_1 A_2 + G_m K_3 S_c \quad (17)$$

$$\tau_y = \left(\frac{\partial v}{\partial z}\right)_{z=0} = -(Q + G_r K_1 + G_m K_3)(b/2) - (G_r K_2 + G_m K_4)\{(1+a)/2\} + G_r K_2 A_2 + G_m K_4 S_c \quad (18)$$

RESULTS AND DISCUSSION

The primary and secondary velocity distributions are tabulated in Table -1 & Table-2 and plotted in Fig. -1 & 2 having six graphs at $G_m = 4$, $P_r = 0.71$, $S_c = 0.6$, S = 0.05, Q = 1 for following different value of G_r , M, K and Ω.

	G_r	M	K	Ω
For Graph-1	5	0.2	1	0
For Graph-2	5	0.2	1	0.5
For Graph-3	10	0.2	1	0.5
For Graph-4	5	0.6	1	0.5
For Graph-5	5	0.2	10	0.5
For Graph-6	5	0.2	1	1

From Graphs - 1 to 6 of Fig.-1, it is found that the primary velocity u increases sharply till z = 1.2 (near the wall) after it primary velocity decreases sharply till z = 3.5 then after it primary velocity decreases continuously with the increase in z. On comparing Graphs -2, 3, 4, 5 and 6 with Graph-1 it is observed that primary velocity increases with the increase in G_r and K, but it decreases with the increase in M and Ω.

From Graph-1 of Fig.-2 it is noticed that secondary velocity is zero in the absence of rotation velocity. From Graphs - 2 to 6 of Fig.-2, it is found that the secondary velocity v decreases sharply till z = 1.2 (near the wall) after it secondary velocity increases sharply till z = 3.5 then after it secondary velocity decreases continuously with the increase in z. On comparing Graphs-3, 4, 5 and 6 with Graph-2 it is observed that secondary velocity increases with the increase in M, but it decreases with the increase in G_r, K and Ω.

The temperature and concentration distribution don't change with the change in parameters taken for velocity.

PARTICULAR CASE

When Ω is equal to zero, this problem reduces to the problem of PRASAD et al [9].

CONCLUSION

The primary and secondary velocities decrease with the increase in Ω (Rotation parameter).

Table-1: Value of primary velocity u for Fig-1 at G_m = 4, P_r = 0.71, S_c = 0.6, S = 0.05, Q = 1 and different values of G_r, M, K and Ω.

z	Graph 1	Graph 2	Graph 3	Graph 4	Graph 5	Graph 6
0	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1	2.41804	1.95340	2.93393	1.84811	2.30128	1.31024
2	1.64872	1.22043	1.86887	1.14834	1.36069	0.68251
3	0.95305	0.66998	1.02648	0.63149	0.68083	0.34570
4	0.52623	0.36045	0.55014	0.34062	0.33843	0.18210
5	0.28639	0.19397	0.29460	0.18359	0.17383	0.09785

Table-2: Value of secondary velocity v for Fig-1 at G_m = 4, P_r = 0.71, S_c = 0.6, S = 0.05, Q = 1 and different values of G_r, M, K and Ω.

z	Graph 1	Graph 2	Graph 3	Graph 4	Graph 5	Graph 6
0	0	0.00000	0.00000	0.00000	0.00000	0.00000
1	0	-0.90538	-1.36107	-0.77868	-1.98622	-1.15181
2	0	-0.72543	-1.10478	-0.61281	-1.70328	-0.82765
3	0	-0.44026	-0.67230	-0.36845	-1.06535	-0.46786
4	0	-0.24624	-0.37533	-0.20528	-0.59891	-0.25285
5	0	-0.13429	-0.20390	-0.11182	-0.32479	-0.13616

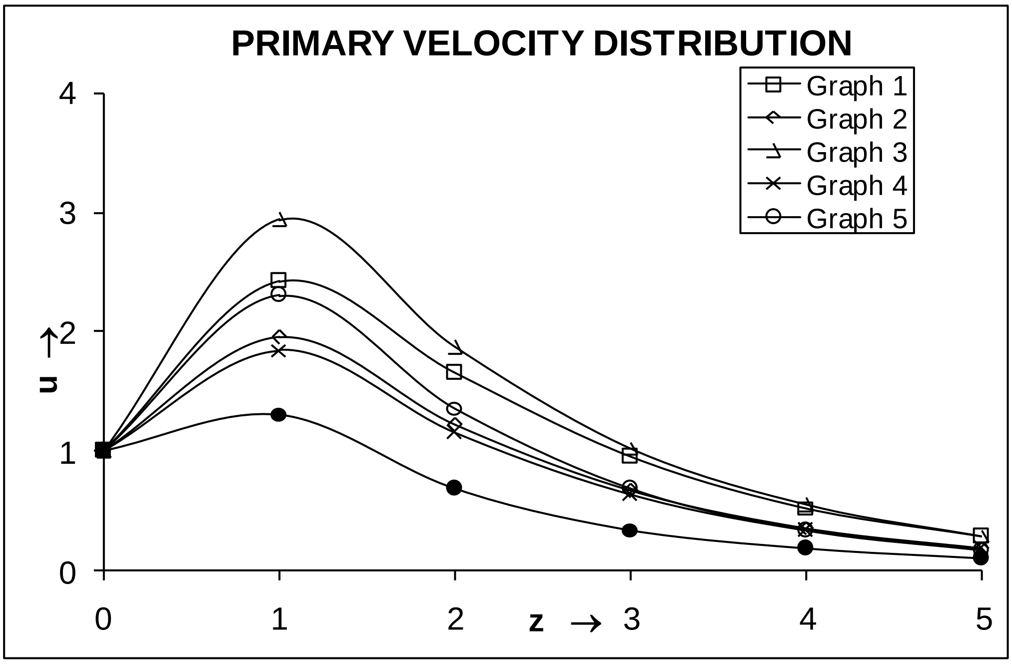

Fig.-1

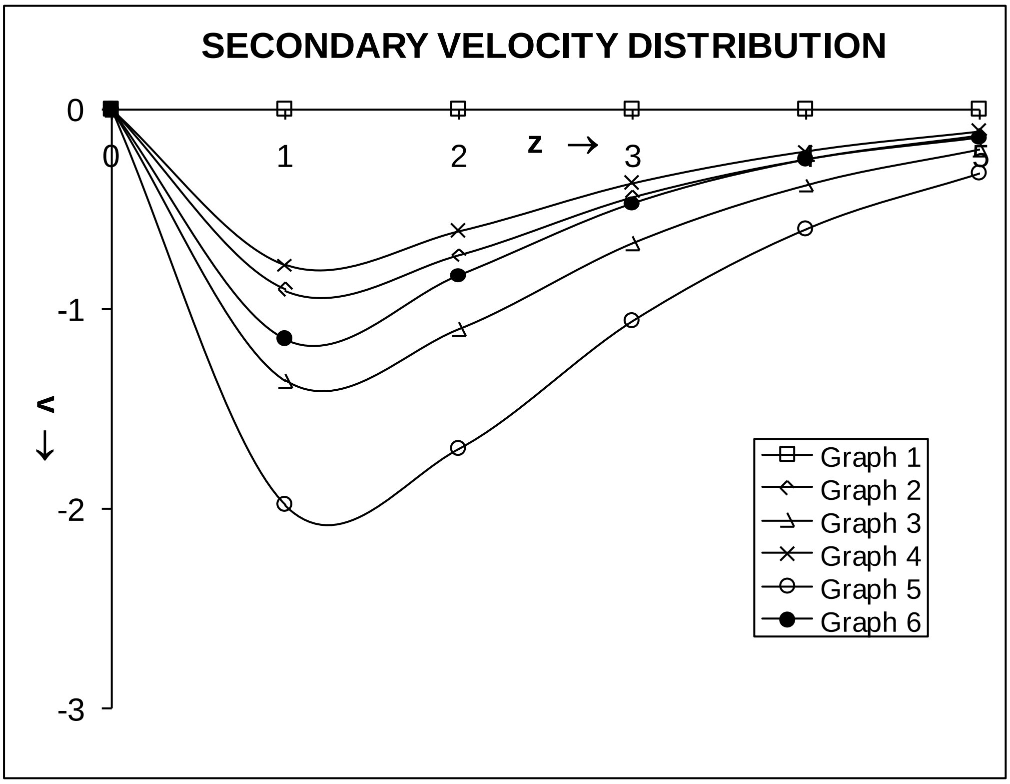

Fig.-2

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